Electrical Current - Review

$$I = \Delta Q/\Delta t$$

Current I

Charge ΔQ enters here during time Δt

wire

ΔQ leaves here during time Δt

Generally not the same charge, but the same <u>amount</u> of charge

Foothold ideas: Currents

- Charge is moving:
  How much?

$$I = \frac{\Delta q}{\Delta t}$$

- How does this relate to the individual charges?

$$I = q\, n\, A\, v$$

- What pushes the charges through resistance? Electric force implies a drop in *V*!

$$F_e = qE$$

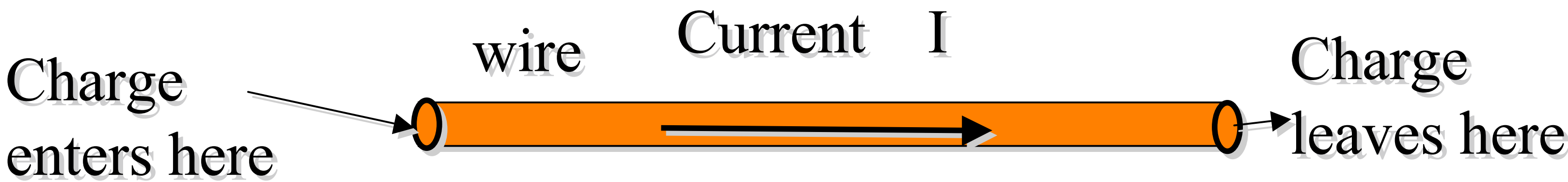
$$\Delta V = -\frac{E}{L}$$

Units

Symbol

■ Current (I)	**Ampere** = Coulomb/sec	A
■ Voltage (V)	**Volt** = Joule/Coulomb	V
■ E-Field (E)	Newton/Coulomb = Volt/meter	V/m
■ Resistance (R)	**Ohm** = Volt/Ampere	Ω
■ Capacitance (C)	**Farad** = Coulomb/Volt	F
■ Power (P)	**Watt** = Joule/sec	W

Current and Fluid Flow



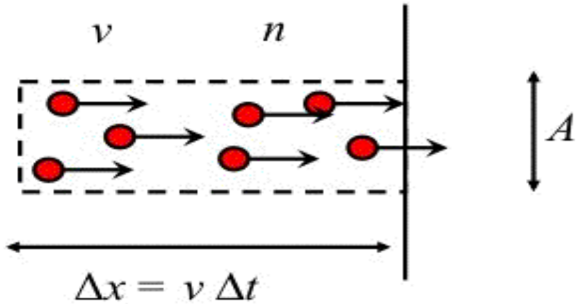
$$V = RI = \frac{l\rho}{A} I$$

The volumetric flow rate of the fluid, I = v*A, will be governed by the Hagen-Poiseuille (H-P) equation:

$$\Delta P \propto \frac{l\mu_{visc}}{A^2} I$$

How are they the same? different? Why?

Whiteboard, TA & LA



Ohm's Law

- Current proportional to change in Electrical Potential

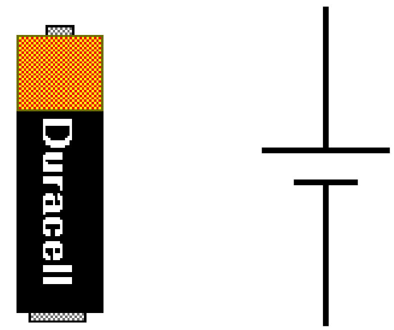
$$\Delta V = IR$$

- Does R depend on the Area of the resistor?
- Does R depend on the length of the resistor?

1. R Increases
2. R decreases
3. R remains the same
4. Depends on material

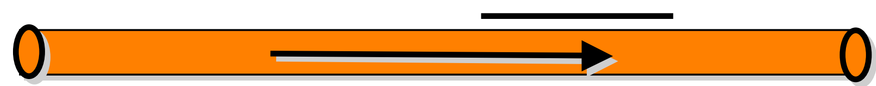
Circuits

Electric circuit elements



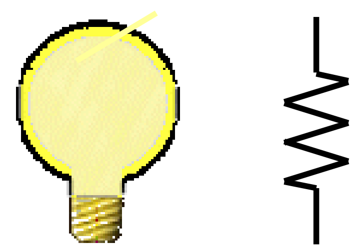
- *Batteries* —devices that maintain a constant electrical potential difference across their terminals

- *Wires* — charges flow quickly need very little forces to move

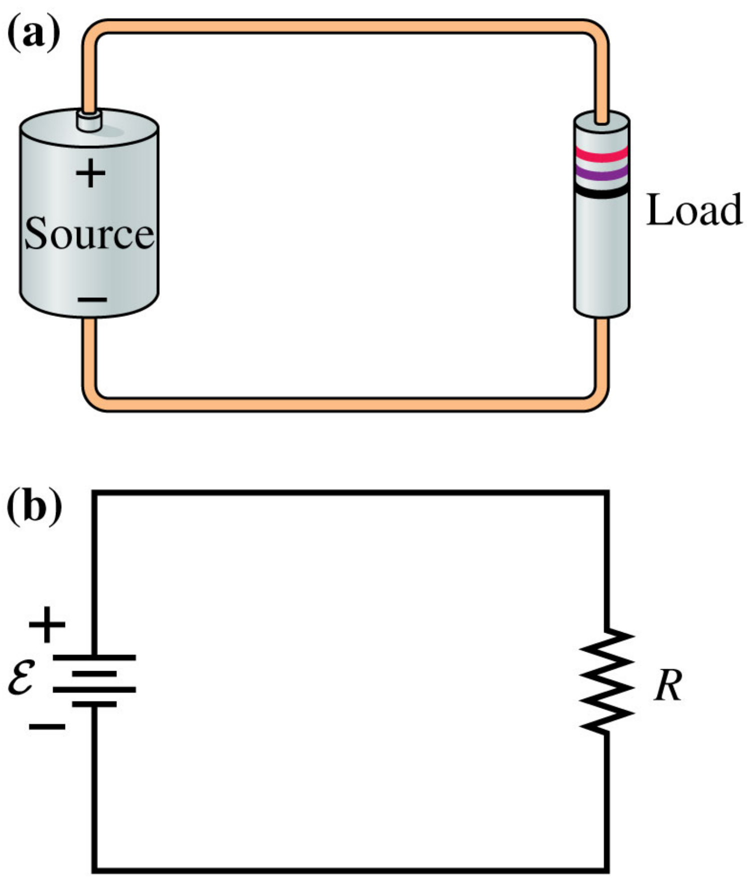


- *Resistors* — charges need a larger force to move. Examples are Resistors and Lightbulbs

- *Capacitors* - You know about these!

