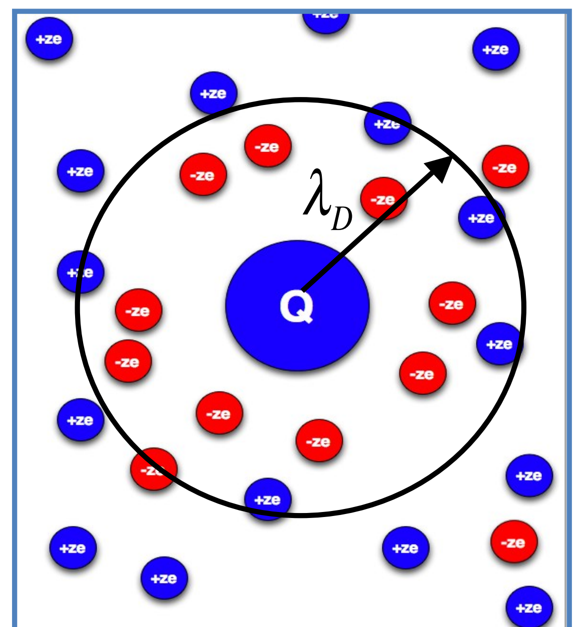
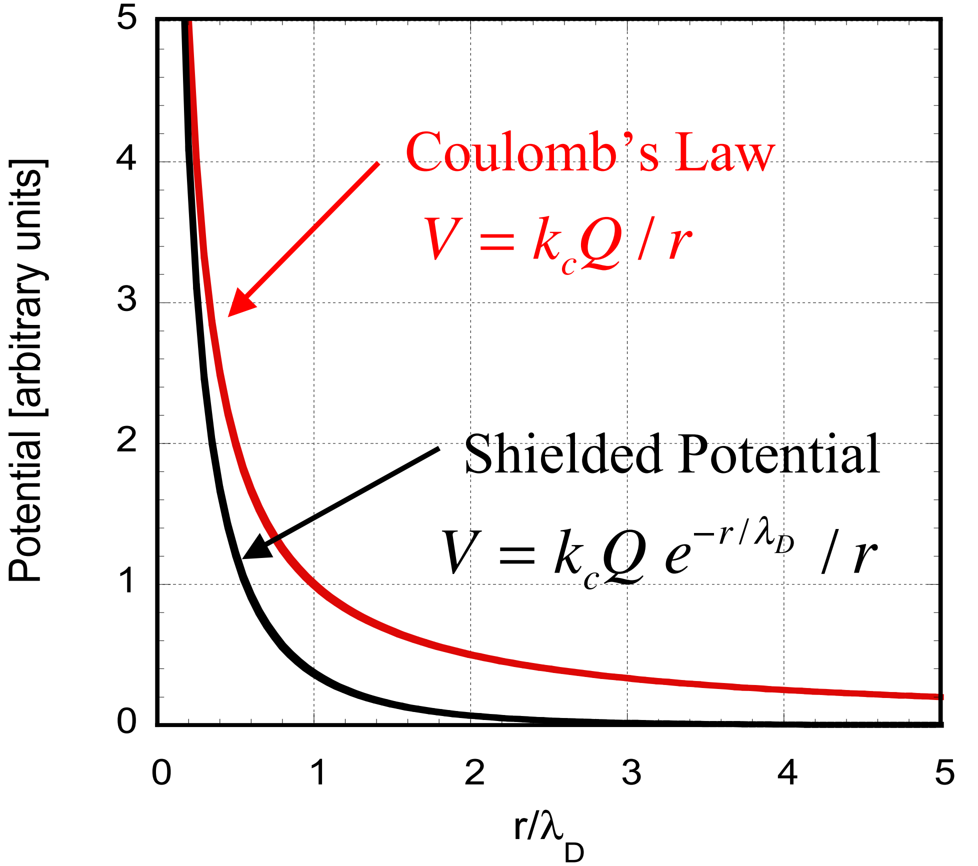
Charged objects in Conducting Fluids

Net charge in a sphere of radius λ_D is approximately zero.

$$\lambda_D^2 = \frac{\varepsilon_0 \kappa k_B T}{c_0 e^2 Z^2}$$

$k_B T$	Thermal energy (Joules)
$k_c = 1/4\pi\varepsilon_0$	Coulomb constant
c_0	Ionic concentration (m^{-3})
Z	Ionic charge state (an integer)
e	elementary charge
κ	relative dielectric constant





Shielding is due to Boltzmann distribution.

Balances kinetic and potential energy

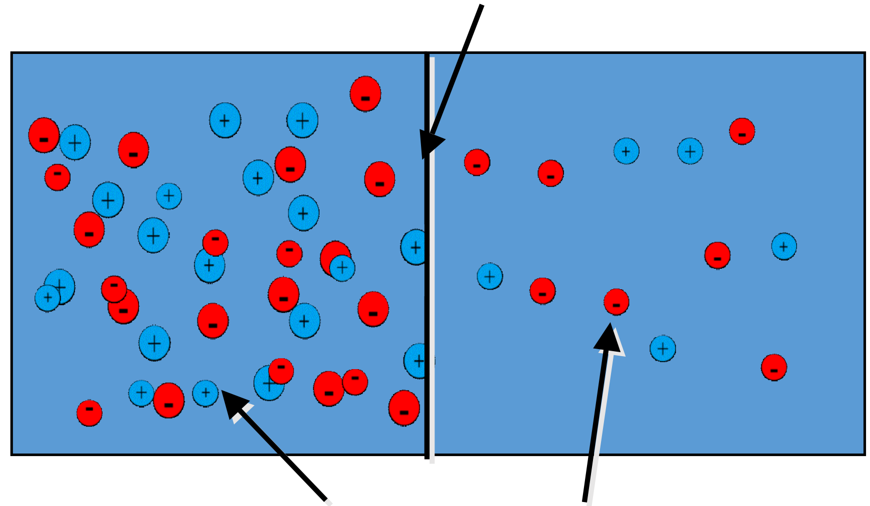
Concentration of ions:

$$c_{\pm}(r) = c_0 \exp[\mp ZeV(r) / k_B T]$$

Nernst Potential

Semi permeable membrane allows blues (+) to pass but not reds (-)

Higher concentration maintained on this side, c_1



Lower concentration, c_2

More blues (+) here than here

Flux of blues (+) due to random walk

Electric field due to excess blues

Potential across membrane

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

Concentration of positive and negative ions in Thermal Equilibrium: Boltzmann distribution

positive ions repelled from region of positive potential

$$c_+(r) = c_0 \exp[-ZeV(r)/k_B T]$$

potential energy of + ion

$$V \uparrow \quad c_+ \downarrow$$

negative ions attracted to region of positive potential

$$c_-(r) = c_0 \exp[+ZeV(r)/k_B T]$$

$$V \uparrow \quad c_- \uparrow$$

Nernst Potential

Difference in electrostatic potential across a membrane. c_1 and c_2 are concentrations of ions on either side of the membrane

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

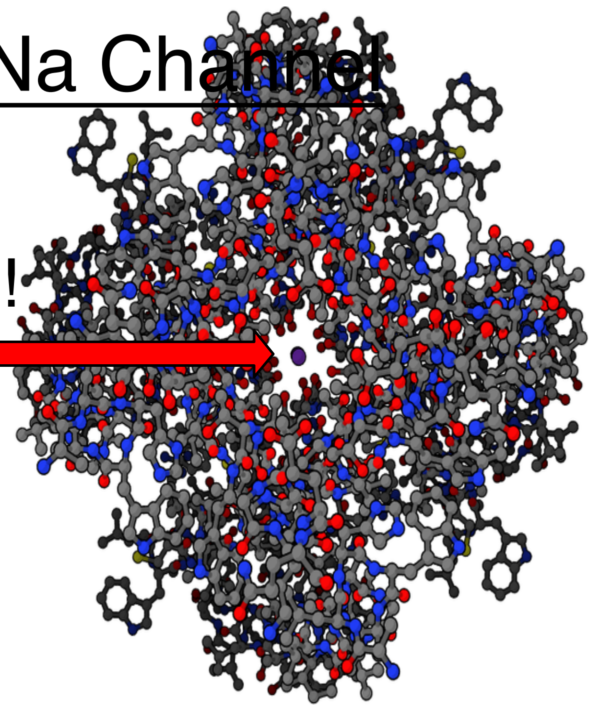
Biology Background:

Ion Channels that only let Potassium through (channels for other types of ions also exist)

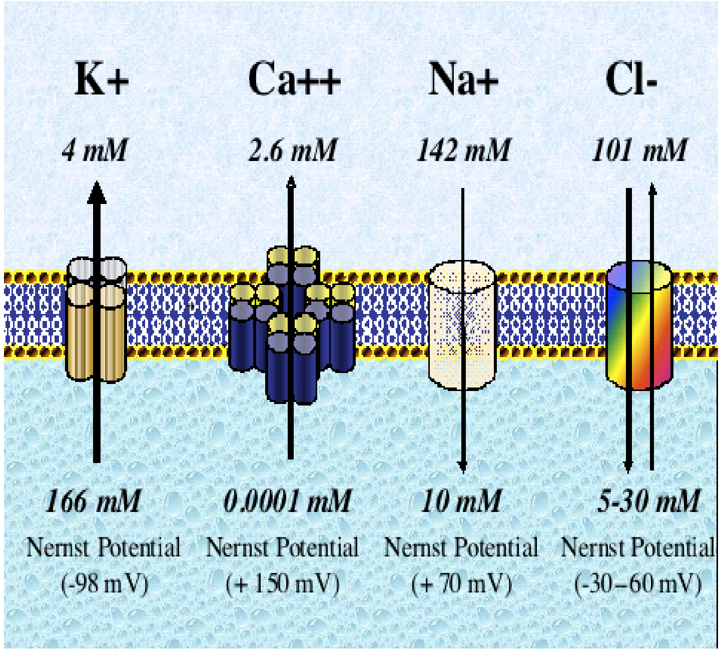
http://www.rcsb.org/pdb/explore/jmol.do?structureId=1BL8&bionumber=1

Top view of Na Channel

Ion in Channel!



