Exam 2

Average: 64.34
Standard Deviation: 13.76
1. (15 points) In the circuit shown at the right, $R_A$ is identical to $R_B$, and their resistance is half of $R_C$:

$$R_A = R_B = \frac{1}{2} R_C.$$ The current through resistor A is $i_A$, $i_B$ is the current through resistor B, and $i_C$ through resistor C. The potential drop across resistor A is $\Delta V_A$ and so on. The battery provides an EMF = $\Delta V_0$

1.1) Which of the following must be true of the currents in the circuit? (5 pts)

a) $i_A = i_B$, only
b) $i_A = i_C$, only
c) $i_A = i_B = i_C$

$$\text{d) } i_A = i_B + i_C$$

$$\text{e) } i_A = i_B - i_C$$

f) None of these.
1. (15 points) In the circuit shown at the right, $R_A$ is identical to $R_B$, and their resistance is half of $R_C$:

$$R_A = R_B = \frac{1}{2} R_C.$$ The current through resistor $A$ is $i_A$, $i_B$ is the current through resistor $B$, and $i_C$ through resistor $C$. The potential drop across resistor $A$ is $\Delta V_A$ and so on. The battery provides an EMF $= \Delta V_0$

1.2) What is the relationship between $i_B$ and $i_C$? (5 pts)

- a) $i_B = \frac{1}{3} i_C$
- b) $i_B = \frac{1}{2} i_C$
- c) $i_B = i_C$
- d) $i_B = 2i_C$
- e) $i_B = 3i_C$
- f) None of these
1. (15 points) In the circuit shown at the right, \( R_A \) is identical to \( R_B \), and their resistance is half of \( R_C \):
\( R_A = R_B = \frac{1}{2} R_C \). The current through resistor A is \( i_A \), \( i_B \) is the current through resistor B, and \( i_C \) through resistor C. The potential drop across resistor A is \( \Delta V_A \) and so on. The battery provides an EMF = \( \Delta V_0 \)

1.2) Which of the following must be true about the potential drops in the circuit? (5 pts)

\[ \alpha) \Delta V_A = \Delta V_B \]
\[ \beta) \Delta V_B = \Delta V_C \]
\[ \chi) \Delta V_A = \Delta V_B \]
\[ \delta) \Delta V_0 = \Delta V_B + \Delta V_C \]
\[ \varepsilon) \Delta V_0 = \Delta V_A + \Delta V_B \]
\[ \phi) \Delta V_0 = \Delta V_A + \Delta V_C \]
1.4a (5 pts). A 50 gram cart is connected to a spring whose unstretched length is 10 cm and whose spring constant is 2.5 N/m. In the list below are described three situations. The motion of the mass is initiated by a person pulling the mass to the right 5 cm from its equilibrium position and releasing it. Ignore both air resistance and internal friction in the spring.

At the time the situation occurs, indicate whether the force vector requested points left (L), right (R), or has magnitude zero (0). (9 pts)

1. The force on the mass exerted by the spring when the mass is at its equilibrium position and is moving to the left.
2. The net force on the mass when the mass is at its equilibrium position and is moving to the right.
3. The force on the mass exerted by the spring when it is at the maximum of its oscillation on the right.
1.4b (5 pts). An oscillating platform is connected to 3 masses: \( m_1 = 4.0 \text{ kg} \), \( m_2 = 1 \text{ kg} \), and \( m_3 = 0.25 \text{ kg} \). They are all connected to the platform with identical springs that have a spring constant, \( k = 100 \text{ N/m} \). There is a small damping with damping constant for each one, \( b = 1.0 \text{ N/(m/s)} \). If the platform is oscillated at a frequency of 3.2 Hz (= 3.2 cycles/s) we find one of the masses oscillates much more than the others. Which one?