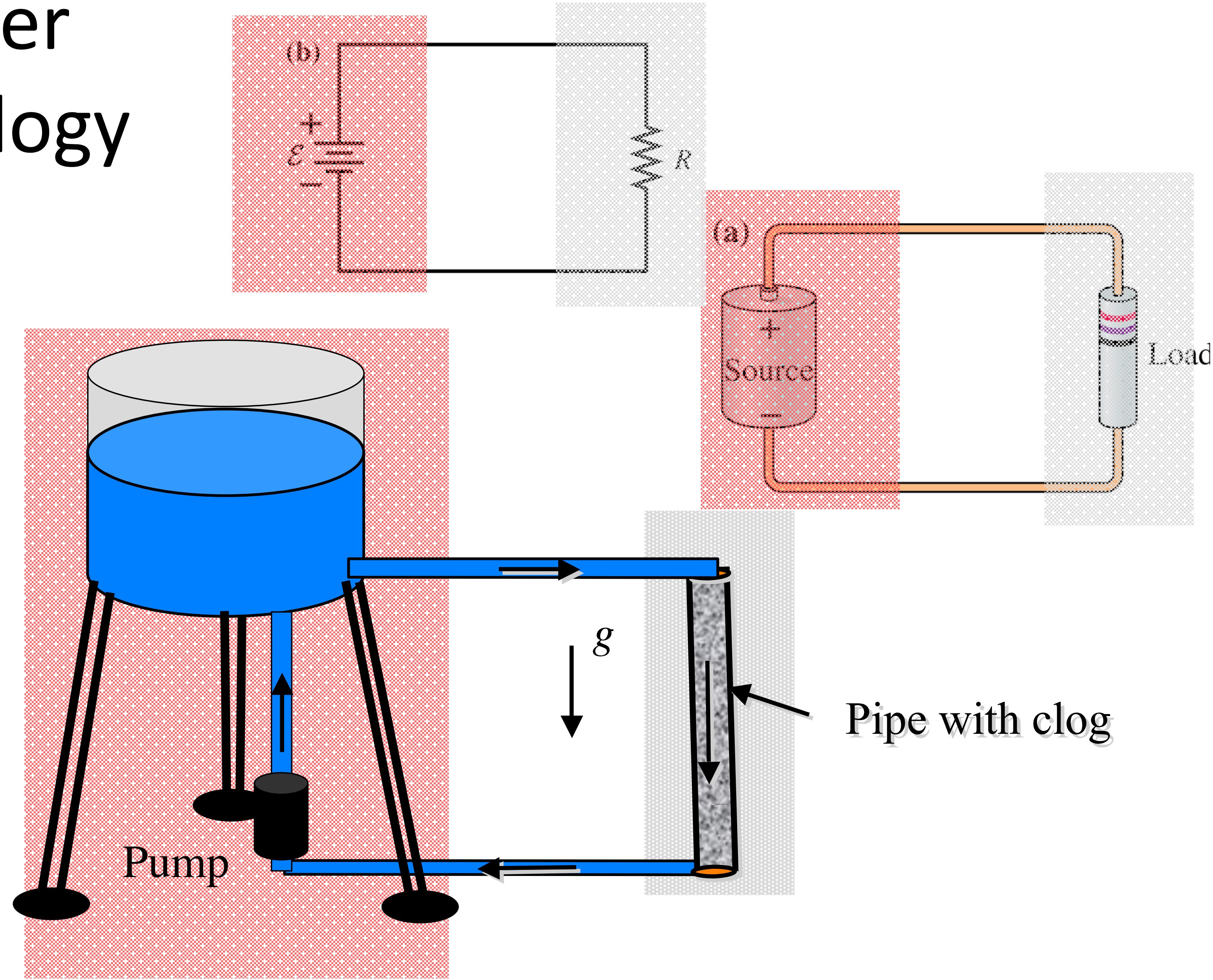


Why do we call them circuits?



- The most basic electric circuit is a single resistor connected to the two terminals of a battery.
- Figure (a) shows a literal picture of the circuit elements and the connecting wires.
- Figure (b) is the circuit diagram.
- This is a **complete circuit**, forming a continuous path between the battery terminals.

Water Analogy



Two types of Rule governing circuits

1. Rules that describe how individual circuit elements work. examples: Ohm's law $V=IR$, also $Q=VC$ for capacitors.

2. Rules about voltage and current that apply due to the way elements are connected together. Kirchhoff's laws

Foothold ideas: Kirchhoff's principles

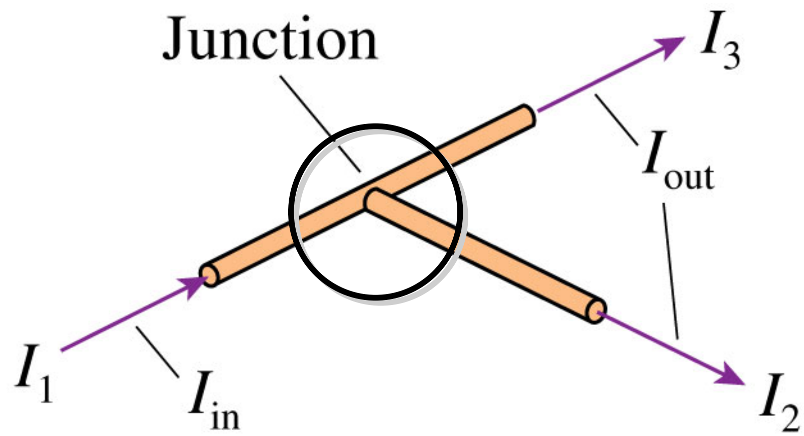
1. ***Junction rule***: The net amount of current entering or leaving any volume in an electrical network is ZERO
2. ***Loop rule***: Following around any loop in an electrical network the potential has to come back to the same value (sum of voltage increases = sum of voltage decreases).

The Junction Rule

For a *junction*, the law of conservation of current requires that:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

where the Σ symbol means summation. This basic conservation statement is called **Kirchhoff's current law. Abbreviated KCL**



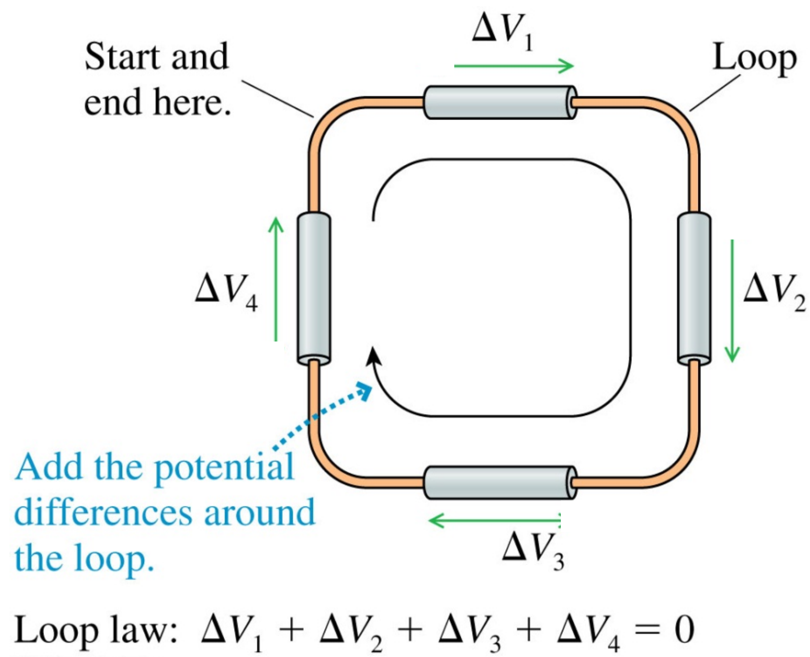
Junction law: $I_1 = I_2 + I_3$

The Loop Rule

- For any path that starts and ends at the same point:

$$\Delta V_{\text{loop}} = \sum(\Delta V)_i = 0$$

- The sum of all the potential differences encountered while moving around a loop or closed path is zero.
- This statement is known as **Kirchhoff's Voltage law. KVL**