Ions in a Cell



http://www.dev.urotoday.com

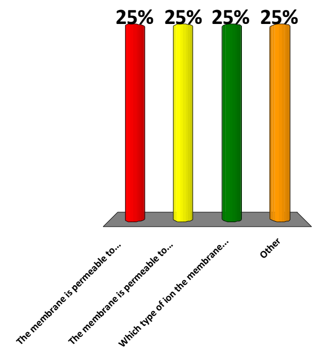
Below you see a membrane that has a channel that is permeable for one of the ions only.

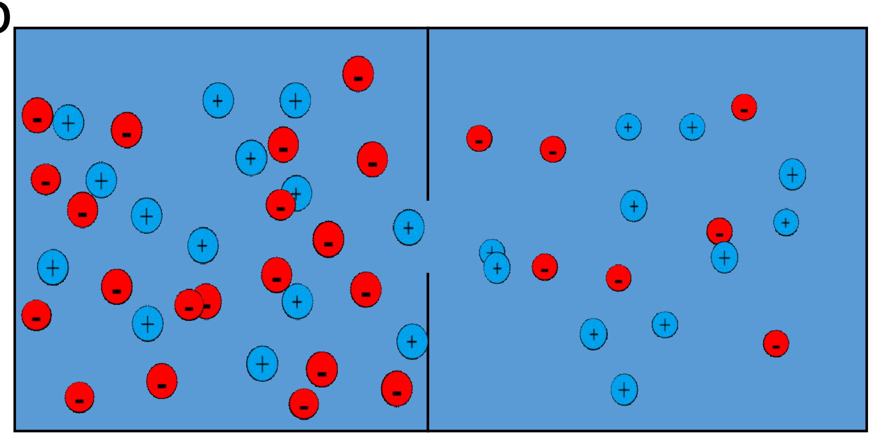
A. The membrane is permeable to positive ions

B. The membrane is permeable to negative ions

C. Which type of ion the membrane is permeable to depends on the initial concentrations

D. Other





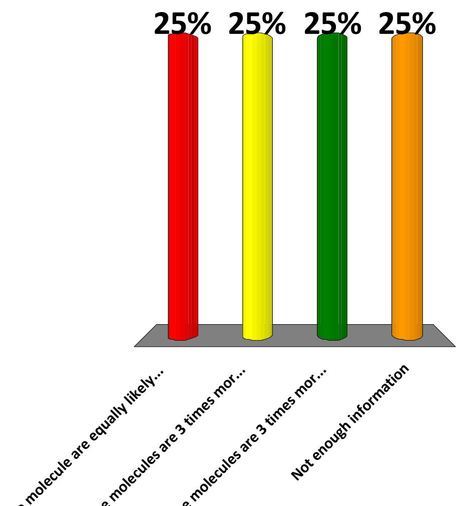
Two boxes one starting with 18 red and blue molecules, the other with 6 of each kind.
Membrane has a channel THAT IS ONLY PERMEABLE to blue molecules. At the start (shown)

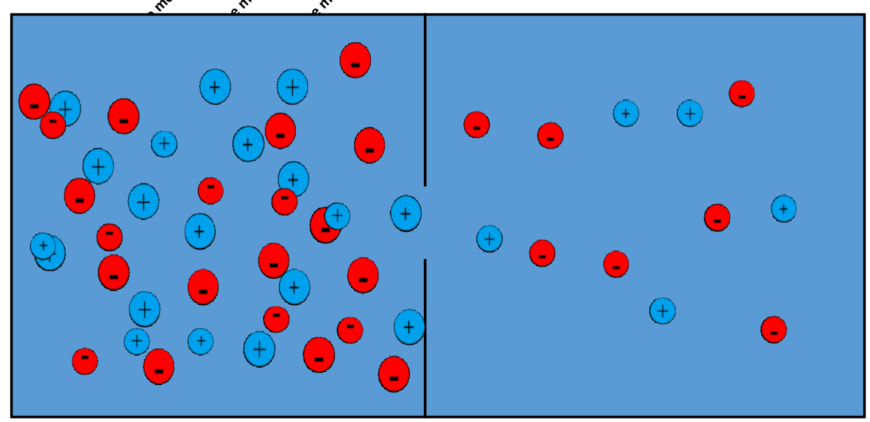
A. An individual blue molecule is equally likely to enter the channel on each side

B. An individual blue molecule is 3 times more likely to enter the channel on the right

C. An individual blue molecule is 3 times more likely to enter the channel on the left

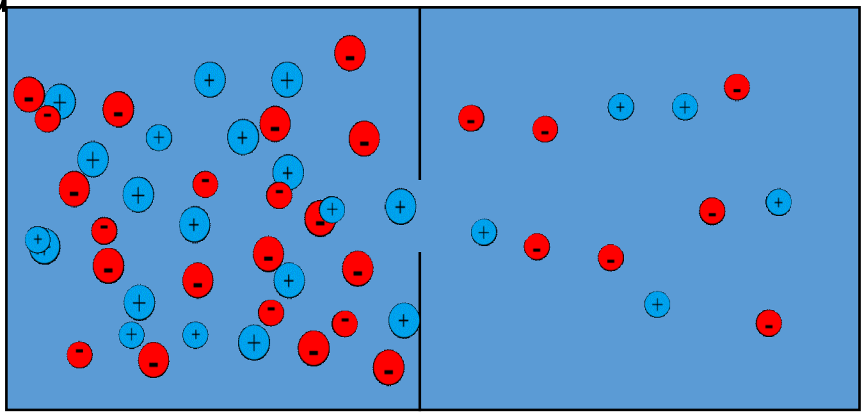
D. Not enough information





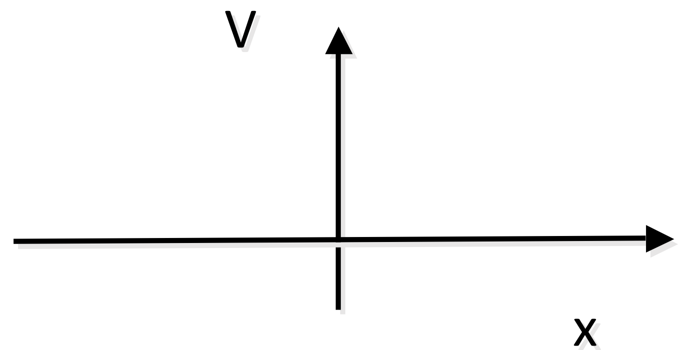
Two boxes one starting with 18 red and blue molecules, the other with 6 of each kind.
Membrane has a channel THAT IS ONLY PERMEABLE to blue molecules. At the start (shown)

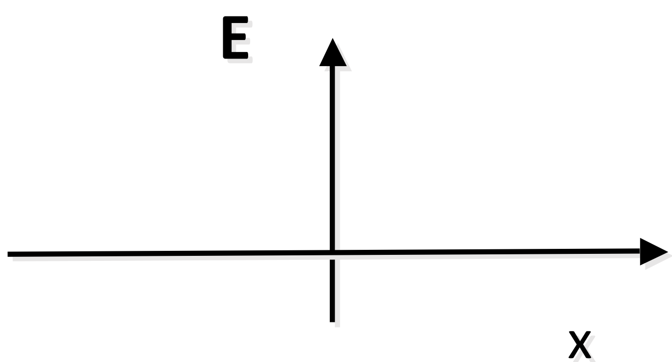
Estimate the potential difference between the two boxes. Assume the gases are at room temperature and the ions are doubly charged



Whiteboard, TA & LA

Sketch equilibrium state



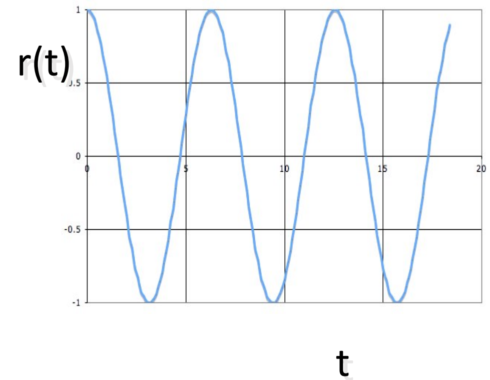


Whiteboard,
TA & LA

Oscillations and Waves

What's the difference?

Separation between two atoms in a molecule r(t)



Oscillations involve a <u>discrete set</u> of quantities that vary in time (usually periodically).

Examples: pendula, vibrations of individual molecules, firefly lights, currents in circuits.