1. Mass 1
2. Mass 2
3. Mass 3
4. None will be much more than the others.

\[
\omega_1 = \sqrt{\frac{100 \text{N/m}}{4.0 \text{ kg}}} = \frac{5 \text{ rad}}{\text{s}} \rightarrow f_1 = \frac{5 \text{s}^{-1}}{2\pi} = 0.80 \text{Hz}
\]

\[
\omega_2 = \sqrt{\frac{100 \text{N/m}}{1.0 \text{ kg}}} = \frac{10 \text{ rad}}{\text{s}} \rightarrow f_2 = \frac{10 \text{s}^{-1}}{2\pi} = 1.6 \text{Hz}
\]

\[
\omega_3 = \sqrt{\frac{100 \text{N/m}}{0.25 \text{ kg}}} = \frac{20 \text{ rad}}{\text{s}} \rightarrow f_3 = \frac{20 \text{s}^{-1}}{2\pi} = 3.2 \text{Hz}
\]
2. (25 points)
A. As part of a lab, a student hooked up two 1.5 Volt batteries and three 0.5 Ohm bulbs as shown. For each question, explain how you know the answer, and if you use one or more of the Kirchhoff principles, state them and tell how you used them.

A.1 If the current in bulb A is equal to \( I \), what is the current in bulb B? (Express your answer in terms of \( I \); don’t find a numerical value.) (5 pts)

\[ I \]  Objects in parallel have the same potential difference. Since the resistances are the same as well, the bulbs have the same current

\[ 2I \]  Using the junction rule \( I_A + I_B = I_C \).

A.2 If the current in bulb A is equal to \( I \), what is the current in bulb C? (Express your answer in terms of \( I \); don’t find a numerical value.) (5 pts)

A.3 Can you find the numerical value of \( I \)? If you can, find it. If you can’t explain why not. (5 pts)

\[ 2A \]  Using the loop rule \( 3V = V_C + V_B \)

\[ 3V = I_C R_C + I_B R_B = (2I)(0.5\Omega) + (1I)(0.5\Omega) = 1.5I \]
B. A second student put the system together as shown in the figure at the right.

B.1 Would the results be any different from those in part A? If so, how? (5 pts)

Yes, there is no current through the circuit.

B.2 Explain how you decided on your answer to B.1. (5 pts)

Using the loop rule. The batteries are in opposition, so the potential difference is zero. That means no current
3. (15 points) In recitation you learned that some of the excitations of a diatomic molecule, like HCl, can be modeled as two masses connected by a spring. Given that the time to go from its equilibrium position to its maximum stretch takes about $3 \times 10^{-15}$ s, estimate the effective spring constant, $k$, that the model will need to use to connect these two masses to get the right oscillation time. Give your result in N/m. Be sure to clearly state your assumptions and how you came to the numbers you estimated, since grading on this problem will be mostly based on your reasoning, not on your answer.

$m_{Cl} = 35m_H$ so let’s treat the chlorine as stationary and hydrogen as a movable mass attached to a spring.

$$t = 3 \times 10^{-15} \text{ s} = \frac{1}{4} T$$

$$T = 1.2 \times 10^{-14} \text{ s} = 2\pi \sqrt{\frac{m}{k}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4(\pi^2)(1.6 \times 10^{-27} \text{ kg})}{(1.2 \times 10^{-14} \text{ s})^2} = 440 \text{ N/m}$$
5. (25 points) In this problem we will consider whether a model of a smooth sheet of charge or of isolated point charges might better represent the charges that lead to the potential across the membrane of an animal cell. A real membrane is quite complex as shown in the figure at the right. There is an excess of positive charge on the top of the membrane and an excess of negative charge on the bottom resulting in an average potential difference between the top and the bottom of the membrane of 70 mV (milliVolts). The thickness of the membrane is about 7 nm (nanometers).

A. For our first model, we will treat the membrane as if it has a network of equally spaced charges separated by a distance $L$. This might look (in a slice through the membrane) something like the figure at the right.

