

The exam is worth 100 points. Some of the equations are given below. **WRITE YOUR NAME ON EVERY SHEET.**

1 dimensional kinematic Equations:

$$v = v_0 + at$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x = x_0 + v_0t + \frac{1}{2}at^2$$
$$x = x_0 + \frac{1}{2}(v + v_0)t$$

Newtons Law

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

$$y(x,t) = A \sin(kx - \omega t), k = 2\pi/\lambda, \omega = 2\pi/T,$$
$$\omega = 2\pi f, v = \lambda/T, v = f\lambda$$

$$v = \sqrt{T/\mu},$$

$$P = \left(\frac{1}{2}\right)\mu\omega^2 A^2 v$$

$$I = P/A, \text{ spherical source } I = P/4\pi r^2$$

$$\beta = 10 \log (I/I_0)$$

$$I_0 = 1.00 \times 10^{12} \text{ W/m}^2$$

$$s(x,t) = s_{\max} \cos(kx - \omega t),$$

$$\Delta P = \Delta P_{\max} \sin(kx - \omega t), \Delta P_{\max} = \rho v \omega s_{\max}$$

$$I = (\Delta P_{\max})^2 / 2\rho v$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$f' = f \frac{(v \pm v_o)}{(v \pm v_s)} \text{ choose right sign}$$

Standing waves:

$$y = 2A \sin(kx) \cos(\omega t)$$

$$\text{String: } f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad \text{Open pipe } f_n = \frac{nv}{2L}$$

$$\text{Closed pipe } f_n = \frac{nv}{4L}$$

Interference:

(constructive)

$$|d_2 - d_1| = m\lambda \quad m = 0, 1, 2,$$

(destructive)

$$|d_2 - d_1| = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2,$$

Electrostatics

$$F = (k_e q_1 q_2)/r^2 \quad E = (k_e q)/r^2$$

$$k_e = 8.99 \times 10^9$$

$$k_e = 1/(4\pi\epsilon_0)$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$KE = \frac{1}{2}mv^2$$

The multiple choice questions are 3 points each.

1. A point source broadcasts sound into a uniform medium. If the distance from the source is tripled, the intensity becomes
- a) $1/3$ b) $1/6$ c) $1/9$ d) $1/4$

Answer the following questions based on the waves given below

- A) $y = 2 \sin(3x - 15t + 2)$ B) $y = 6 \sin(3x + 15t - 2)$
 C) $y = 8 \sin(4x + 20t)$ D) $y = 4 \sin(3x - 15t)$
 E) $y = 8 \sin(2x + 15t)$ F) $y = 7 \sin(6x - 24t)$

2. Which of these waves are right going and which are left?

Left: B, C, E

Right: A, D, F

3. Rank the waves according to their velocities from maximum to minimum. (Something like $A > B = C \dots$)

Velocity = ω/k where $y = A \sin(kx - \omega t)$ so velocities are as follows:

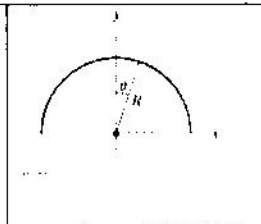
- A) 5 B) -5 C) -5 D) 5 E) -7.5 F) 4

Therefore $A = D > F > B = C > E$

4. A long rope is hung from a ceiling and waves are sent up the rope from its lower end. The speed of the wave
- a. Increases as the wave travels upward due to a decrease in the tension in the string.
 b. Increases as the wave travels upward due to an increase in the tension in the string.
 c. Decreases as the wave travels upward due to a decrease in the tension in the string.
 d. Decreases as the wave travels upward due to an increase in the tension in the string.
- 5.

The non-conducting arc has a uniform charge spread over it. The net electric field at the origin will be

- a) in the x direction due to cancellation of the y components.
 b) in the y direction due to cancellation of the x components.
 c) at 45 degrees in the third quadrant.
 d) at 45 degrees in the fourth quadrant.



6. A free electron and a proton are separated by a distance d and released simultaneously. Which of the following statements is true?
- a) Both particles will experience the same acceleration and reach the midway point at the same time.
 b) The proton will experience a higher acceleration and reach the midway point earlier.
 c) The electron will experience a higher acceleration and reach the midway point earlier.
 d) Both particles feel the same force due to Newton's third law, and therefore reach the midway point at the same time.

Problem # 1 (18 points)

A bat moving at 5.00 m/s is chasing an insect flying with a speed of v_{ins} . The bat emits a 40.0 kHz chirp and receives back an echo at 40.4 kHz. (use speed of sound as 340 m/s)

- a) Find an expression for the frequency the insect hears (call it f_{ins}) in terms of v_{ins} . Explain your steps and indicate what the source is and what the observer is. (6)

Stationary observer, source moving w/ velocity v_A :

$$f' = f \frac{v}{v - v_A} = f \frac{340}{340 - v_{bat}} = \frac{340}{335} f = 1.015f$$

Someone on the ground would hear the bat chirping at $1.015f = 40.60 \text{ kHz}$ as it chases the insect. For the insect,

Stationary source, observer moving w/ velocity v_{ins} :

$$f_{ins} = f' \frac{v - v_{ins}}{v} = f \left(\frac{v}{v - v_{bat}} \right) \left(\frac{v - v_{ins}}{v} \right) = \boxed{f \frac{v - v_{ins}}{v - v_{bat}}}$$

- b) Now considering the insect as the source (it reflects the sound waves it receives from the bat at the same frequency it hears) find an expression for frequency of the echo the bat hears in terms of f_{ins} and v_{ins} . Explain the signs in the formula. (6)

Insect is now the source and bat is the observer.