Waves involve <u>continuous</u> quantities the vary in <u>both space and time</u>. (variation may be peri

Examples: light waves, sound waves, elastic waves, surface waves, electro-chemical waves c

Learning about Oscillations and waves

- Why to learn it
  - How the ear senses sound
  - Sound itself
  - Brain waves
  - Heart contraction waves
  - Molecule oscillations

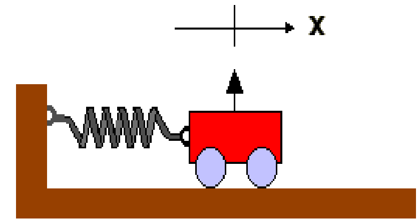
- What to learn
  - How to describe oscillations mathematically (sin, cos)
  - How to think about waves
  - Resonances

- Heart beat
- http://www.youtube.com/watch?annotation_id=annotation_611436&feature=iv&src_vid=Pes9O5z8efk&v=uR4t__B-Zwg

- Ventricular Fibrillation

http://www.youtube.com/watch?v=riUAFkV7HCU

Model system: Mass on a Spring

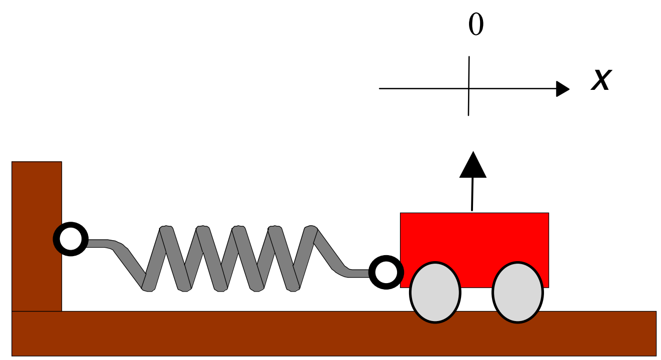


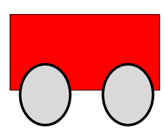
- Consider a cart of mass *m* attached to a light (mass of spring << *m*) spring.
- Choose the coordinate system so that when the cart is at 0 the spring it at its rest length
- Recall the properties of an ideal spring.
  - When it is pulled or pushed on both ends it changes its length.
  - T is tension, T>0 means the spring is being stretched

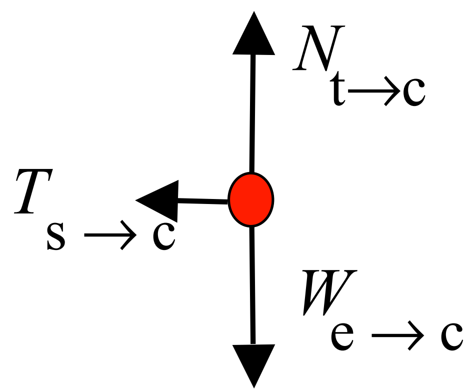
$$T = k\Delta l$$

Analyzing the forces: cart & spring

- What are the forces acting on the cart?





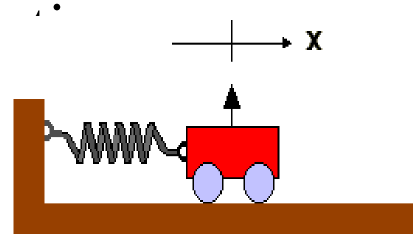


What do the subscripts mean ?

A mass connected to a spring is oscillating back and forth. Consider two possibilities:

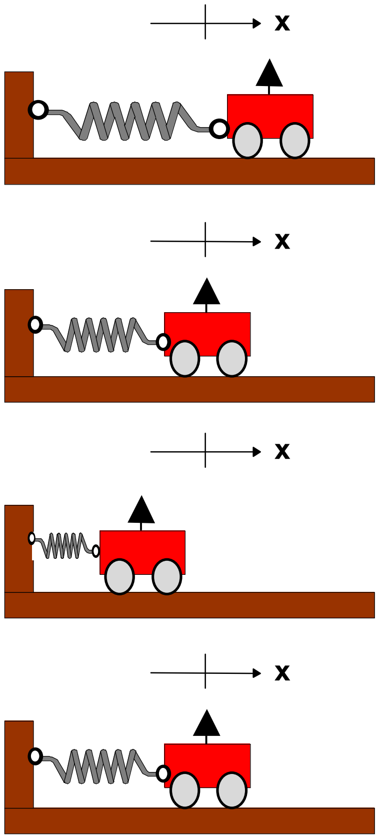
(i) at some point during the oscill
the mass has v = 0 but $a \neq 0$

(ii) at some point during the oscil
the mass has v = 0 and $a = 0$.



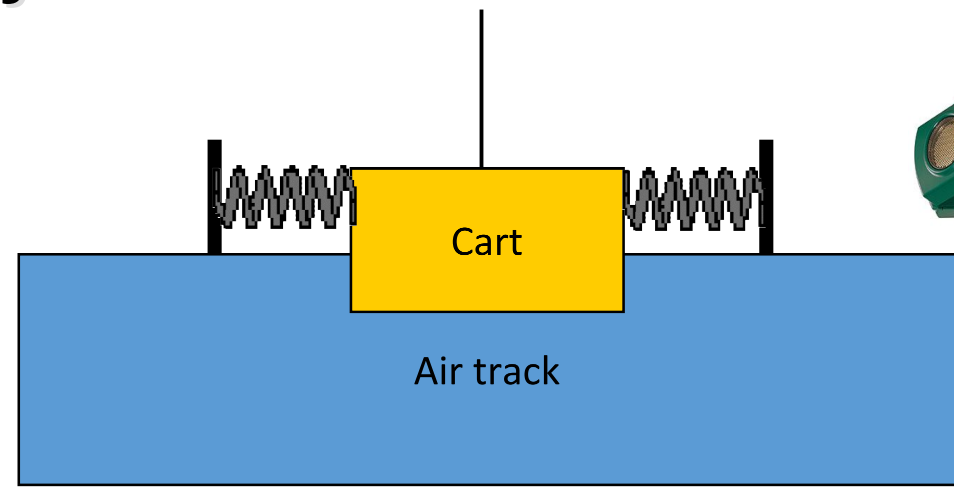
1. Both occur sometime during the oscillation.
2. Neither occurs during the oscillation.
3. Only (i) occurs.
4. Only (ii) occurs.

Tracking the motion



x	F^{net}	a	v

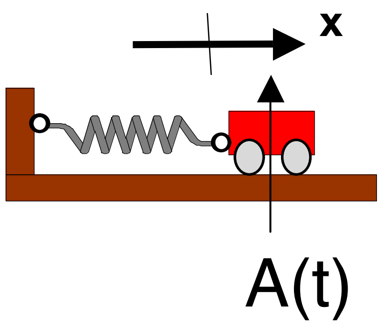
Let’s try it



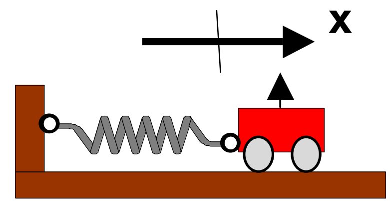
Sonic ranger

Why do we have two springs?

- Position of the cart depends on time t
- Lets call the x position of the cart: A(t)



Doing the Math: The Equation of Motion



- Newton’s equation for the cart is

$$a = {F_{net}}\Big/{m} = {-kA(t)}\Big/{m} = -\left(\frac{k}{m}\right)A(t)$$

- What kind of a quantity is k/m? (i.e. what are its units?)

$$\left[\frac{k}{m}\right] = \frac{[}{\;} \qquad \frac{1}{\Gamma^{2}}$$

Mathematical structure

$$a = \frac{d^2 A(t)}{dt^2} = -\left(\frac{k}{m}\right) A(t)$$

- Express *acceleration a* as a derivative of A(t).

$$\frac{d^2 A(t)}{dt^2} = -\omega_0^2 A(t)$$

- Except for the constant, this is like having a function that is(the negative of) its own second derivative.

$$\frac{d^2 f}{dt^2} = -f$$

- In calculus, we learn that sin(t) and cos(t) work like this. How about:

?

$$A(t) = \cos t$$

- How do we define A=0 ?
    1. The origin (where A=0) is chosen at the initial state of the spring
    2. The origin is chosen at the unstretched state of the spring
    3. The origin can be chosen arbitrarily

- How do we define t
    1. t=0 is chosen at the initial state of the spring
    2. t=0 is chosen when the string is not stretched.
    3. t=0 can be chosen arbitrarily

Interpreting the Result

- We'll leave it to our friends in math to show that these results actually satisfy the N2 equations.
- What do the various terms mean?
  - A_{max} is the maximum displacement — the *amplitude* of the oscillation.
  - What is ω_0? If T is the *period* (how long it takes to go through a full oscillation) then

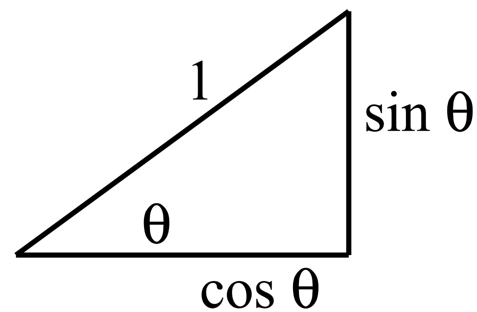
$$\omega_0 t : 0 \rightarrow 2\pi$$

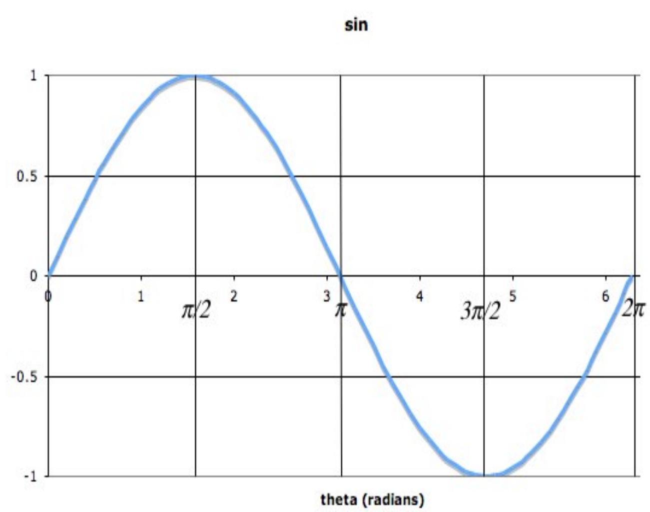
$$t \quad : 0 \rightarrow T$$

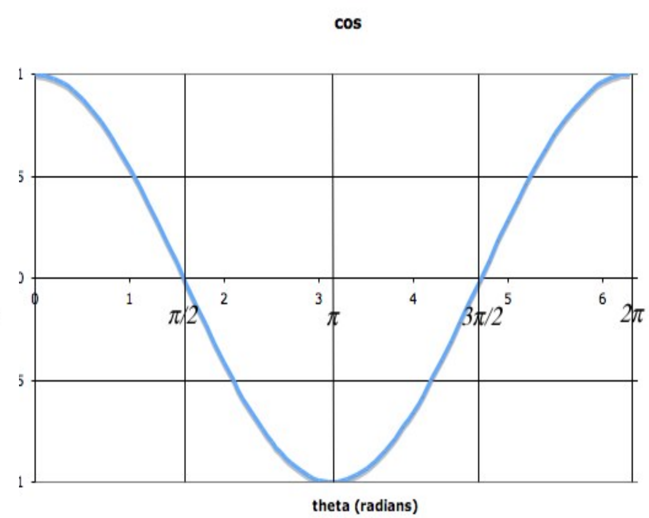
$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