A.1 In actuality, we should sum the effects of all the charges to get the potentials, but to get an estimate, let’s assume as a first approximation that only the neighboring charges on the same level matter since $L$ is much smaller than $d$. This means, for example, that to get the potential at the top “x” you would only add the contribution of the two charges next to it on the same level:

Assuming that each of the positive and negative charges have magnitude $e$, write an equation for the potential difference, $\Delta V$, between the top $x$ and the bottom $x$ in terms of the symbols $e$, $L$, and whatever constants you need. Put your result in the box, showing what you used to get it in the space below. (8 pts)

$$V_{x_{\text{top}}} = \frac{2 \left( kCe \right)}{L/2} = \frac{4kCe}{L} = -V_{x_{\text{bottom}}}$$

$$\Delta V = V_{x_{\text{top}}} - V_{x_{\text{bottom}}} = \frac{8kCe}{L}$$
A.2 Find what has to be the spacing between the charges, $L$, in order to get the right potential difference between the top and bottom of the membrane in this model. Is this consistent with our assumption about the size of $L$ compared to $d$? (7 pts)

$$\Delta V = 70 \text{mV} = \frac{8 k_C e}{L}$$

$$L = \frac{8 \left( 9 \times 10^9 \, \text{Nm}^2/\text{C}^2 \right) \left( 1.6 \times 10^{-19} \, \text{C} \right)}{7 \times 10^2 \, \text{Nm}/\text{C}} = 1.6 \times 10^{-7} \, \text{m}$$

$L$ is too large compared to $d$, so this is not consistent

B. For our second model, treat the membrane as if it is two sheets of smooth charges of a density $+\sigma$ and $-\sigma$ (Coulombs/m$^2$) as shown in the figure at the right. The E field between two such plates is uniform and given by $E = 4\pi k_c \sigma$

If we are to describe the membrane by this model we need to choose $\sigma$ to give the correct voltage difference across the membrane. What value should you choose for $\sigma$? Show your work and explain (briefly) your reasoning. (10 pts)

$$\Delta V = Ed$$

$$E = \frac{70 \text{mV}}{7 \, \text{nm}} = 10^7 \, \text{N}/\text{C} = 4\pi k_c \sigma$$

$$\sigma = 8.8 \times 10^{-5} \, \text{C}/\text{m}^2$$
Quiz 7

Average: 7.88
Standard Deviation: 2.01
1. (4 pts)

A pulse traveling along a long spring is shown in the figure at the right. (The pulse is sharpened up and flattened for simplicity of analysis. Ignore friction and damping.) A spot of paint marks a bit of the spring as indicated by the down arrow. For the items below, put all the correct answers in the box at the right. (Note: you will lose credit for including wrong answers.) If none are correct, put N.

1.1 Which of the following graphs would correctly represent the y-displacement of the spot of paint as a function of time? (2 pts)

- [ ] A
- [x] B
- [ ] C
- [ ] D
A pulse traveling along a long spring is shown in the figure at the right. (The pulse is sharpened up and flattened for simplicity of analysis. Ignore friction and damping.) A spot of paint marks a bit of the spring as indicated by the down arrow. For the items below, put all the correct answers in the box at the right. (Note: you will lose credit for including wrong answers.) If none are correct, put N.

1.2. Which of the following graphs would correctly represent the velocity of the bits of the string as a function of position at the time shown in the figure at the top? (2 pts)
2. (6 pts) In the figure below is shown a pulse sent down a long taut spring by first dipping your hand and then raising it quickly, and finally bringing it back to your starting point. The pulse moves to the left. Four points on the spring (imaginary “beads”) are identified by black dots and by letters underneath them.

Rank the speeds of the four beads at the instant shown. Use the coordinate system shown and remember that all positive numbers are greater than all negative numbers. Use only “>” (greater than) signs and “=” signs. (Do NOT use “<” or you will lose credit.) Put your answer in the box below. It should be a string of letters that looks something like H > E = F > G meaning H is the biggest, E and F are equal, and G is the smallest. (You, of course, should use A, B, C, and D instead of E, F, G, and H.)

\[ B > A = D > C \]
Sinusoidal waves

□ Suppose we make a continuous wiggle. When we start our clock (t = 0) we might have created shape something like

\[ y(x, 0) = A \sin kx \]

□ If this moves in the +x direction, at later times how will the function look?