Source is moving away from the emitted sound:

$$f'' = f_{ins} \frac{v}{v + v_{ins}}$$

Observer moving toward the sound:

$$f_{bat \text{ echo}} = f'' \frac{v + v_{bat}}{v} \Rightarrow \boxed{f_{bat \text{ echo}} = f_{ins} \frac{v + v_{bat}}{v + v_{ins}}}$$

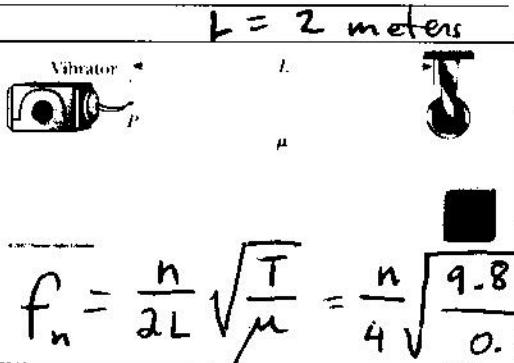
- c) Find the speed of the insect. (6)

$$40.4 \text{ kHz} = 40 \text{ kHz} \left(\frac{340 - v_{ins}}{340 - 5} \right) \left(\frac{340 + 5}{v + v_{ins}} \right)$$

$$\boxed{\begin{aligned} &\text{Solve for } v_{ins} \\ &= 3.3087 \text{ m/s} \end{aligned}}$$

Problem # 2 (32 points)

An object is hung from a string (with linear mass density $\mu = 0.002 \text{ kg/m}$) that passes over a light pulley. The string is connected to a vibrator (of constant frequency f), and the length of the string between point P and the pulley is $L = 2.00 \text{ m}$. When the mass m of the object is either 16.0 kg or 25.0 kg , standing waves are observed. NO standing waves are observed with any mass between these two values however.



- a) What happens to the number of modes if the tension in the string (or the mass attached) increases? Explain using the formula for frequency of an n th harmonic for standing waves in a string. (3 points)

As Tension increases, the number of modes decreases

- b) Based on what you said above, if there are " n " harmonics when the 25 kg mass is hung, how many harmonics would be seen when 16 kg mass is hung ($n+1$) or ($n-1$)? (2 points)

- c) What is the constant natural frequency of the vibrator? (12 points. Hint: A ratio between the expressions for frequency in two cases will simplify calculations.)

$$f_{n_1} = \frac{n_1}{2L} \sqrt{\frac{(9.8)16}{.002}} = 70 n_1 \quad \rightarrow n_1 = n_2 + 1$$

$$f_{n_2} = \frac{n_2}{2L} \sqrt{\frac{(9.8)25}{.002}} = 87.5 n_2$$

$$70 n_1 = 87.5 n_2$$

$$70 (n_2 + 1) = 87.5 n_2 \Rightarrow n_2 = 4, n_1 = 5 \quad \text{so } f = 70 n_1 = \boxed{350}$$

- d) What is the largest object mass for which standing waves should be observed? (3 points)

400 kg

- e) Using the value of n you found for 25 kg mass, find the wavelength λ , velocity, angular frequency ω of the wave. (2+4+2). If the amplitude of the waves is 4 cm, write an expression for the left and right going waves. (4) $n = 4$

$$\lambda = 1 \text{ meter}$$

$$v = 350 \text{ m/s}$$

$$\omega = 700 \pi \text{ rad/s} = 2,200 \text{ rad/s}$$

right-going $y_R(x,t) = (0.02 \text{ m}) \sin[2\pi x - 700\pi t]$

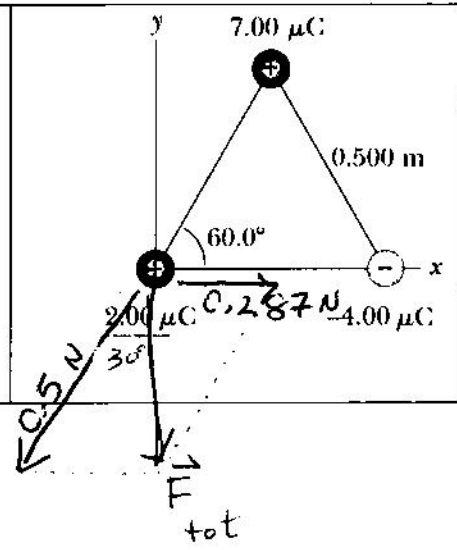
left-going $y_L(x,t) = (0.02 \text{ m}) \sin[2\pi x + 700\pi t]$

sum $y_R + y_L = (0.04 \text{ m}) \sin[2\pi x] \cos[700\pi t]$
 \uparrow the amplitudes sum up to 0.04 m