Graphs: sin(θ) vs cos(θ)

- Which is which? How can you tell?
- The two functions sin and cos are derivatives of each other (slopes), but one has a minus sign.
  Which one?
  How can you tell?





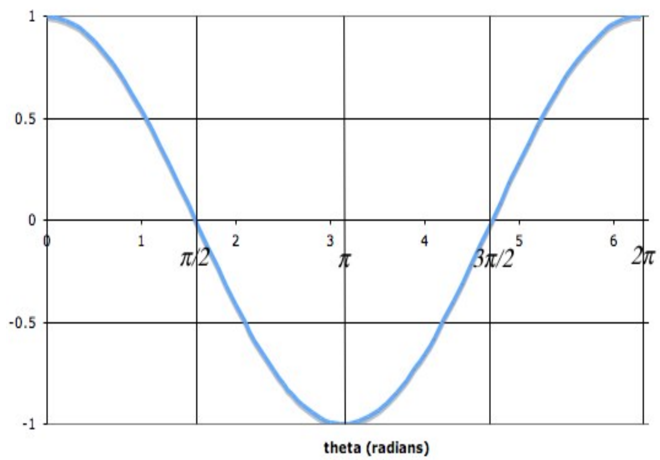


Graphs: sin(θ) vs sin(ω_0t)

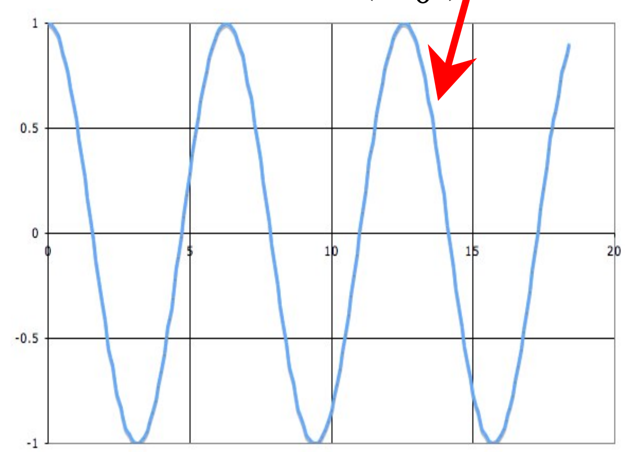
- For angles, $\theta = 0$ and $\theta = 2\pi$ are the same
  so you only get one cycle.
- For time, *t* can go on forever so the cycles repeat.

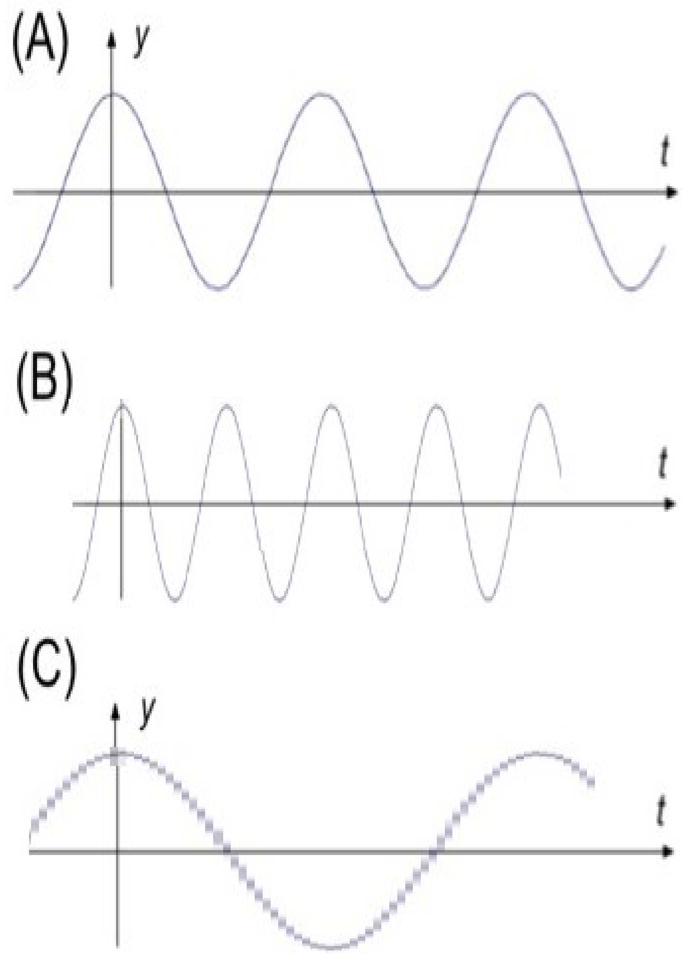
What does changing _0 do to this graph?

cos(θ)



cos($\omega_0 t$)





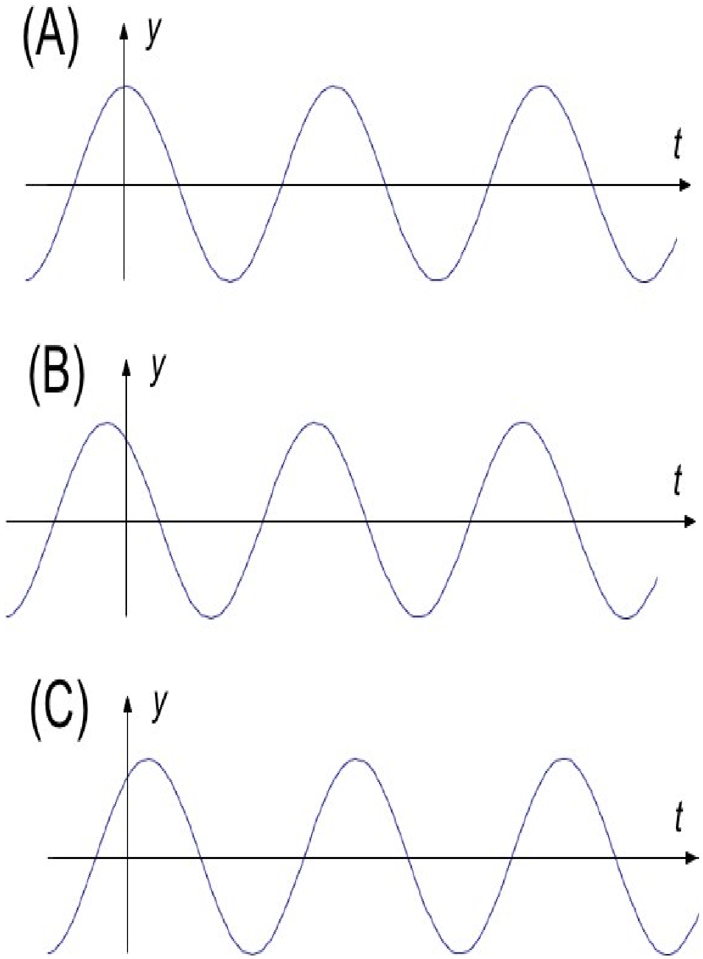
If curve (A) is

$$A\cos(\omega_0 t)$$

which curve is

$$A\cos(2\omega_0 t)?$$

1. (A)
2. (B)
3. (C)
4. None of the above.



Which of these curves is described by

$$A\cos(\omega_0 t + \phi)$$

with $\phi > 0$ (and $\phi << 2\pi$)?

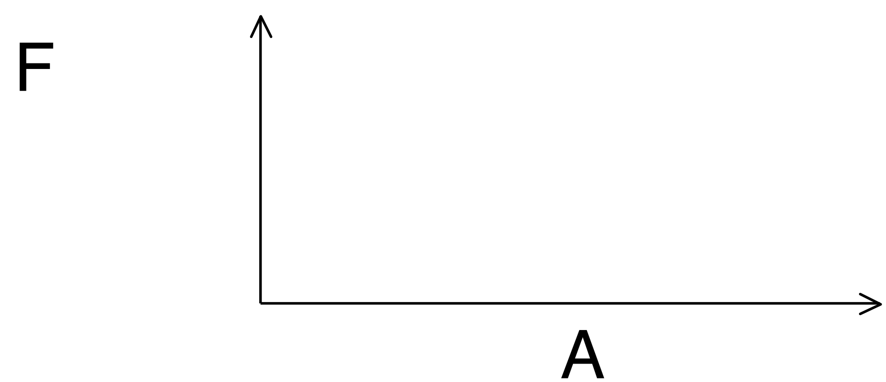
1. (A)
2. (B)
3. (C)
4. None of the above.