Why do we need a “k”?
If this moves in the +x direction, at later times it will look like

\[ y(0, t) = A \sin kx \]

A. \[ y(x, t) = A \sin(kx - v_0 t) \]

B. \[ y(x, t) = A \sin(kx + v_0 t) \]

C. \[ y(x, t) = A \sin[k(x - v_0 t)] \]

D. \[ y(x, t) = A \sin[k(x + v_0 t)] \]
Sinusoidal waves

☐ Suppose we make a continuous wiggle. When we start our clock \((t = 0)\) we might have created shape something like

\[
y(x, 0) = A \sin kx
\]

☐ If this moves in the +x direction, at later times it would look like

\[
y(x, t) = A \sin k(x - v_0 t)
\]
Interpretation – Wavelength and Period

\[ y = A \sin(kx - \omega t) \quad \omega \equiv kv_0 \]

Fixed time: Wave goes a full cycle when

\[ kx : 0 \rightarrow 2\pi \]

\[ x : 0 \rightarrow \frac{2\pi}{k} \equiv \lambda \quad \text{(wavelength)} \]

Fixed position: Wave goes a full cycle when

\[ \omega t : 0 \rightarrow 2\pi \]

\[ t : 0 \rightarrow \frac{2\pi}{\omega} \equiv T \quad \text{(period)} \]
How does $T$, $f$ and $\omega$ connect to $v_0$?

$\omega = kv_0$?

Interpret

$$\omega = 2\pi f = \frac{2\pi}{T} \quad k = \frac{2\pi}{\lambda}$$

$\omega = kv_0 \quad \Rightarrow \quad 2\pi f = \frac{2\pi}{\lambda} v_0$ \quad or

$f\lambda = v_0$ \quad (famous wave formula)

Interpret

$$\frac{1}{T} \lambda = v_0 \quad \Rightarrow \quad \lambda = v_0 T$$
Sinusoidal Waves: \( y(x, t) = A \sin k(x - v_0 t) \)

\[ \text{http://phet.colorado.edu/simulations/sims.php?sim=Wave_on_a_String} \]
An elastic string (modeled as a series of beads) driven by a wheel driving one of the beads up and down sinusoidally. The driving wheel has generated a traveling wave of amplitude 10 cm moving to the right. (The string continues on for a long way to the right as indicated by its going “out the window.”) The figure shows \( t = 0 \), when the green bead marked “II” is passing through its equilibrium point.

Which of the graphs could serve as the graph of **the vertical displacement of bead II** as a function of **time**?
An elastic string (modeled as a series of beads) driven by a wheel driving one of the beads up and down sinusoidally. The driving wheel has generated a traveling wave of amplitude 10 cm moving to the right. (The string continues on for a long way to the right as indicated by its going “out the window.”) The figure shows $t = 0$, when the green bead marked “II” is passing through its equilibrium point.

Which of the graphs could serve as the graph of the **vertical displacement of bead III** as a function of time?
This is the state of the PhET wave-on-a-string simulation when the string is very long so reflection can be ignored. What is the speed of the wave (assuming that the frequency is given in cycles/min?)
What happens when the pulse reaches a fixed end?

A. Pass through like before
B. Stop and die
C. Bounce back right-side-up
D. Bounce back upside-down
E. C but delayed
F. D but delayed
What happens when the pulse reaches a loose end?

A. Pass through like before
B. Stop and die
C. Bounce back right-side-up
D. Bounce back upside-down
E. C but delayed
F. D but delayed
Foothold principles:
Superposition of Mechanical waves

- **Superposition**: when two or more waves (or pulses) overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)

- **Beats**: When sinusoidal waves of different frequencies travel in the same direction, you get variations in amplitude (when you fix either space or time) that happen at a rate that depends on the difference of the frequencies.

- **Standing waves**: When sinusoidal waves of the same frequency travel in opposite directions, you get a stationary oscillating pattern with fixed nodes.
Beats

Describing the motion of the beads

• Sketch the velocity of each bead in the top figure at the time shown.

