March 4, 2016  Physics 132  Prof. E. F. Redish

- **Theme Music:** Duke Ellington
  *Take the A Train*

- **Cartoon:** Lynn Johnson
  *For Better or for Worse*
Foothold principles: Newton’s Laws

• Newton 0:
  – An object responds only to the forces it feels and only at the instant it feels them.

• Newton 1:
  – An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).

• Newton 2:
  – An object that is acted upon by other objects changes its velocity according to the rule

\[
\vec{a}_A = \frac{\vec{F}_{\text{net}}}{m_A}
\]

• Newton 3:
  – When two objects interact the forces they exert on each other are equal and opposite.

\[
\vec{F}_{A\rightarrow B}^{\text{type}} = -\vec{F}_{B\rightarrow A}^{\text{type}}
\]
Foothold ideas: Kinetic Energy and Work

- Newton’s laws tell us how velocity changes. The Work-Energy theorem tells us how speed (independent of direction) changes.

- Kinetic energy = \( \frac{1}{2}mv^2 \)

- Work done by a force = \( F_x \Delta x \) or \( F_\parallel \Delta r \) (part of force \( \parallel \) to displacement)

- Work-energy theorem:

  \[
  \Delta(\frac{1}{2}mv^2) = F_\parallel^{\text{net}} \Delta r \quad \text{(small step)}
  \]

  \[
  \Delta(\frac{1}{2}mv^2) = \int_i^f F_\parallel^{\text{net}} \, dr \quad \text{(any size step)}
  \]
Foothold ideas:
Potential Energy

• The work done by some forces only depends on the change in position. Then it can be written

\[ \vec{F} \cdot \Delta \vec{r} = -\Delta U \]

\( U \) is called a potential energy.

• For gravity, \( U_{\text{gravity}} = mgh \)

For a spring, \( U_{\text{spring}} = \frac{1}{2} kx^2 \)

For electric force, \( U_{\text{electric}} = k \frac{Q_1 Q_2}{r_{12}} \)

• Potential to force:

\[ \vec{F} = -\frac{\Delta U}{\Delta \vec{r}} = -\left( \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right) = -\nabla U \]

The force associated with a PE at a given place points “downhill” – in the direction where the PE falls the fastest.
Foothold ideas: Energy

• Kinds of energy (macro)
  – Kinetic
  – Potential
  – Thermal
  – Chemical

• Kinds of energy (micro)?

• First law of thermodynamics
  – Conservation of total energy

\[
\Delta U = Q - W \\
\Delta H = \Delta U + p\Delta V
\]
Foothold ideas: Bound states

• When two objects attract, they may form a *bound state* – that is, they may stick together.

• If you have to do positive work to pull them apart in order to get to a separated state with $KE = 0$, then the original state was in a state with negative energy.
Foothold ideas: Inter-atomic interactions

• The interaction between atoms arises from the combination of the electrical forces of its components (electrons and nuclei).
  – It can be quite complex and involve electron sharing and chemical bonds.
  – The complexity arises from the quantum character of electrons.
• Despite this complexity, a simple potential model summarizes many features of a two-atom interaction.
Foothold ideas: 
Inter-atomic potentials

• The interaction between neutral atoms includes an attraction at long-range that arises from the fluctuating charge distribution in each atom; the PE behaves like $1/r^6$.

• When the atoms are pressed close, they repel each other strongly; both because the +nuclei repel and because of the Pauli principle (two electrons cannot be in the same state).

• Two commonly used models are:
  – The Lennard-Jones potential $(A/r^{12}-B/r^6)$
  – The Morse potential (exponentials)
Foothold ideas: Energy conservation with chemical energy

- Consider the reaction of a gas of A and B molecules that react (fully) to C and D molecules $A + B \rightarrow C + D$

  - If the initial and final states both have the two molecules far apart, $U_{AB} \sim U_{CD} \sim 0$.

$$U^i_{\text{thermal}} + U^i_{\text{chemical}} = U^f_{\text{thermal}} + U^f_{\text{chemical}}$$

$$\Delta U_{\text{thermal}} + \Delta U_{\text{chemical}} = 0$$
Foothold ideas:
Thermal Equilibrium & Equipartition

- Degrees of freedom – where energy can reside in a system.
- Thermodynamic equilibrium is dynamic. Changes keep happening, but equal amounts in both directions.
- Equipartition – At equilibrium, the same energy density in all space and in all DoFs.
Foothold ideas: Enthalpy

• When a chemical reaction takes place at a constant $T$ and $p$ (especially in a gas), the gas may have to do work to “make room for itself”. This affects the energy balance between the chemical energy change and the thermal energy change.