Oscillations

- What is the solution to the following equation?

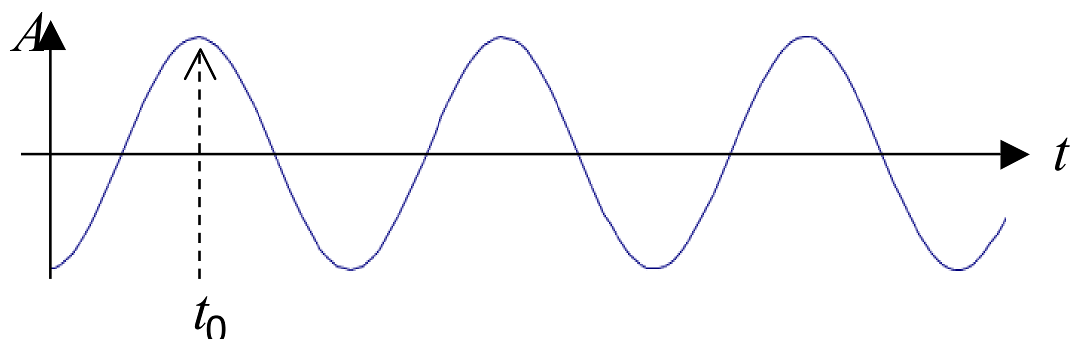
$$\frac{d^2 A(t)}{dt^2} = -\omega_0^2 A(t)$$

- Interpretation: DRAW graph
  - Sketch in A=0



Oscillations

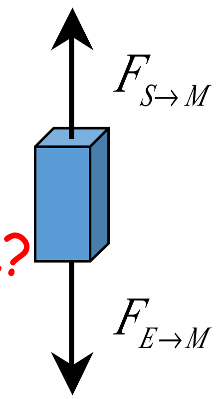
- A typical oscillation



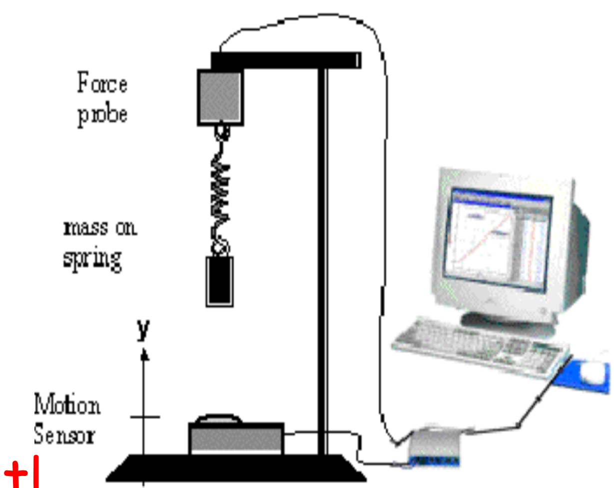
Examples of oscillations

- How general are the equations we just studied?

Summary with Equations: Mass on a spring



Measured from where?

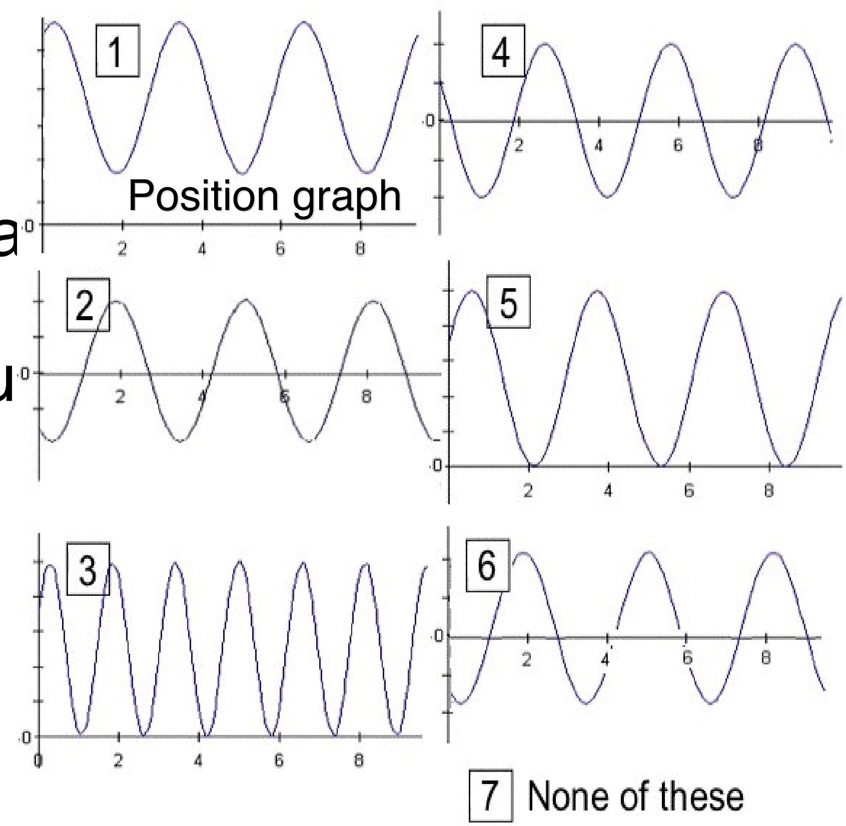


Interpret!

A mass is hanging from a spring. The position of the mass is measured by a sonic ranger sitting 25 cm under the mass's equilibrium position. At some time, the mass is started oscillating.

At a later time, the sonic ranger begins to take da

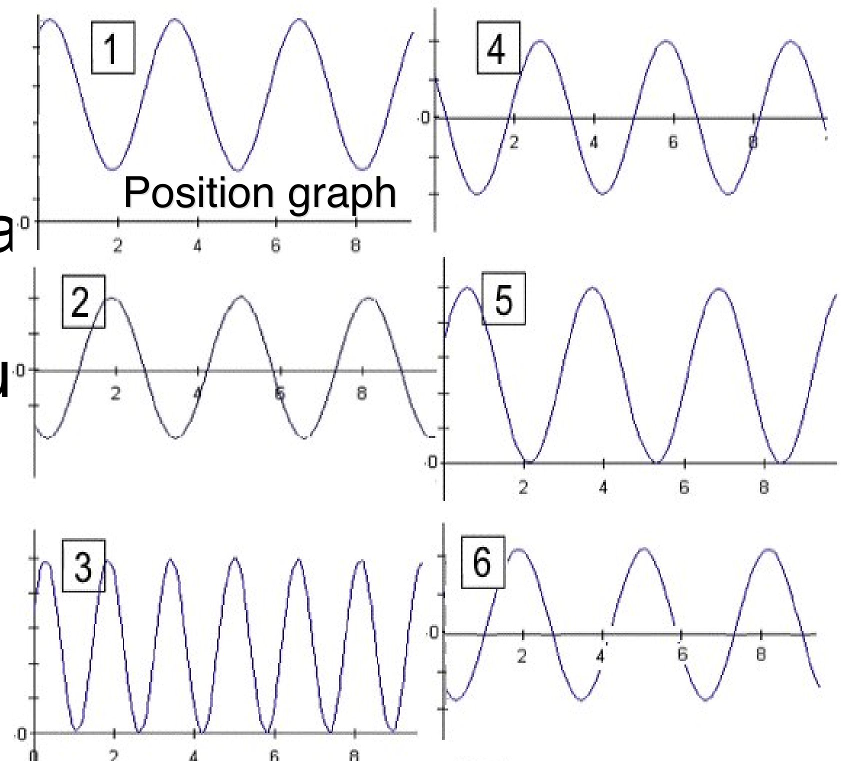
Which graph cou represent the **velocity** of the mass?



A mass is hanging from a spring. The position of the mass is measured by a sonic ranger sitting 25 cm under the mass's equilibrium position. At some time, the mass is started oscillating.

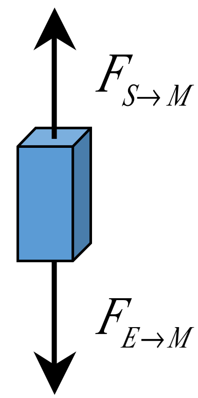
At a later time,
the sonic ranger
begins to take da

Which graph cou
represent the
net force
on the mass?