KVL and KCL

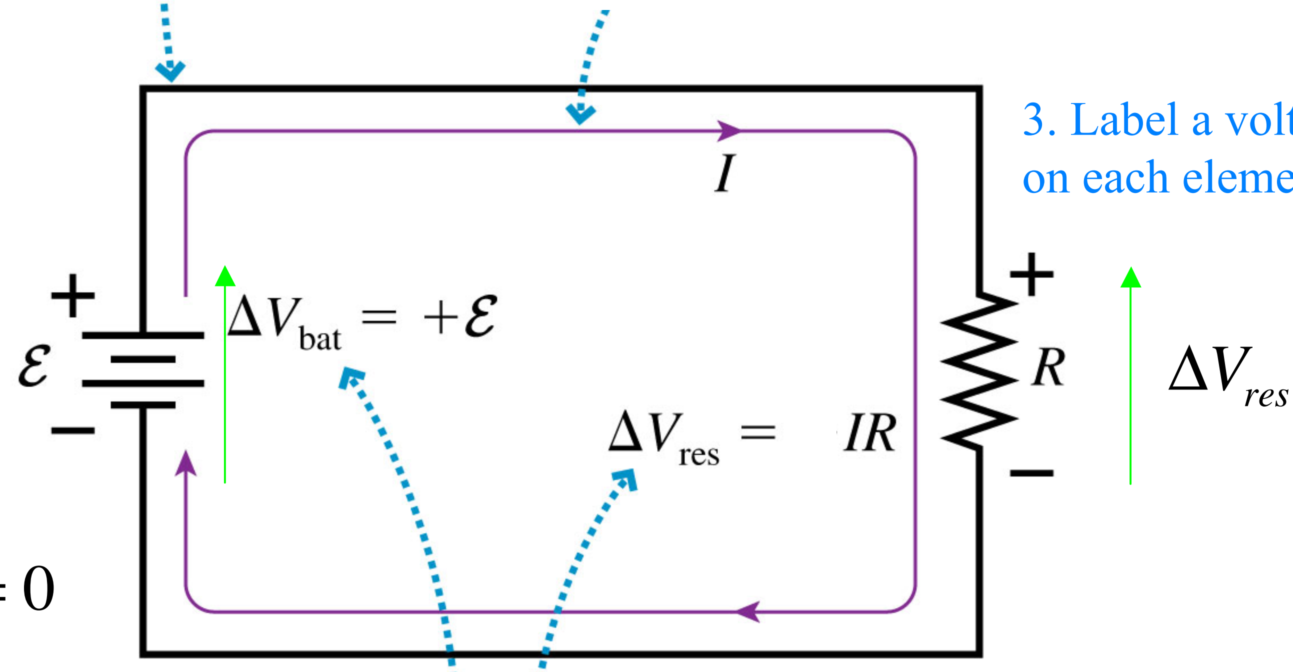
Kirchhoff's voltage law and Kirchhoff's current law are restatements of principles you already know. Can you name them?

Underlying Principle	Kirchhoff Law
?	KCL - junction rule
?	KVL- loop rule

1. Energy is conserved.
2. Momentum is conserved
3. Charge is conserved
4. Electrostatic force is conservative
5. Entropy increases

❶ Draw a circuit diagram.

❷ Pick a direction to label the current, its your choice.



3. Label a voltage on each element

4. Apply KVL

$$\Delta V_{Bat} - \Delta V_{res} = 0$$

5. Supply ΔV for each circuit element.

Ohm's Law for resistors.
Prescribed Voltage for batteries

$$\Delta V_{Bat} - \Delta V_{res} = 0$$

$$\varepsilon - IR = 0$$

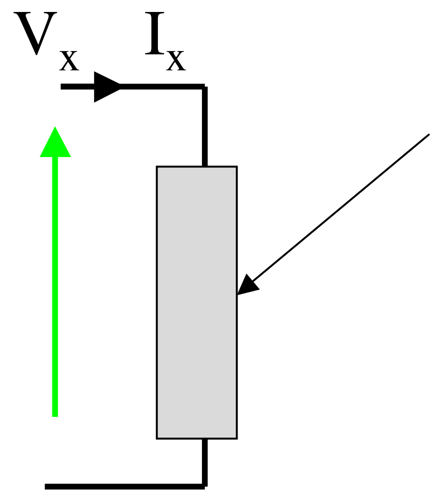
$$I = \varepsilon / R$$

Engineering Convention for Labeling Voltages and Currents

1. Pick one terminal and draw an arrow going in.

2. Label the current I_x.

3. Label the Voltage at that terminal V_x. This is the potential at that terminal relative to the other terminal.



Two terminal device
Resistor
Capacitor
Inductor (not discussed)
Ideal Battery

Device laws

$$V_R = RI_R$$

$$I_C = dQ / dt = CdV_c / dt$$

$$V_B = const$$ Independent of I_B

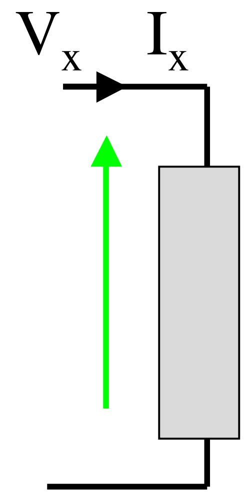
A Word about Voltage and Current

Voltage is “across”.

Current is “through”.

Voltage is the potential difference between the two terminals.

Current is the amount of charge per unit time flowing through the device.



If you catch yourself saying:

“Voltage through..”.

or

“Current across…”.

You are probably confused.

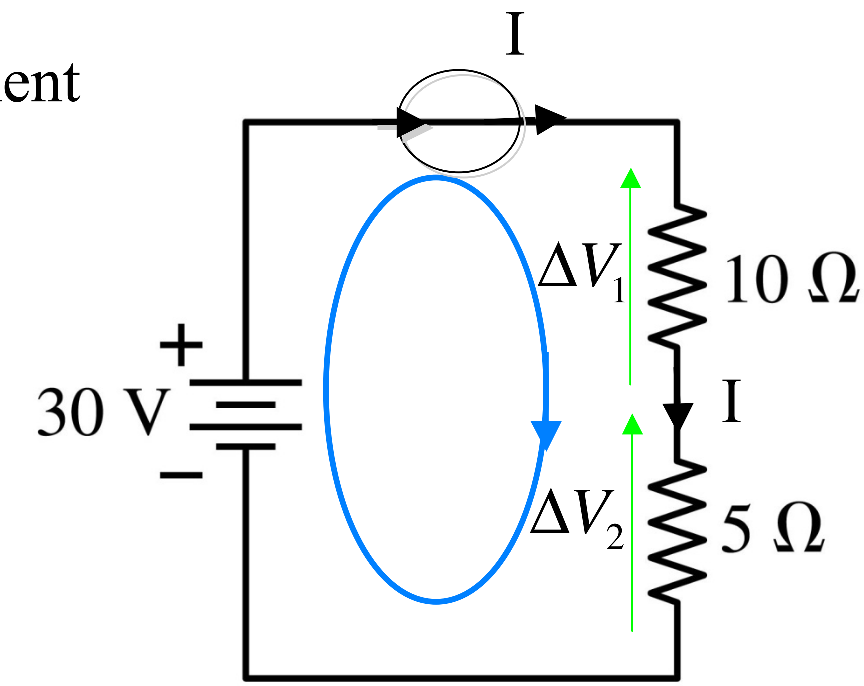
Now apply the rules to this circuit

KCL: same *I* through each element

Apply KVL

What is the current through each resistor?

What is the voltage across each resistor?



Whiteboard, TA & LA

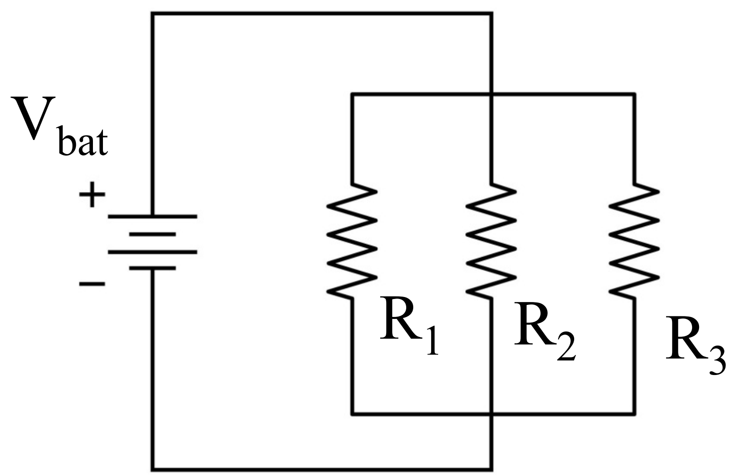
Parallel Circuits

KCL at junctions?

How many loops?

What can you say about the voltage across each resistor?

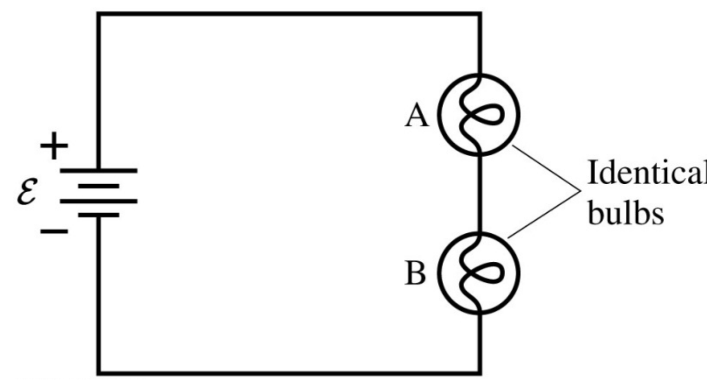
What is the total current?