Pulse moving to the right
Standing waves: Sinusoidal Waves, same frequency, going in opposite directions

\[ y(x, t) = A \sin(kx - \omega t) + A \sin(kx + \omega t) \]

Using trig identities (sc+cs...) we can show

\[ y(x, t) = 2A \sin(kx) \cos(\omega t) \]

For each point on the string labeled “x” it oscillates with an amplitude that depends on where it is — but all parts of the string go up and down together.
Adding Sinusoidal Waves – an example

\[ y = A \sin(kx - \omega t) + A \sin(kx + \omega t) \]

\[ y = 2A \sin(kx) \cos(\omega t) \]

Is there a position for which this function is zero at all times?

\[ y(x = 0, t) = A \sin(-\omega t) + A \sin(\omega t) \]

\[ y(kx = \pi, t) = A \sin(\pi - \omega t) + A \sin(\pi + \omega t) \]

The function is also zero wherever \(kx\) is a multiple of \(\pi\)
Standing Waves

• Some points in this pattern (values of \( x \) for which \( kx = n\pi \)) are always 0. (NODES)

• To wiggle like this (all parts oscillating together in a “standing wave”) we need to have the end fixed

\[
L = n \frac{\lambda}{2}
\]

• We still have \( v_0 = \frac{\omega}{k} \) that is \( v_0 = \lambda f \)
For what frequencies will I generate a large (resonant) standing wave if I drive it with a small amplitude?
- The figure shows a string of length $L$ tied at $x = 0$ and $x = L$.
- Reflections at the ends of the string cause waves of equal amplitude and wavelength to travel in opposite directions along the string.
- These are the conditions that cause a standing wave!
For a string of fixed length $L$, the boundary conditions can be satisfied only if the wavelength has one of the values:

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \ldots$$

Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength $\lambda_m$ is:

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m \frac{v}{2L} \quad m = 1, 2, 3, 4, \ldots$$

The lowest allowed frequency is called the fundamental frequency: $f_1 = v/2L$. 
Shown are the first four possible standing waves on a string of fixed length $L$.

These possible standing waves are called the modes of the string, or sometimes the normal modes.

Each mode, numbered by the integer $m$, has a unique wavelength and frequency.
- $m$ is the number of antinodes on the standing wave.
- The fundamental mode, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency $f_1$ can be found as the difference between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.
What is the mode number for this standing wave?

A. 4

B. 5

C. 6

D. Can’t say without knowing what kind of wave it is.
A standing wave on a string vibrates as shown. Suppose the string tension is reduced to 1/4 its original value while the frequency and length are kept unchanged. Which standing wave pattern is produced?

A. 
B. 
C. 
D.

The frequency is $f_m = m \frac{v}{2L}$.

Quartering the tension reduces $v$ by one half. Thus $m$ must double to keep the frequency constant.
Explore with a simulation

http://phet.colorado.edu/en/simulation/normal-modes
If we start our beaded string off in a sinusoidal shape \( y(x) = A \sin(\pi x/L) \) it will oscillate with a frequency \( f_0 \). If we start it out with a shape \( y(x) = A \sin(2\pi x/L) \) with what frequency will it oscillate?

A. \( f_0 \)

B. \( 2f_0 \)

C. \( f_0/2 \)

D. Something else
In the figure below is shown a picture of a string at a time $t_1$. The pieces of the string are each moving with velocities indicated by arrows. (Vertical displacements are small and don't show up in the picture.) If the shape of the string at time $t_1$ is that shown below (displacement magnified by X100) then the motion of the string is

A. Left travelling wave
B. Right travelling wave
C. Standing wave increasing in amplitude
D. Standing wave decreasing in amplitude
E. None of these
In the figure below is shown a picture of a string at a time $t_1$. The pieces of the string are each moving with velocities indicated by arrows. (Vertical displacements are small and don't show up in the picture.) If the shape of the string at time $t_1$ is that shown below (displacement magnified by X100) then the motion of the string is

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A. Left travelling wave
B. Right travelling wave
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D. Standing wave decreasing in amplitude
E. None of these
Two overlapping water waves create an interference pattern.
A circular or spherical wave can be written:

\[ D(r; t) = a \sin(kr - \omega t + \phi_0) \]

where \( r \) is the distance measured outward from the source.

The amplitude \( a \) of a circular or spherical wave diminishes as \( r \) increases.
- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.

- Points of destructive interference. A crest is aligned with a trough of another wave.
The mathematical description of interference in two or three dimensions is very similar to that of one-dimensional interference.