7 None of these

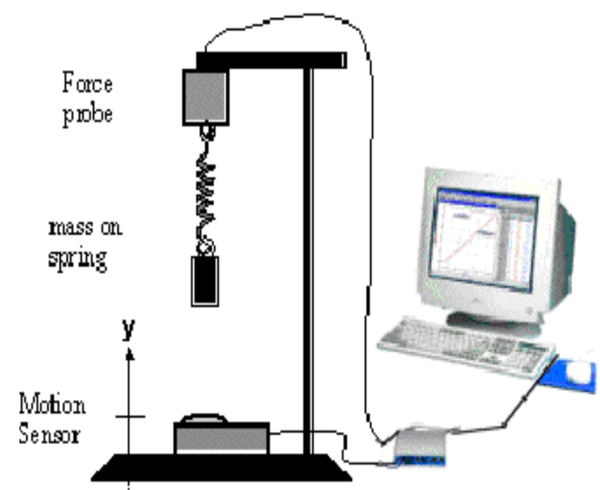
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k(\Delta l)^2$$

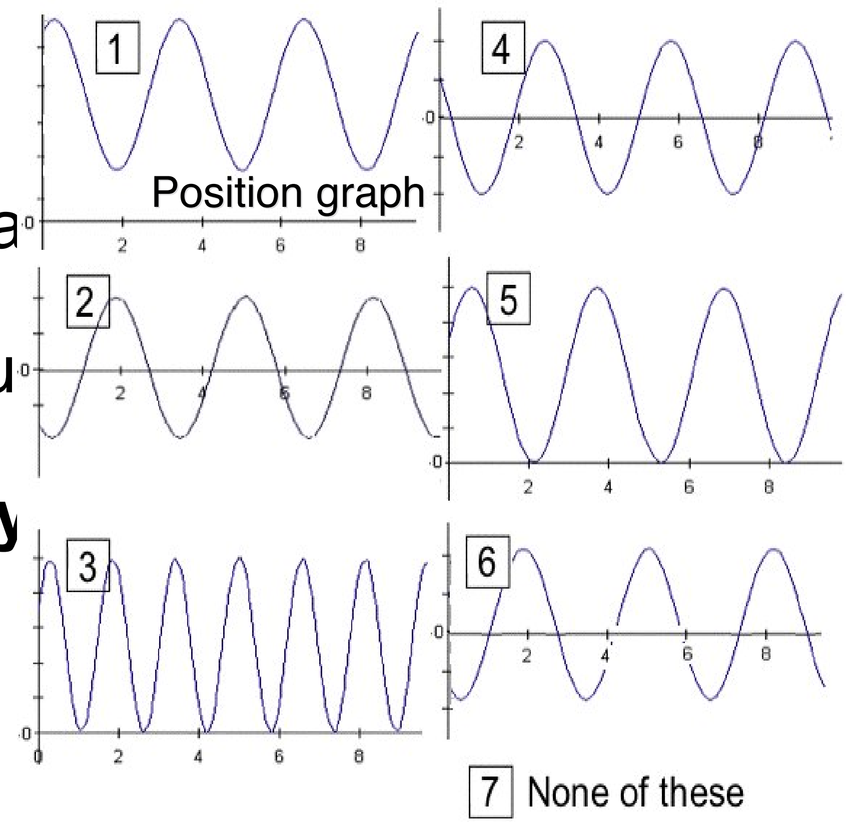
$$E_i = E_f$$



A mass is hanging from a spring. The position of the mass is measured by a sonic ranger sitting 25 cm under the mass's equilibrium position. At some time, the mass is started oscillating.

At a later time, the sonic ranger begins to take da

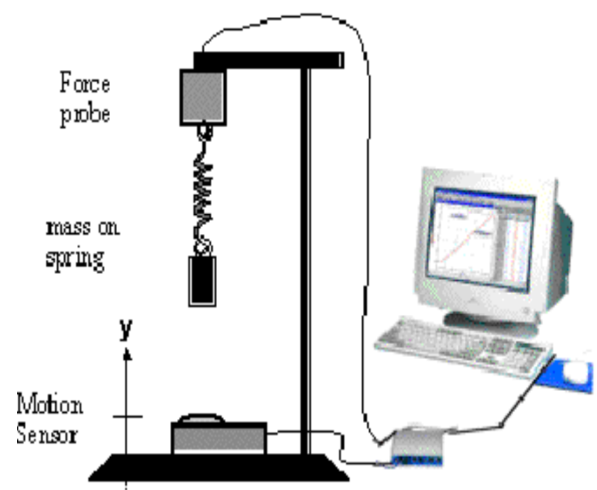
Which graph cou represent the **potential energy** of the spring?



When we pull the mass down from its equilibrium, what happens to the energies?

The gravitational PE

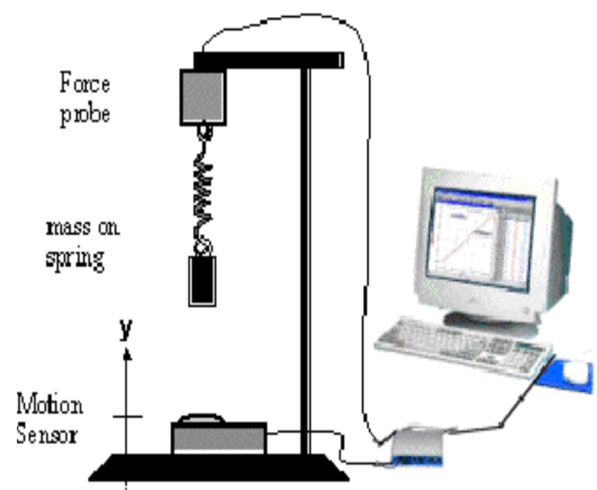
A. increases
B. decreases
C. remains the same
D. you can’t tell from the information given.



When we pull the mass down from its equilibrium, what happens to the energies?

The spring PE

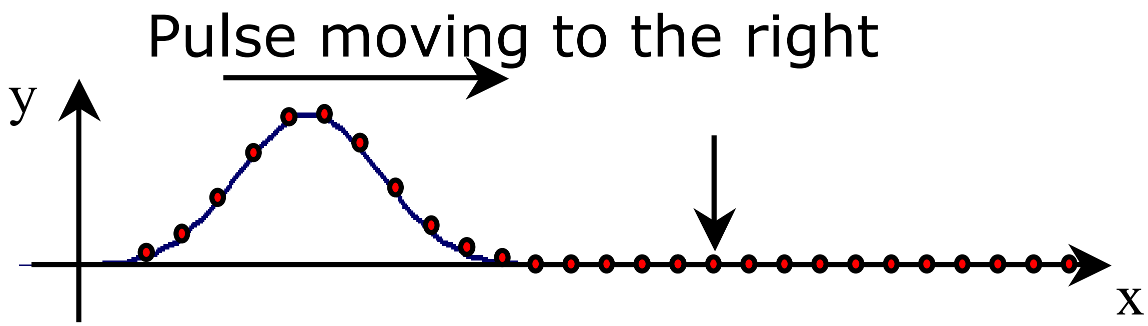
A. increases
B. decreases
C. remains the same
D. you can't tell from the information given.



Displacements on an elastic string / spring

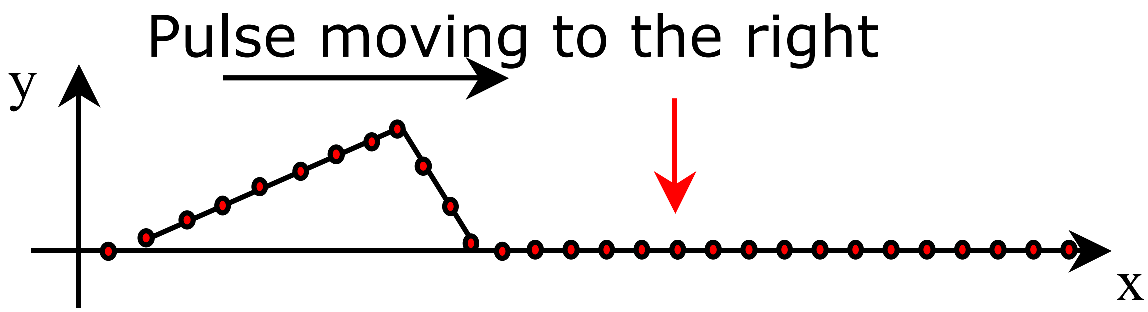
- Each bit of the string can move up or down (perpendicular to its length) – transverse waves
- Each bit of string can also move toward/away along the string length if the string is elastic (most notable on very deformable strings such as slinky, rubber band). – longitundinal waves

How do the beads move?



- Sketch the y position of the bead indicated by the arrow as a function of time

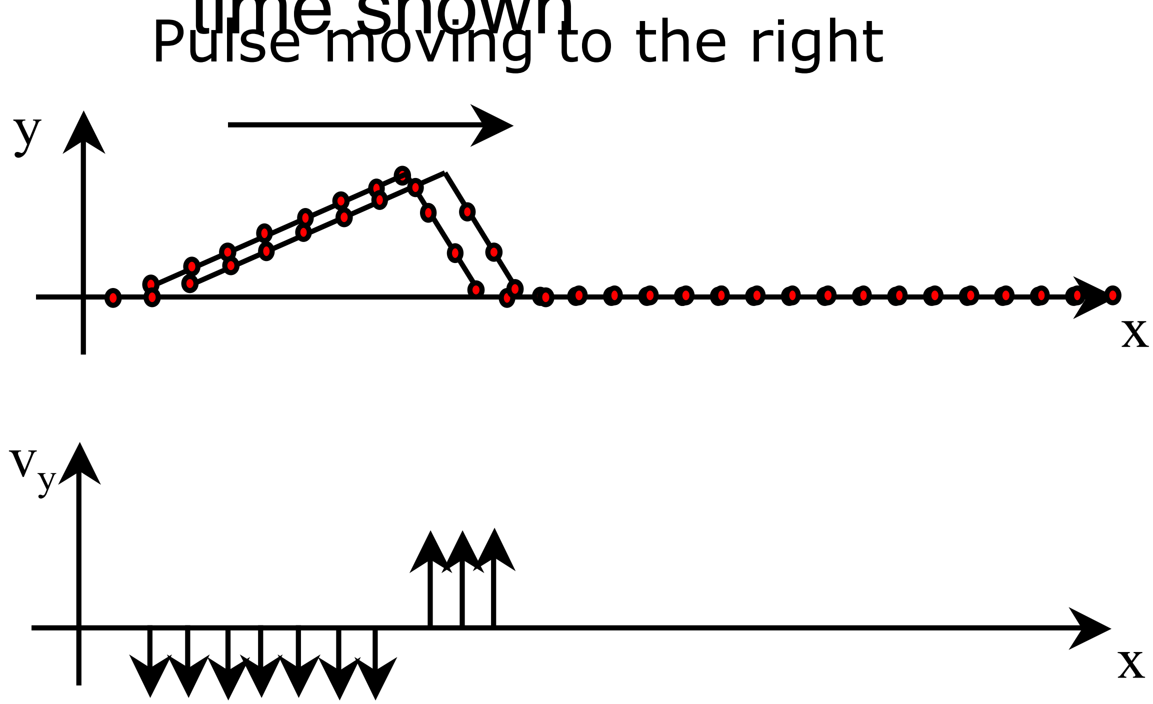
If this is the space-graph (photo at an instant of time) what does the time-graph look like for the bead marked with a red arrow?