V_{bat} = 10V, R1=10 ohms, R2=5 ohms, R3=2 ohms.

Whiteboard, TA & LA

- The figure shows two identical lightbulbs in a circuit.
- The current through both bulbs is *exactly the same! (KCL)*
- It's not the current that the bulbs consume, it's *energy.*



- The battery creates a potential difference, which supplies potential energy to the charges.
- As the charges move through the lightbulbs, they lose some of their potential energy, transferring the energy to the bulbs.

- The power <u>supplied</u> by a battery is (where *I* is current <u>out</u> of Battery):

$$P_{\text{bat}} = I\mathcal{E} \qquad \text{(power delivered by an emf)}$$

- The units of power are J/s or W.
- The power <u>dissipated</u> by a resistor is:

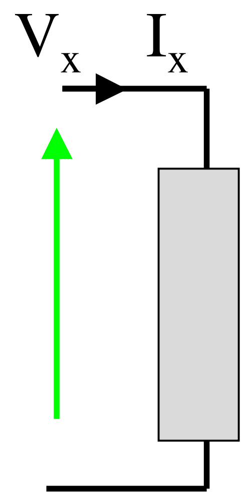
$$P_{\text{R}} = \frac{dE_{\text{th}}}{dt} = \frac{dq}{dt}\Delta V_{\text{R}} = I\,\Delta V_{\text{R}}$$

- Or, in terms of the potential drop across the resistor:

$$P_{\text{R}} = I\,\Delta V_{\text{R}} = I^2R = \frac{(\Delta V_{\text{R}})^2}{R} \qquad \text{(power dissipated by a resistor)}$$

Electric Power

- The rate at which electric energy <u>enters</u> a device is: $P_x = I_x V_x$

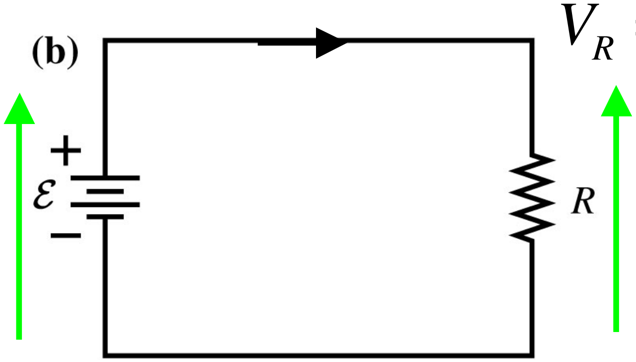


Is it possible to have negative power? If so, what would this mean?**

$$I = \mathcal{E} / R$$

$$V_B = \mathcal{E}$$

$$V_R = \mathcal{E}$$



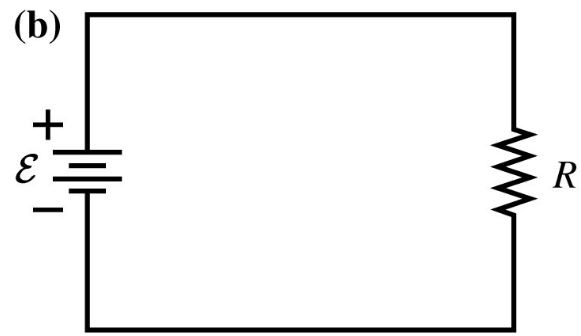
What is the rate power enters the resistor?, the battery?

Whiteboard, TA & LA

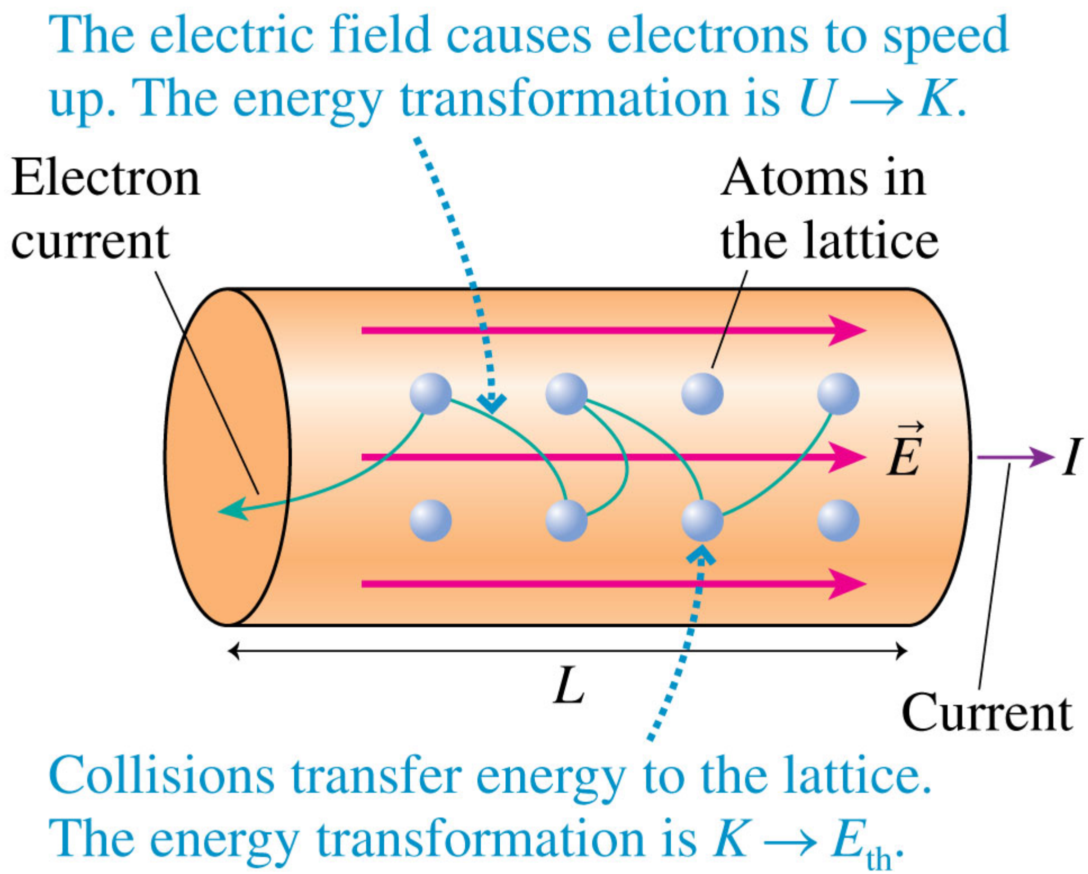
A 90 Ω load is connected to a 120 V battery. How much power is delivered by the battery?

$I = 120\ V / 90\ \Omega = 1.33\ A$

$P = (1.33\ A)(120\ V) = 160\ W$

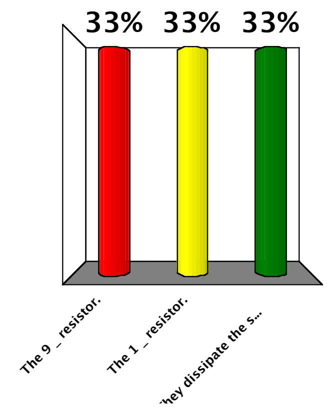


Whiteboard, TA & LA



A current-carrying resistor dissipates power because the electric force does work on the charges.