• Define enthalpy, $H$

$$\Delta H = \Delta U_{\text{thermal}} + \Delta U_{\text{chemical}} + p\Delta V$$
Foothold ideas:
Thermal Equilibrium & Equipartition

• *Degrees of freedom* – places energy can reside in a system.

• *Thermodynamic equilibrium is dynamic* – Changes keep happening, but equal amounts in both directions.

• *Equipartition* – At equilibrium, there is the same energy density in all space and in all DoFs – on the average.
Foothold ideas:
Connecting micro and macro

• **Microstate** – A specific arrangement of energy among all the degrees of freedom of the system

• **Different microstates may not be distinguishable when you are looking at many molecules** – At the macro level (even as small as nm$^3$) some microstates look the same.

• **Macrostate** – A specification of things we care about at the macro level: pressure, temperature, concentration.
Foothold ideas: Entropy

- **Entropy** – an extensive* measure of how well energy is spread in an object.

- Entropy measures
  - The number of microstates in a given macrostate
  - The amount that the energy of a system is spread among the various degrees of freedom

- Change in entropy upon heat flow

$$ S = k_B \ln(W) $$

$$ \Delta S = \frac{Q}{T} $$
Foothold ideas:
The Second Law of Thermodynamics

• Systems composed of a large number of particles spontaneously move toward the thermodynamic (macro)state that correspond to the largest possible number of particle arrangements (microstates).
  – The 2\textsuperscript{nd} law is probabilistic. Systems show fluctuations – violations that get proportionately smaller as N gets large.

• Systems that are not in thermodynamic equilibrium will spontaneously transform so as to increase the entropy.
  – The entropy of any particular system can decrease as long as the entropy of the rest of the universe increases more.

• The universe tends towards states of increasing chaos and uniformity. (Is this contradictory?)
Foothold ideas: Transforming energy

• Internal energy: \( \Delta U \) thermal plus chemical

• Enthalpy: \( \Delta H = \Delta U + p\Delta V \) internal plus amount needed to make space at constant \( p \)

• Gibbs free energy: \( \Delta G = \Delta H - T\Delta S \) enthalpy minus amount associated with raising entropy of the rest of the universe due to energy dumped

• A process will go spontaneously if \( \Delta G < 0 \).
The sign of the Gibbs Free Energy change indicates spontaneity!

\[ \Delta G = \Delta H - T \Delta S \]

- \( -T \Delta S_{\text{total}} \)
- \( -T \Delta S_{\text{surroundings}} \)
- \( T \Delta S_{\text{system}} \)

\[ \Delta G < 0 \rightarrow \Delta S_{\text{total}} > 0 \rightarrow \text{spontaneous} \]

\[ \Delta G > 0 \rightarrow \Delta S_{\text{total}} < 0 \rightarrow \text{not spontaneous} \]
Foothold ideas: Energy distribution

• Due to the randomness of thermal collisions, even in (local) thermal equilibrium a range of energy is found in each degree of freedom.

• The probability of finding an energy $E$ is proportional to the Boltzmann factor

$$P(E) \propto e^{-E/k_B T} \text{ (for one DoF)}$$

$$P(E) \propto e^{-E/RT} \text{ (for one mole)}$$

• At 300 K, $k_B T \sim 1/40$ eV

$$N_A k_B T = RT \sim 2.4 \text{ kJ/mol}$$
The Boltzmann probability

• The probability of finding an additional energy $\Delta E$ in a DoF is proportional to the number of ways that that energy can be distributed, $W$.
• The overall probability has to be normalized so that the sum (integral) over all energies is 1.

$$P(\Delta E, T) = P_0 W(\Delta E, T) e^{-\frac{\Delta E}{k_B T}}$$
Foothold ideas: Exponents and logarithms

- Power law: \( f(x) = x^2 \quad g(x) = Ax^7 \)
a variable raised to a fixed power.

- Exponential: \( f(x) = e^x \quad g(N) = 2^N \quad h(z) = 10^z \)
a fixed constant raised to a variable power.