1. Choice One
2. Choice Two
3. Choice Three
4. Choice Four
5. Choice Five
6. Choice Six
7. Choice Seven
8. Choice Eight

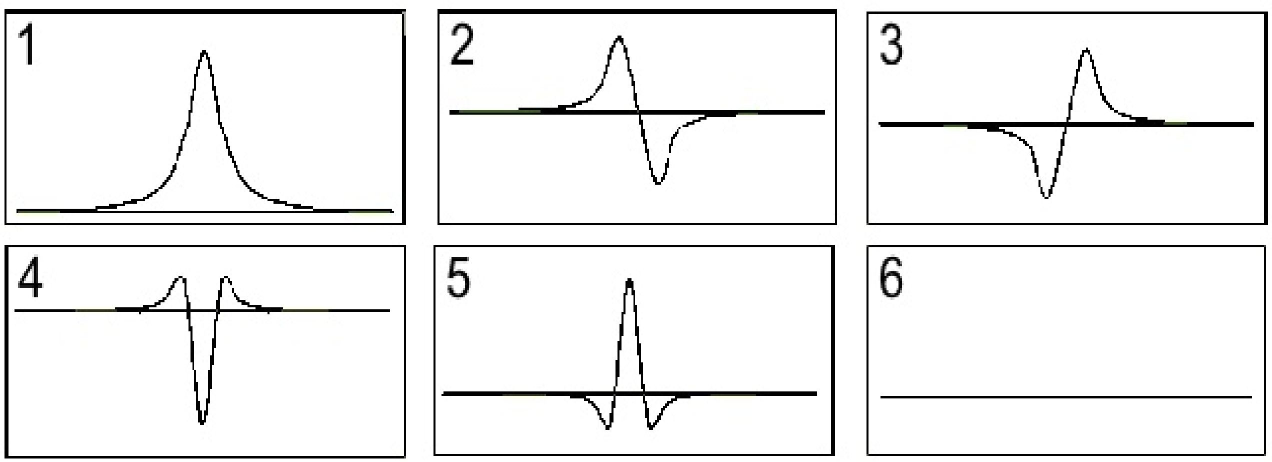
Describing the motion of the beads

- Sketch the velocity of each bead in the top figure at the time shown



the
right. At a time t_0 a photograph of the string
would look like figure 1 below. A point on the string
to the right of the pulse is marked by a
spot of paint. *(x is horizontal and right, y is vertical and up)*

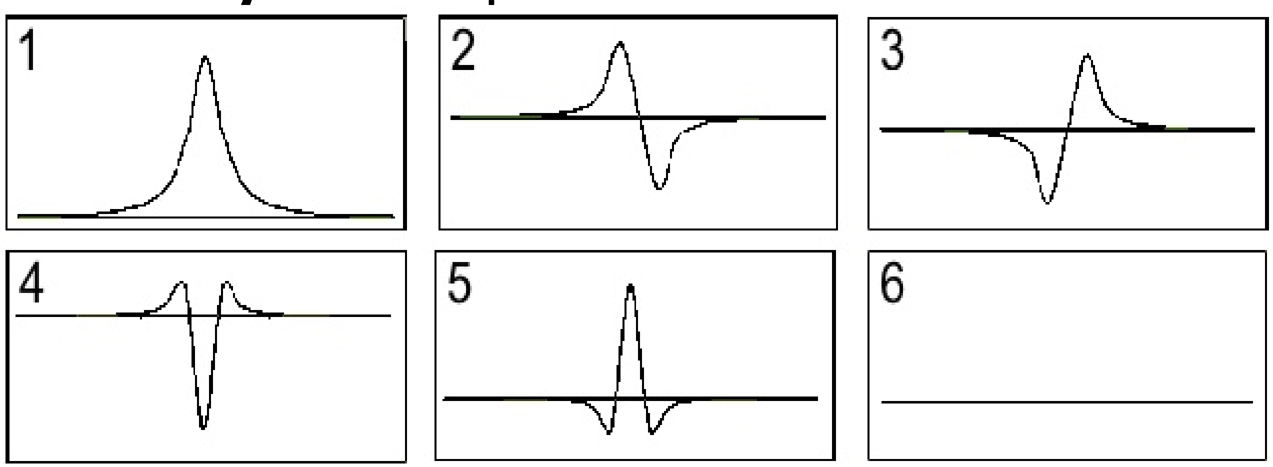
Which graph would look most like a graph of the
y displacement of the spot as a function of



7 None of these

the
right. At a time t_0 a photograph of the string
would look like figure 1 below. A point on the string
to the right of the pulse is marked by a spot of paint. *(x is horizontal and right, y is vertical and up)*

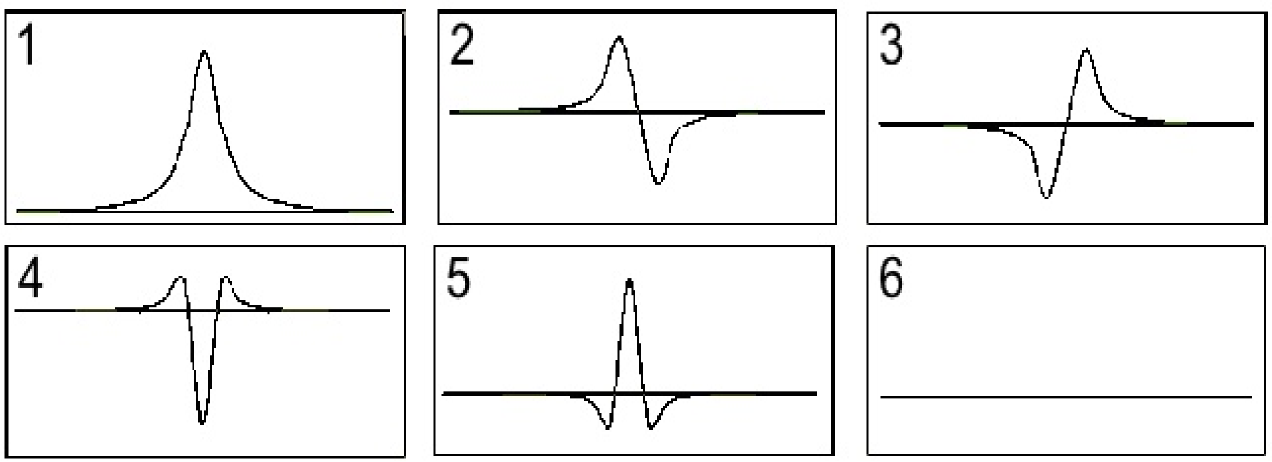
Which graph would look most like a graph of the
x velocity of the spot as a function of time?



7 None of these

the
right. At a time t_0 a photograph of the string
would look like figure 1 below. A point on the string
to the right of the pulse is marked by a spot of paint. *(x is horizontal and right, y is vertical and up)*

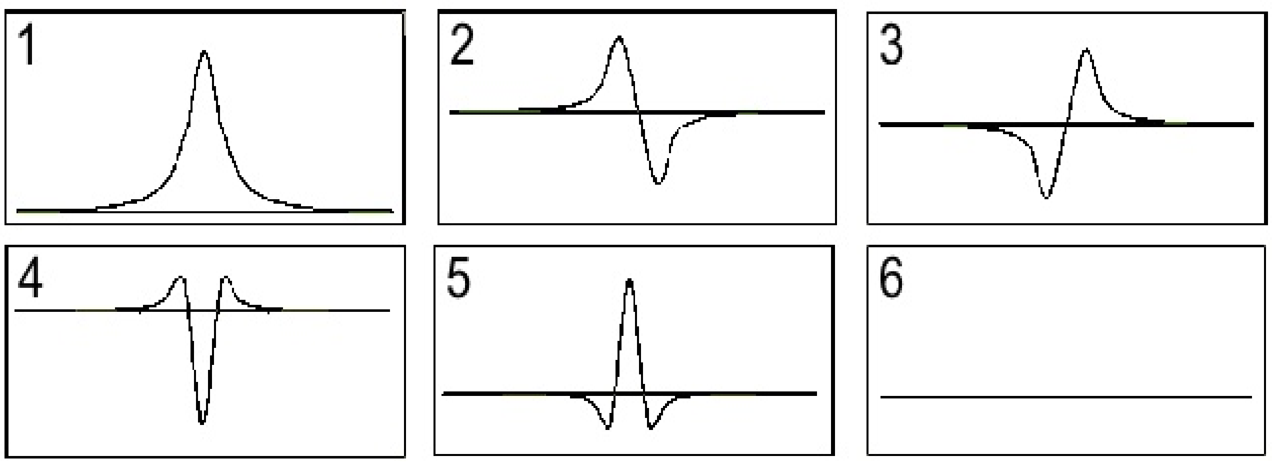
Which graph would look most like a graph of the
y velocity of the spot as a function of time?