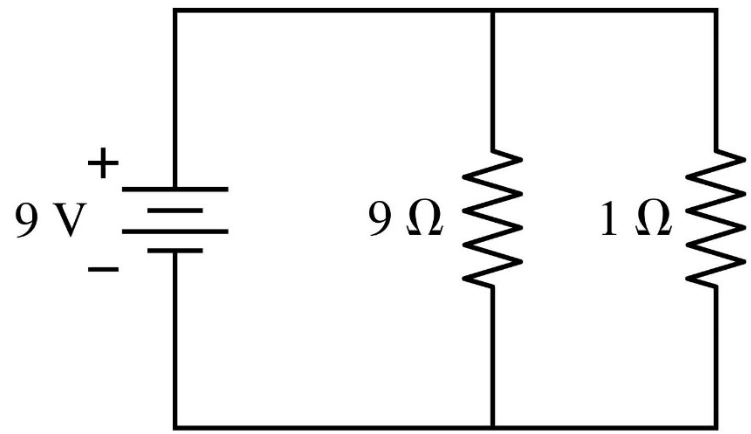
Which resistor dissipates more power?



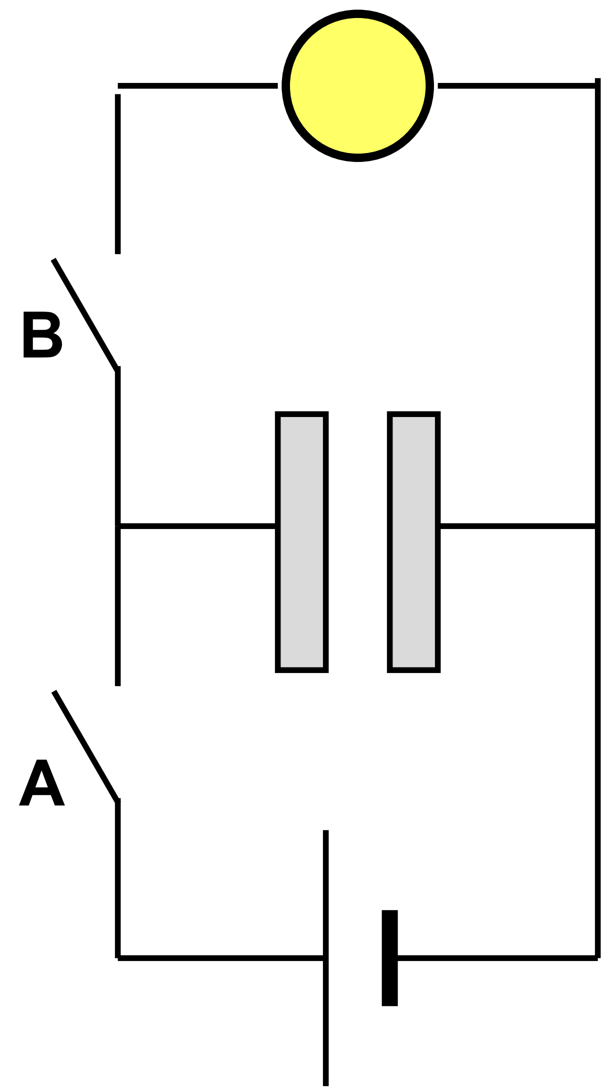
A. The 9 Ω resistor.

B. The 1 Ω resistor.

C. They dissipate the same power



- Suppose we:
  - Close A for a few seconds
  - Open A
  - Close B
- What happens to the bulb?
  - 1. It stays off.
  - 2. It stays on after you close A
  - 3. It stays on after you close B
  - 4. It flashes when you close A
  - 5. It flashes when you open A
  - 6. It flashes when you close B



Electric Fields in Materials - Screening

What happens when we attempt to introduce/apply an electric field in a material?

It depends on the material.

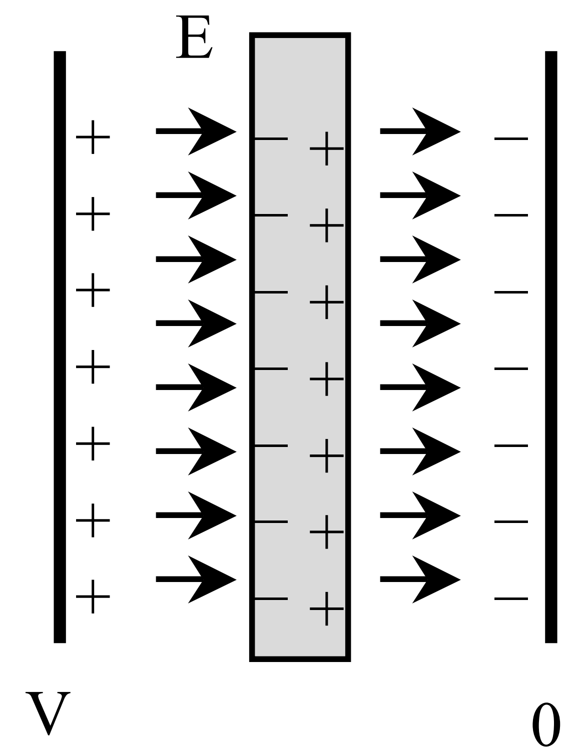
Conductors - current flows (until E is reduced to zero)

Insulators - material becomes polarized

Ionic solutions (plasmas) - some of both of the above

Conductors

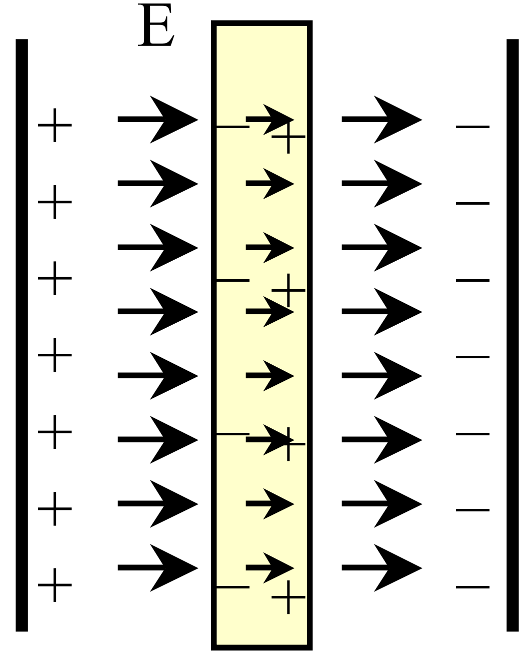
- Putting a conductor inside a capacitor eliminates the electric field inside the conductor.
- The distance, d', used to calculate the ΔV is only the place where there is an E field, so putting the conductor in reduces the ΔV for a given charge.



$$C = \frac{1}{4\pi k_C}\frac{A}{d'}$$

Consider what happens with an insulator

- We know that charges separate even with an insulator.
- This reduces the field inside the material, just not to 0.
- The field reduction factor is defined to be κ - relative dielectric constant.



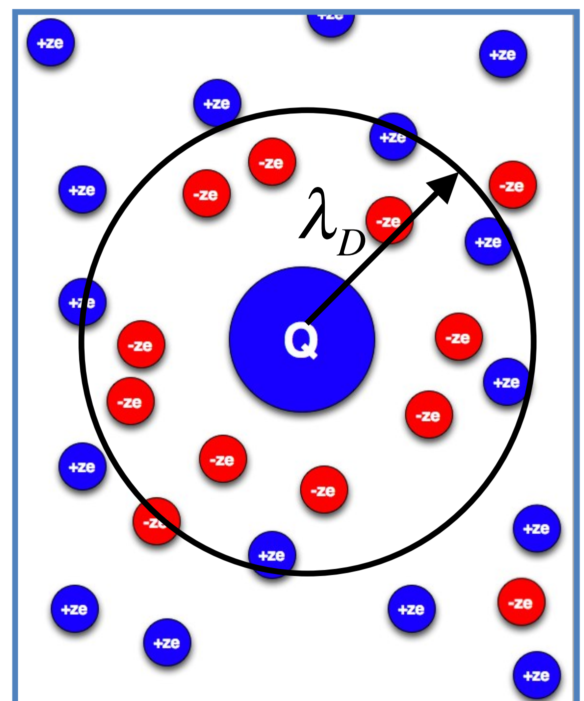
$$E_{\text{inside material}} = \frac{1}{\kappa} E_{\text{if no material were there}}$$

Charged objects in Conducting Fluids

- What happens if place a charged object into a neutral fluid with ions?
  - Opposite charged ions are attracted to object
  - Like charged ions are repelled
  - Thermal energy keeps ions moving