The conditions for constructive and destructive interference are:

\[
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = m \cdot 2\pi
\]

Maximum constructive interference:

\[
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = m \cdot 2\pi \\
\text{Perfect destructive interference:} \\
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = m + \frac{1}{2} \cdot 2\pi
\]

where \( \Delta r \) is the \textit{path-length difference}. 

\[ m = 0, 1, 2, \ldots \]
- The figure shows two identical sources that are in phase.
- The path-length difference $\Delta r$ determines whether the interference at a particular point is constructive or destructive.

\[\text{At A, } \Delta r_A = \lambda, \text{ so this is a point of constructive interference.}\]

\[\text{At B, } \Delta r_B = \frac{1}{2}\lambda, \text{ so this is a point of destructive interference.}\]
Interference in Two and Three Dimensions

\[ \Delta r = 2\lambda \]
\[ \Delta r = \frac{3}{2}\lambda \]
\[ \Delta r = \frac{1}{2}\lambda \]
\[ \Delta r = 0 \]

- Antinodal lines, constructive interference, oscillation with maximum amplitude. Intensity is at its maximum value.

- Nodal lines, destructive interference, no oscillation. Intensity is zero.
Two in-phase sources emit sound waves of equal wavelength and intensity. At the position of the dot,

A. the interference is constructive.  

B. the interference is destructive.  

C. the interference is somewhere between constructive and destructive.  

D. There’s not enough information to tell about the interference.
Two in-phase sources emit sound waves of equal wavelength and intensity. How many antinodal lines (lines of constructive interference) are in the interference pattern?

A. 1  
B. 2  
C. 3  
D. 4  
E. 5

Sources are 1.5\(\lambda\) apart, so no point can have \(\Delta r\) more than 1.5\(\lambda\).
Two-dimensional interference between two loudspeakers

Two loudspeakers in a plane are 2.0 m apart and in phase with each other. Both emit 700 Hz sound waves into a room where the speed of sound is 341 m/s. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of the center. Is the interference at this point maximum constructive, perfect destructive, or in between? How will the situation differ if the loudspeakers are out of phase?

**Model** The two speakers are sources of in-phase, spherical waves. The overlap of these waves causes interference.
Two-dimensional interference between two loudspeakers

**VISUALIZE** The figure shows the loudspeakers and defines the distances $r_1$ and $r_2$ to the point of observation. The figure includes dimensions and notes that $\Delta \phi_0 = 0$ rad.
Two-dimensional interference between two loudspeakers

**SOLVE** It’s not $r_1$ and $r_2$ that matter, but the difference $\Delta r$ between them. From the geometry of the figure we can calculate that

$$ r_1 = \sqrt{(5.0 \text{ m})^2 + (1.0 \text{ m})^2} = 5.10 \text{ m} $$

$$ r_2 = \sqrt{(5.0 \text{ m})^2 + (3.0 \text{ m})^2} = 5.83 \text{ m} $$

Thus the path-length difference is $\Delta r = r_2 - r_1 = 0.73 \text{ m}$. The wavelength of the sound waves is

$$ \lambda = \frac{v}{f} = \frac{341 \text{ m/s}}{700 \text{ Hz}} = 0.487 \text{ m} $$
Two-dimensional interference between two loudspeakers

In terms of wavelengths, the path-length difference is \( \Delta r/\lambda = 1.50 \), or

\[
\Delta r = \frac{3}{2} \lambda
\]

Because the sources are in phase (\( \Delta \phi_0 = 0 \)), this is the condition for destructive interference. If the sources were out of phase (\( \Delta \phi_0 = \pi \) rad), then the phase difference of the waves at the listener would be

\[
\Delta \phi = 2\pi \frac{\Delta r}{\lambda} + \Delta \phi_0 = 2\pi \left(\frac{3}{2}\right) + \pi \text{ rad} = 4\pi \text{ rad}
\]

This is an integer multiple of \( 2\pi \) rad, so in this case the interference would be constructive.
Two-dimensional interference between two loudspeakers

**ASSESS** Both the path-length difference *and* any inherent phase difference of the sources must be considered when evaluating interference.
Picturing Interference: Two Identical Sources

\[ \Delta r = \lambda \]

\[ \Delta r = 0 \]

\[ \Delta r = \lambda \]

4/16/2014

Intensity at \( x = 4 \) m

Legend:
- Red: Crest
- Green: Zero
- Blue: Trough
Picturing Interference: Two Out-of-Phase Sources