7 None of these

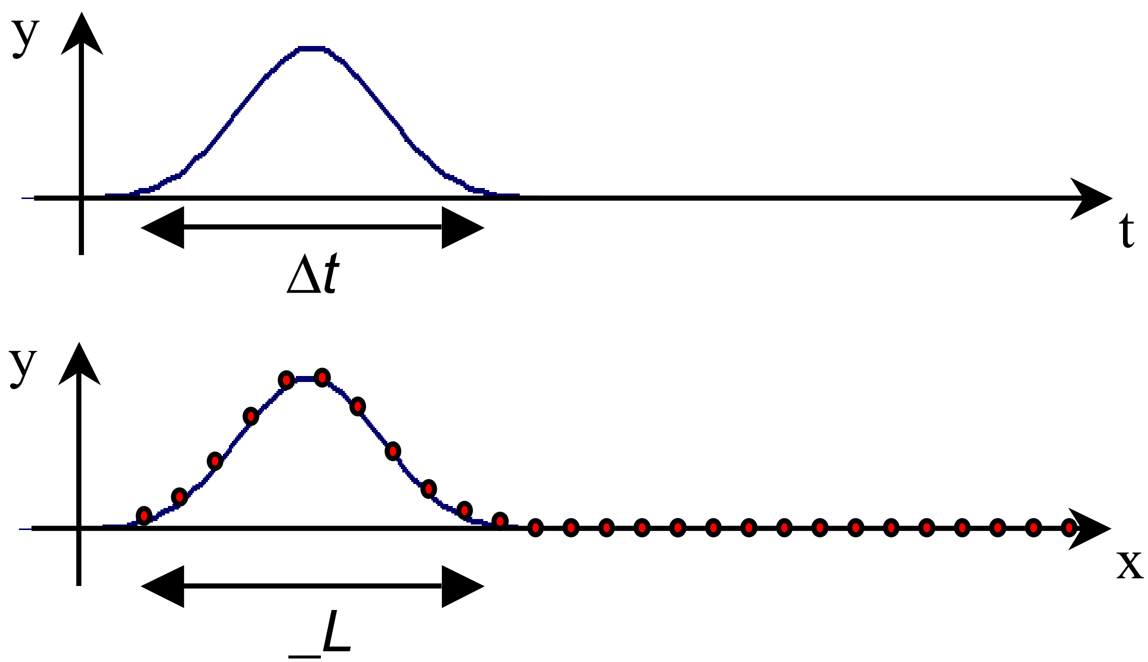
the
right. At a time t_0 a photograph of the string
would look like figure 1 below. A point on the string
to the right of the pulse is marked by a spot of paint. *(x is horizontal and right, y is vertical and up)*

Which graph would look most like a graph of the
y force on the spot as a function of time?



7 None of these

What controls the widths of the pulses in time and space?

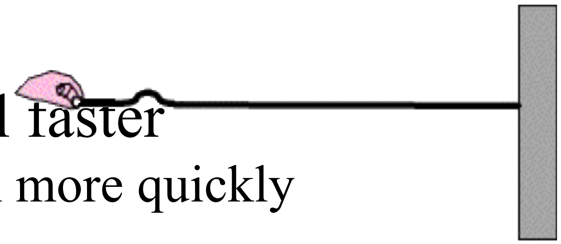


Width of a pulse

- The amount of time the demonstrator's hand was displaced up and down determines the time width of the t-pulse, Δt.
- The speed of the signal propagation on the string controls the width of the x-pulse, ΔL.
  - The leading edge takes off with some speed, v_0.
  - The pulse is over when the trailing edge is done.
  - The width is determined by "how far the leading edge got to" before the displacement was over.

$$\Delta L = v_0 \Delta t$$

What Controls the Speed of the Pulse on a Spring?

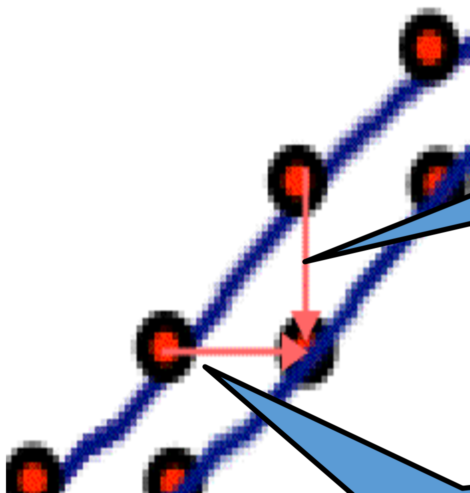


To make the pulse go to the wall faster

1. Move your hand up and down more quickly (but by the same amount).
2. Move your hand up and down more slowly (but by the same amount).
3. Move your hand up and down a larger distance in the same time.
4. Move your hand up and down a smaller distance in the same time.
5. Use a heavier string of the same length under the same tension.
6. Use a string of the same density but decrease the tension.
7. Use a string of the same density but increase the tension.
8. Put more force into the wave,

Speed of a bead

- The speed the bead moves depends on how fast the pulse is moving and how far it needs to travel to stay on the string.



slope of pulse

$$v(x,t) = \frac{dy(x,t)}{dt} = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{dy}{dx}\right)v_0$$

speed of bead

speed of pulse

Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)
- *Matter speed*: the speed of the bits of matter depend on both the **Amplitude** and **shape of the pulse** and **pattern speed**.

Dimensional analysis

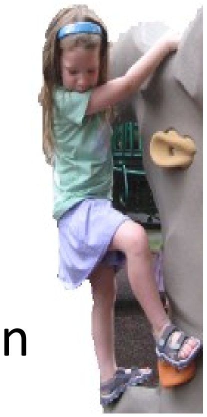
- Square brackets are used to indicate a quantities dimensions – mass ($\mathcal{M}$), length ($\mathcal{L}$), or time ($\mathcal{T}$)
  - $[m] = \mathcal{M}$
  - $[L] = \mathcal{L}$
  - $[t] = \mathcal{T}$
  - $[F] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$

- Build a velocity using mass (m), length (L), and tension (T) of the string:
  - $[v] = \mathcal{L}/\mathcal{T}$
  - $[T] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$
  - $[T/m] = \mathcal{L}/\mathcal{T}^2$
  - $[TL/m] = \mathcal{L}^2/\mathcal{T}^2$

$$v_0^2 = \frac{TL}{m}$$

or, using $\mu = m/L$ $\qquad v_0 = \sqrt{\frac{T}{\mu}}$

Foothold principles: Mechanical waves



- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium
  with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

$$v_0 = \sqrt{T/\mu}$$

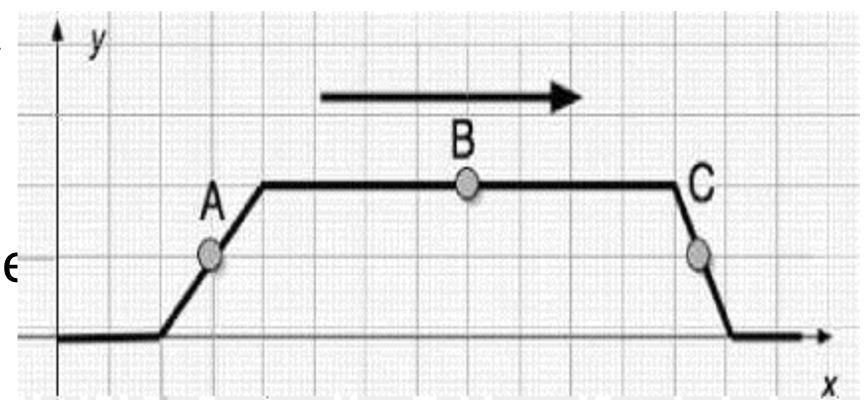
v_0 = speed of pulse
T = tension of spring
_ = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse

Below is a snapshot of a piece of an elastic spring on which a pulse is traveling. Each square of the grid represents 1 cm. In a video of the motion of the pulse, the pulse is observed to move a distance of 2 cm in 0.04 seconds.

Three pieces of tape are attached to the spring. They are marked by small circles in the figure and are labeled A , B , and C .

Graph the velocity of each of the 3 pieces
of tape as the pulse moves past them.



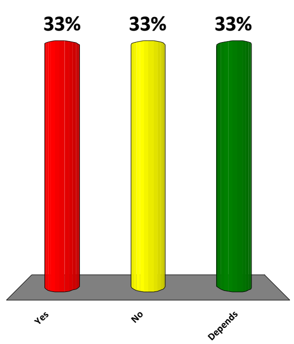
(Take this instant as t = 0)

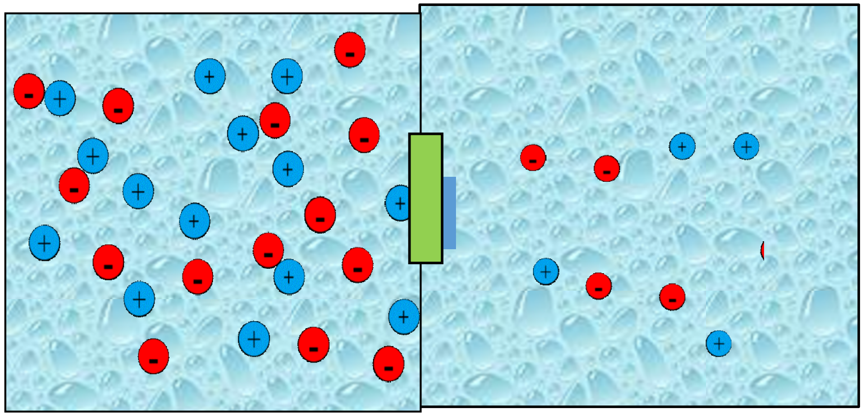
Does this concentration difference lead to an electrostatic potential?

A. Yes

B. No

C. Depends





Does opening a channel lead to a potential difference?

A. Yes

B. No

C. Depends