Net charge in a sphere of radius λ_D is close to zero.

$$\lambda_D^2 = \frac{\varepsilon_0 \kappa \, k_B T}{c_0 e^2 Z^2}$$

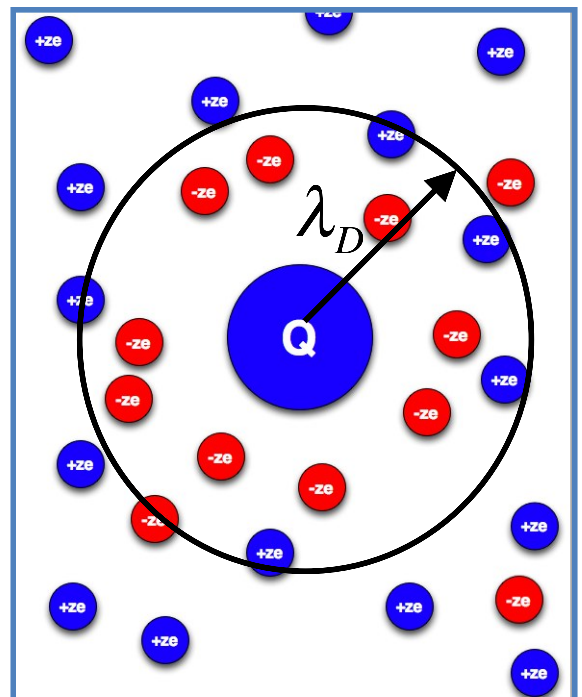


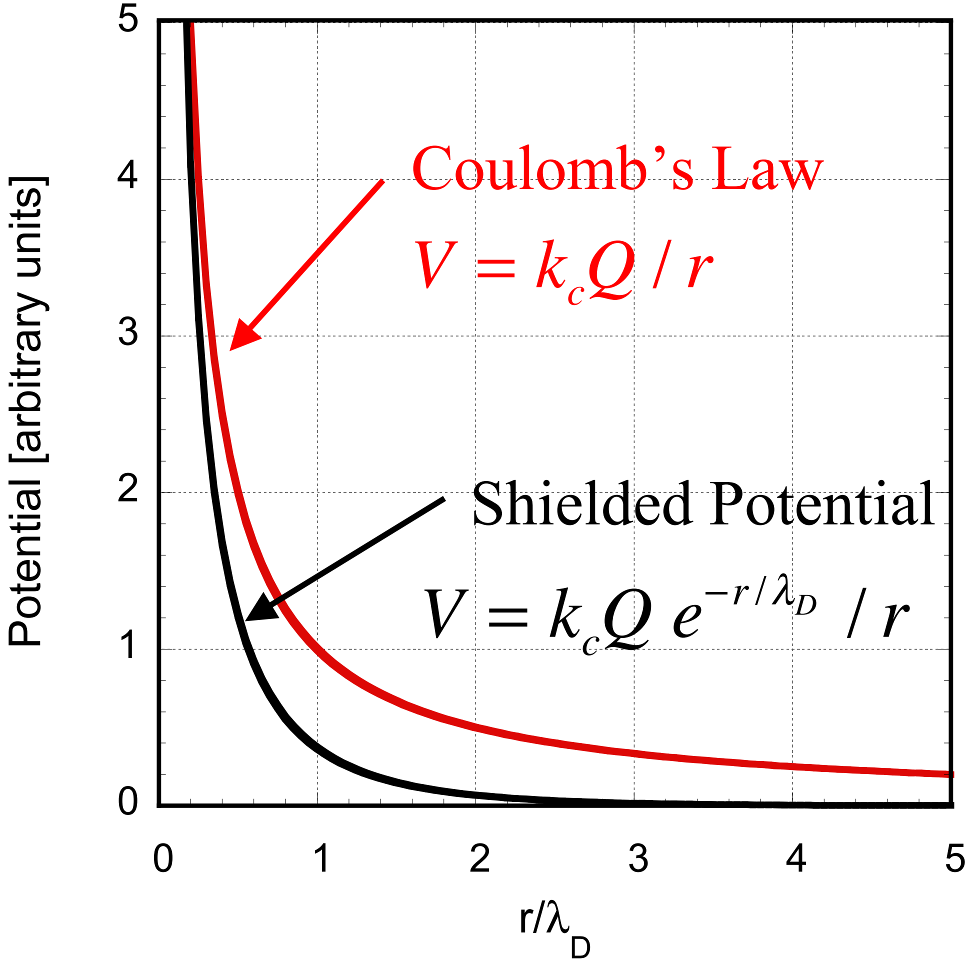
Charged objects in Conducting Fluids

Net charge in a sphere of radius λ_D is approximately zero.

$$\lambda_D^2 = \frac{\varepsilon_0 \kappa k_B T}{c_0 e^2 Z^2}$$

$k_B T$	Thermal energy (Joules)
$k_c = 1/4\pi\varepsilon_0$	Coulomb constant
c_0	Ionic concentration (m^{-3})
Z	Ionic charge state (an integer)
e	elementary charge
κ	relative dielectric constant





Shielding is due to Boltzmann distribution.

Balances kinetic and potential energy

Concentration of ions:

$$c_{\pm}(r) = c_0 \exp[\mp ZeV(r) / k_B T]$$

Concentration of positive and negative ions in Thermal Equilibrium: Boltzmann distribution

positive ions repelled from region of positive potential

$$c_+(r) = c_0 \exp[-ZeV(r)/k_BT]$$

potential energy of + ion

$$V \uparrow \quad c_+ \downarrow$$

negative ions attracted to region of positive potential

$$c_-(r) = c_0 \exp[+ZeV(r)/k_BT]$$

$$V \uparrow \quad c_- \uparrow$$

Multiple Ion Species

MAMMALIAN CELL

Ion	Cell (mM)	Blood (mM)
K^+	139	4
Na^+	12	145
Cl^-	4	116
HCO_3^-	12	29
X^-	138	9
Mg^{2+}	0.8	1.5
Ca^{2+}	<0.0002	1.8

⇩

$$\lambda_D \approx 1\, nm$$

Sum over all ions

$$\frac{1}{\lambda_D^2} = \sum_{ions} \frac{c_i e^2 Z_i^{\,2}}{\varepsilon_0 \kappa k_B T}$$

$$\lambda_D[cm] = \frac{9.6 \times 10^{-7} \left[T\,[eV]\right]^{1/2}}{\left[\sum_{ions} c_i [mM] Z_i^2\right]^{1/2}}$$

Nernst Potential

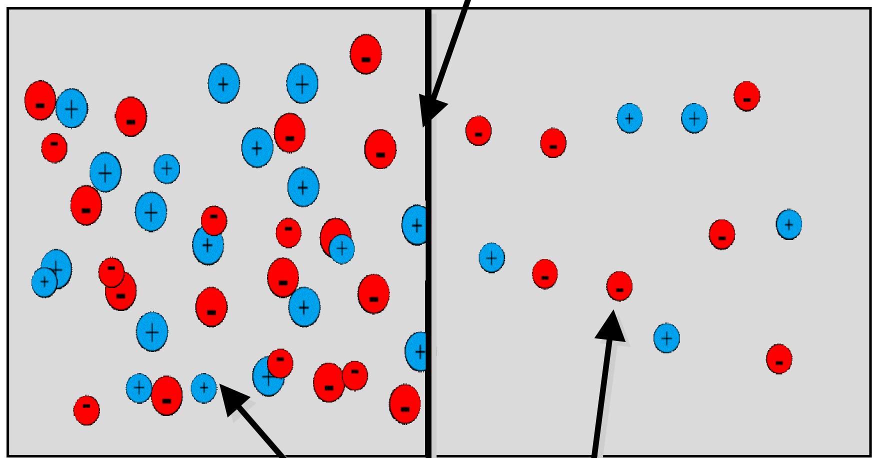
Difference in electrostatic potential across a membrane. c_1 and c_2 are concentrations of ions on either side of the membrane