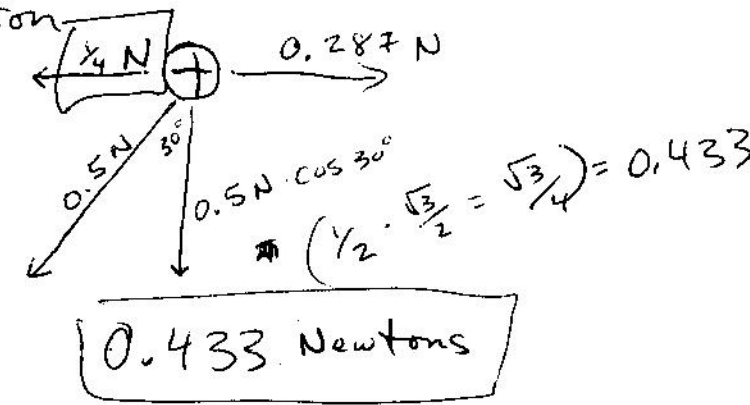
Problem 3 (15 points)

Three charged particles are located at the corners of an equilateral triangle as shown in Figure 23.7.

- Calculate the total (magnitude and direction) electric force on the $2.00 \mu\text{C}$ charge. Show the forces (& label them) and the resultant force in the figure or a separate sketch. (8+2)
- From value of the force calculated above, find the electric field at the $2.00 \mu\text{C}$ charge. How is the direction of electric field related to the direction of the resultant force? Explain. (3+2)



$$F = k \frac{7 \mu\text{C} \times 5 \mu\text{C}}{(0.5 \text{ m})^2} = 0.5 \text{ Newton}$$



Final Force

$$\theta = \text{ArctAN}\left(\frac{-0.433}{0.037}\right) = -85^\circ$$

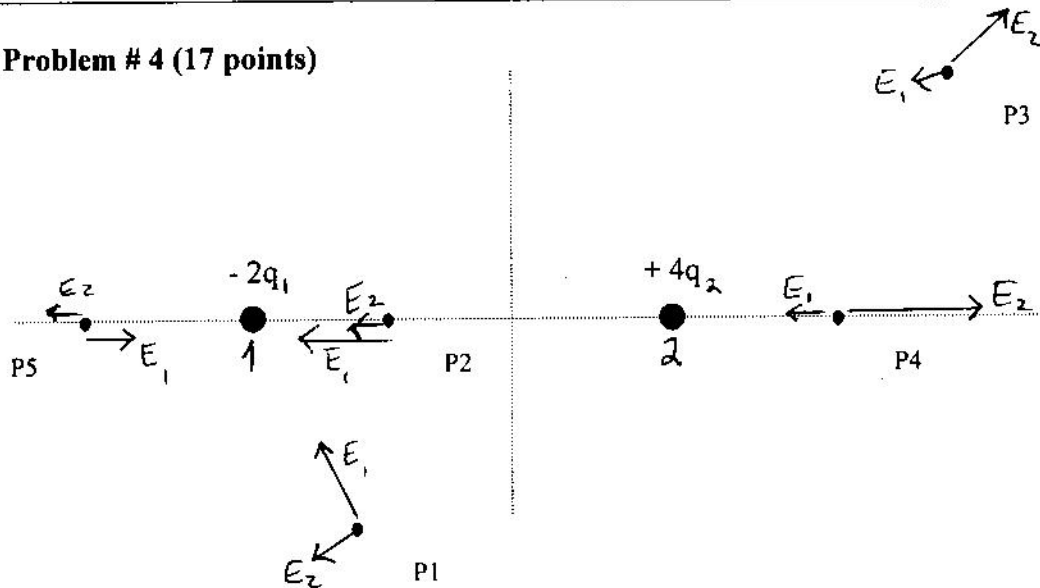
0.435 N = Force (a)

So $\vec{E} = \vec{F} / q$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{0.435 \text{ N}}{2 \times 10^{-6} \text{ C}} = 2.17 \times 10^5 \text{ N/C}$$

(b)

Problem # 4 (17 points)



The figure shows two charges located at the positions shown. Calling the electric field due to the $-2q$ charge as E_1 the $+4q$ charge as E_2 , answer the following questions.

- Draw the electric fields at the points P1, P2, P3, P4 and P5. When you draw the arrows for any point, pay particular attention to which field is bigger. You don't have to ensure consistency in the length of the arrows across two points. (10 points)
- Argue why the resultant field can never be zero anywhere above and below the x axis. (2)
- At which of the points P5, P2, P4 can/cannot the electric field be zero and why? (5)

a) See picture

b) Two vectors can never cancel each other unless they lie on the same parallel line

c) Cannot equal zero at P2 b/c both vectors point in the same direction, cannot cancel each other.
 Cannot equal zero at P4, field E_1 is too weak to cancel E_2