- Logarithm: the inverse of the exponential.

\[
x = e^{\ln(x)} \quad x = \ln(e^x)
\]

\[
y = 10^{\log(y)} \quad y = \log(10^y)
\]

\[
\log(2) = 0.3010 \quad \log(e) = 0.4343
\]

\[
2^N = \left(10^{0.3010}\right)^N \approx 10^{0.3N}
\]

\[
e^x = \left(10^{0.4343}\right)^x \approx 10^{0.4x}
\]

\[
2^N = B
\]

\[
N \log 2 = \log B \Rightarrow N = \frac{\log B}{\log 2}
\]
Foothold ideas:
Charge – A hidden property of matter

• Matter is made up of two kinds of electrical matter (positive and negative) that usually cancel very precisely.
• Like charges repel, unlike charges attract.
• Bringing an unbalanced charge up to neutral matter polarizes it, so both kinds of charge attract neutral matter
• The total amount of charge (pos – neg) is constant.
Foothold ideas: Conductors and Insulators

• Insulators
  – In some matter, the charges they contain are bound and cannot move around freely.
  – Excess charge put onto this kind of matter tends to just sit there (like spreading peanut butter).

• Conductors
  – In some matter, charges in it can move around throughout the object.
  – Excess charge put onto this kind of matter redistributes itself or flows off (if there is a conducting path to ground).
Foothold idea: Coulomb’s Law

• All objects attract each other with a force whose magnitude is given by

\[
\vec{F}_{q\rightarrow Q} = -\vec{F}_{Q\rightarrow q} = \frac{k_C qQ}{r_{qQ}^2} \hat{r}_{q\rightarrow Q}
\]

• \(k_C\) is put in to make the units come out right.

\[
k_C = 9 \times 10^9 \text{ N-m}^2 / \text{C}^2
\]
Foothold ideas: Energies between charge clusters

• Atoms and molecules are made up of charges.
• The potential energy between two charges is

\[
U_{12}^{\text{elec}} = \frac{k_c Q_1 Q_2}{r_{12}}
\]

• The potential energy between many charges is

\[
U_{12...N}^{\text{elec}} = \sum_{i<j=1}^{N} \frac{k_c Q_i Q_j}{r_{ij}}
\]
Foothold idea: 
Fields

• Test particle
  – We pay attention to what force it feels. We assume it does not have any affect on the source particles.

• Source particles
  – We pay attention to the forces they exert and assume they do not move.

• Physical field
  – We consider what force a test particle would feel if it were at a particular point in space and divide by its coupling strength to the force. This gives a vector at each point in space.

\[
\vec{g} = \frac{1}{m} \vec{W}_{E \rightarrow m} \quad \vec{E} = \frac{1}{q} \vec{F}_{\text{all charges} \rightarrow q} \quad V = \frac{1}{q} U_{\text{elec} \text{ all charges} \rightarrow q}
\]
Foothold ideas: Electric potential energy and potential

- The potential energy between two charges is
  \[ U_{12}^{elec} = \frac{k_c Q_1 Q_2}{r_{12}} \]

- The potential energy of many charges is
  \[ U_{12...N}^{elec} = \sum_{i<j=1}^{N} \frac{k_c Q_i Q_j}{r_{ij}} \]

- The potential energy added by adding a test charge \( q \) is
  \[ \Delta U_{q}^{elec} = \sum_{i=1}^{N} \frac{k_c q Q_i}{r_{iq}} = qV \]

= the voltage at the position of the test charge
Units

• Gravitational field
  units of $g = \text{Newtons/kg}$
• Electric field
  units of $E = \text{Newtons/C}$
• Electric potential
  units of $V = \text{Joules/C} = \text{Volts}$
• Energy = $qV$ so $e\Delta V =$ the energy gained
  by an electron (charge $e = 1.6 \times 10^{-19} \text{C}$)
  in moving through a change of $\Delta V$ volts.
  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
Representations

• Representing $E$
  – Arrows (length shows $|E|$)
  – Arrows (fixed length, color or width shows $|E|$)
  – Field lines (show direction only)
  – Field lines (color shows $|E|$)

• Representing $V$
  – 1D: Graph
  – 2D: Isoclines (lines of equal value)
  – 3D: Equipotential surfaces (surfaces of $= \text{value}$)
Foothold ideas: Electric charges in materials

• The electric field inside the body of a static conductor (no moving charges) is zero.
• The entire body of a static conductor (no charges moving through it) is at the same potential.
• The average electric field in an insulator is reduced (due to the polarization of the material by the field) by a factor that is a property of the material: the dielectric constant, $\kappa$.
  (Sometimes written in biology as $\varepsilon$) Since $\kappa$ is the ratio of two fields, it is dimensionless.