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

Nernst Potential

Semi permeable membrane allows blues (+) to pass but not reds (-)

Higher concentration maintained on this side, c_1

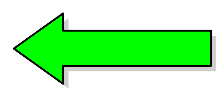


Lower concentration, c_2

More blues (+) here than here

Flux of blues (+) due to random walk

Electric field due to excess blues

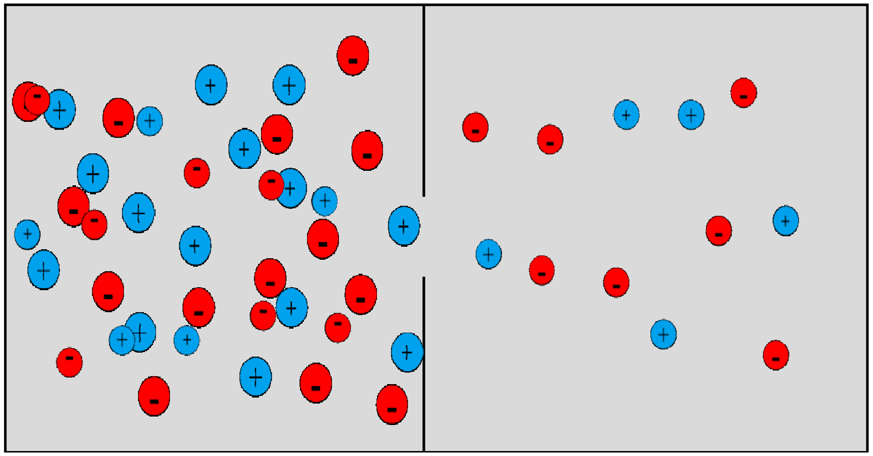


Potential across membrane

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

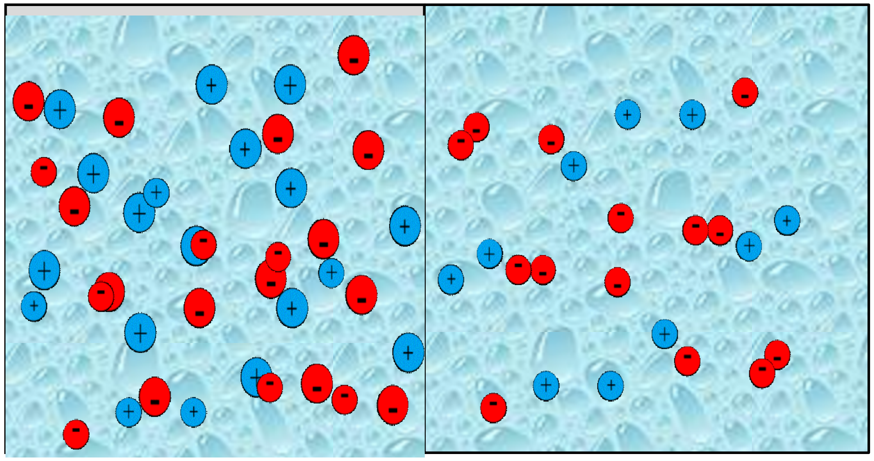
Two boxes one starting with 18 red and blue molecules, the other with 6 of each kind. Membrane has a channel THAT IS ONLY PERMEABLE to blue molecules. At the start (shown)

1. Blue molecules are equally likely to enter the channel on each side
2. Blue molecules are 3 times more likely to enter the channel on the right
3. Blue molecules are 3 times more likely to enter the channel on the left
4. Not enough information



Two boxes starting with different concentrations of ions are separated by a Membrane that is only permeable to blue (+) molecules. When concentrations come to equilibrium as shown. Which is true

1. Potential V on the right is <u>positive</u> w.r.t. the left
2. Potential V on the right is <u>negative</u> w.r.t. the left
3. There is no potential difference.
4. Not enough information to say anything about potential.



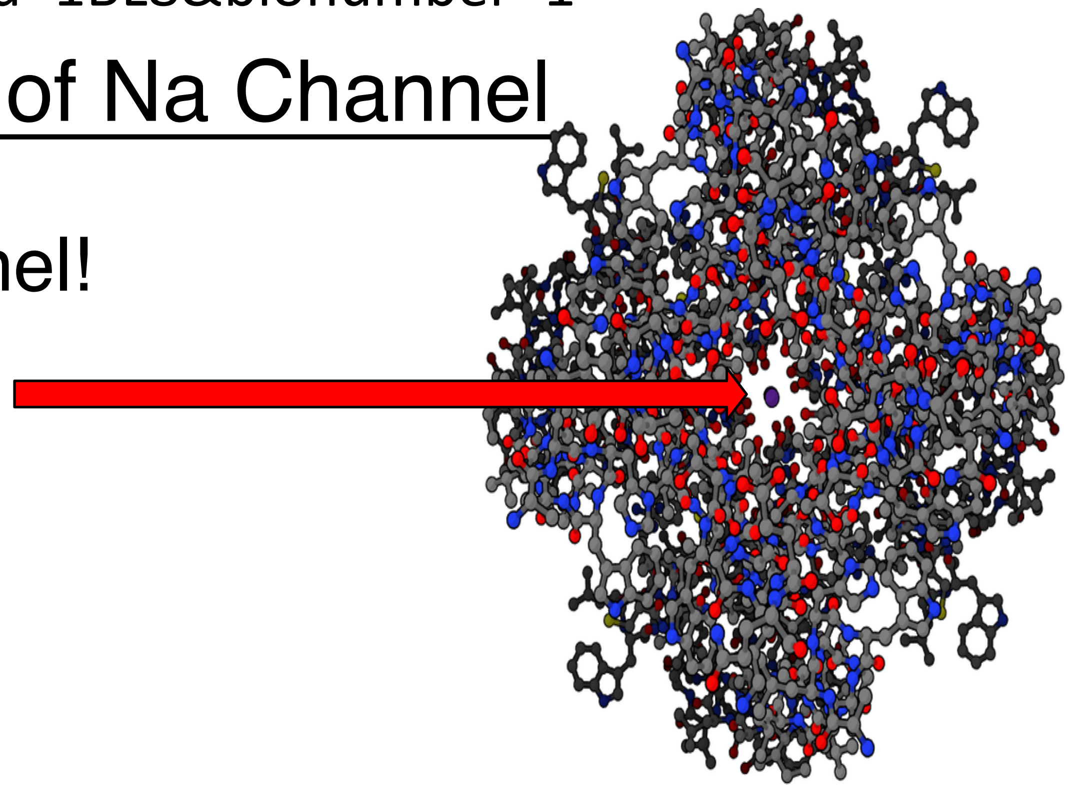
Biology Background:

Ion Channels that only let Potassium through (channels for other types of ions also exist)

http://www.rcsb.org/pdb/explore/jmol.do?structureId=1BL8&bionumber=1

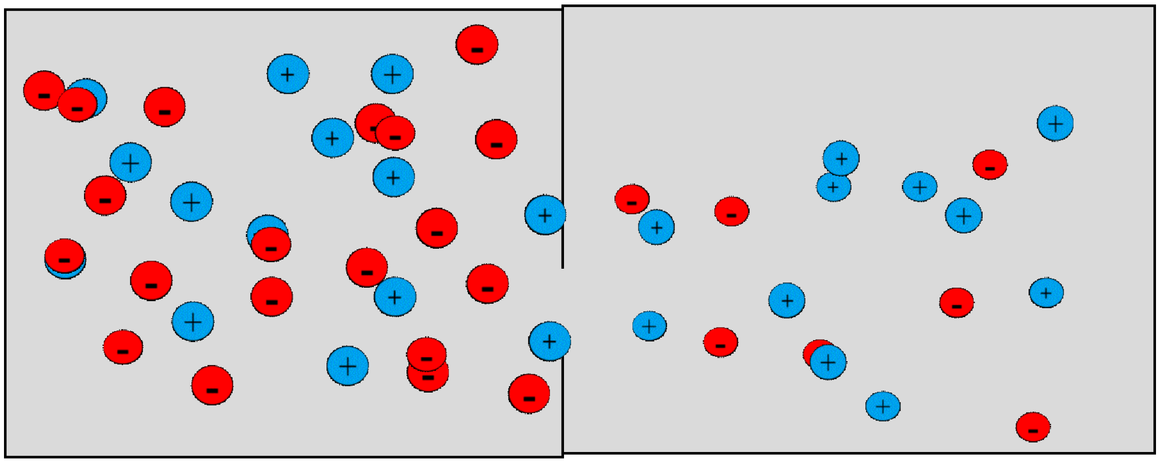
Top view of Na Channel

Ion in Channel!



Below you see a membrane that has a channel that is permeable for one of the ions only.

1. The membrane is permeable to positive ions
2. The membrane is permeable to negative ions
3. Depends on the initial distribution of ions
4. other



Sketch equilibrium state

- Electric fields?
  1. None
  2. Near membrane
  3. everywhere

Quantifying the electrostatic energy penalty: how much more (or less) likely is it for an ion to have an electrostatic energy of E_1 compared to E_0

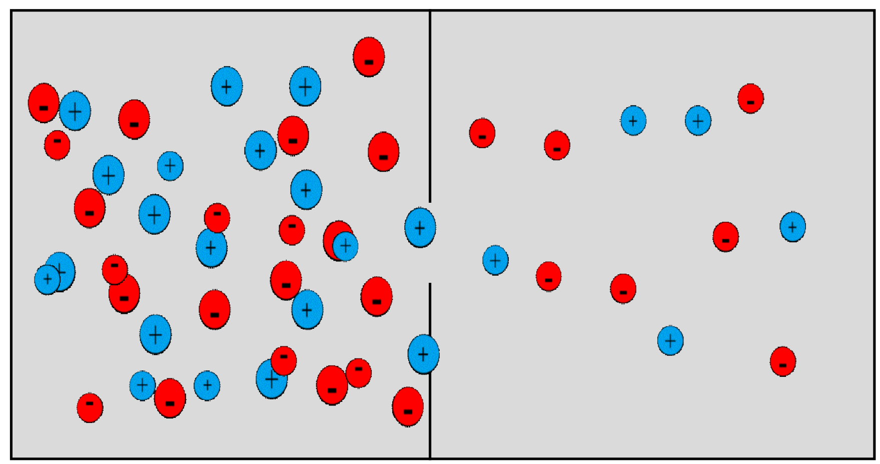
1. $$P = e^{\frac{E_1 - E_0}{k_B T}}$$

2. $$P = e^{-\frac{E_1 - E_0}{k_B T}}$$

3. $$P \sim e^{\frac{E_1}{E_0}}$$

4.

5. Need more information



Nernst Equation

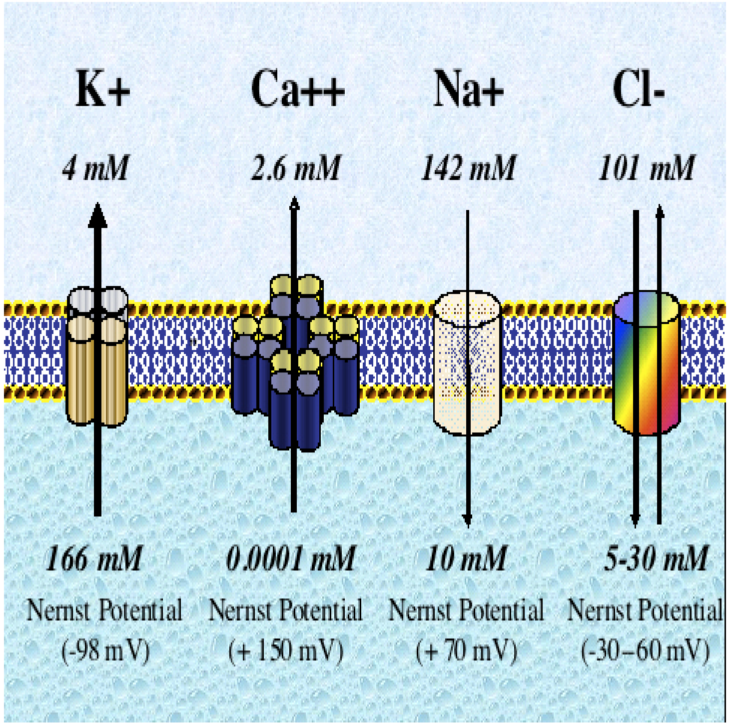
- Diffusion: Concentration gradient in the presence of ion channel -> ions flow to equilibrate concentration
- Electrostatic potential: only one ion species can flow -> electrostatic potential builds up -> makes it less likely for ions to keep flowing across channel

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

Nernst

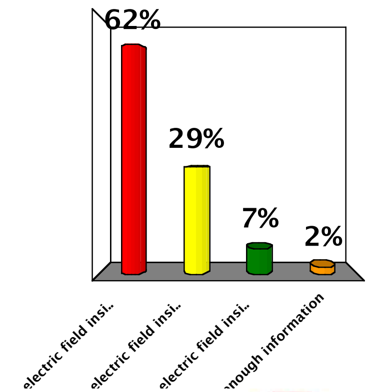
- Depends on the potential difference
- Requires selective ion channels

Ions in a Cell



http://www.dev.urotoday.com

What happens if we fill half the gap between plates with a conductor?

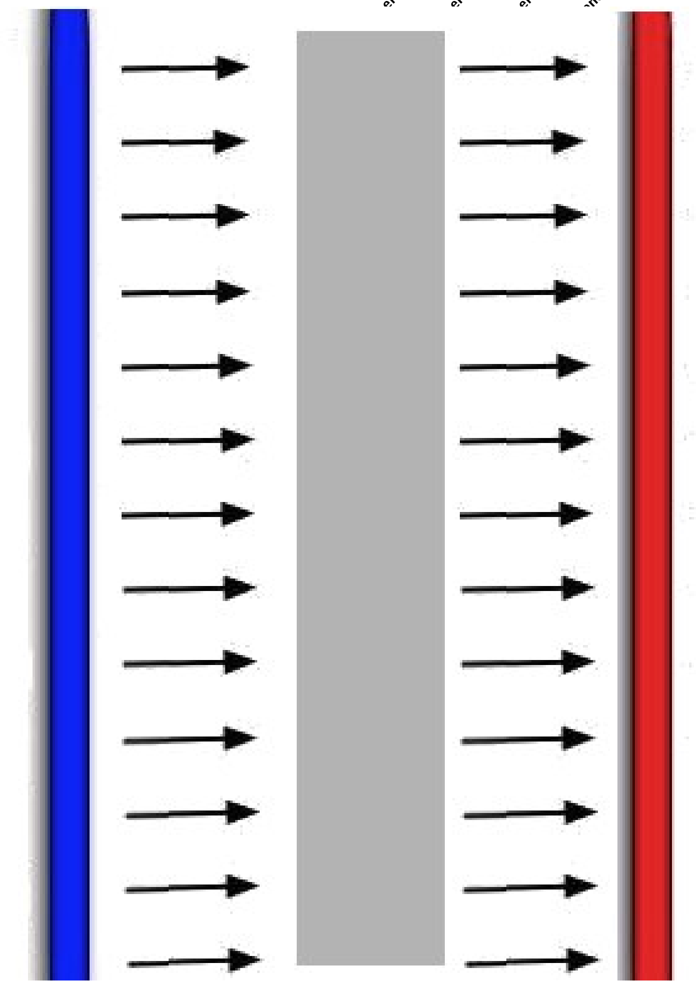


A. The electric field inside the conductor is the same as outside

B. The electric field inside the conductor is opposite to the field outside

C. The electric field inside the conductor is zero

D. Not enough information



As the lightbulb flashes which of the following is true

1. **Positive** charges move through the lightbulb, they move at roughly constant speed
2. **Positive** charges move through the lightbulb, they move slowest at the lightbulb
3. **Negative** charges move through the lightbulb, they move at roughly constant speed
4. **Negative** charges move through the lightbulb, they move slowest at the lightbulb
5. None of the above