Distributions of Charge

Often we encounter distributions of charge - not single charges

<u>Examples:</u>

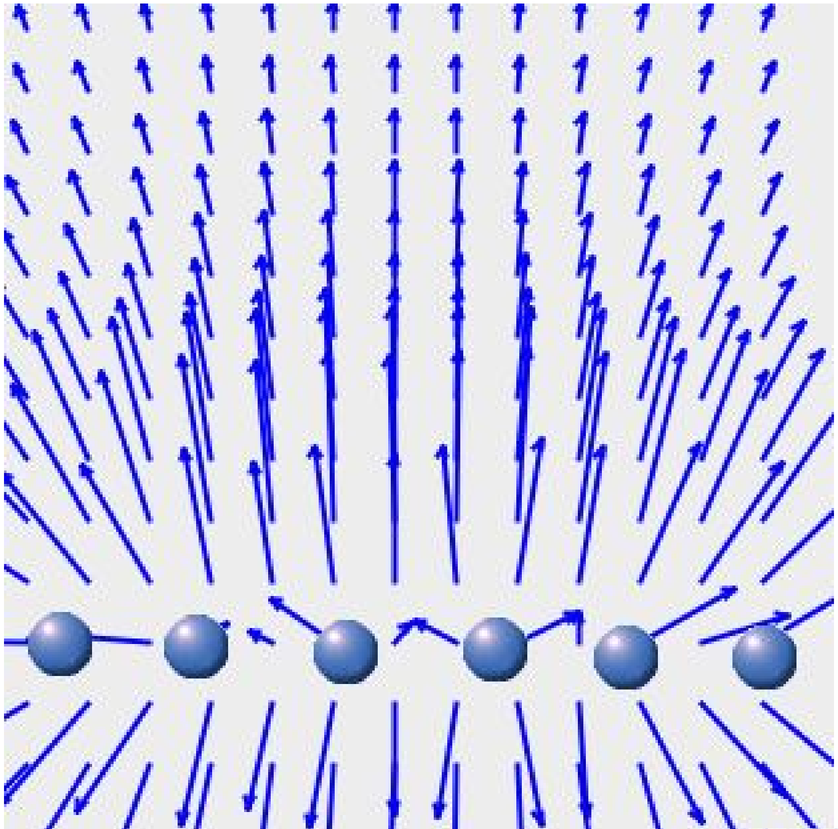
Charges on wires in circuits

Charges in electronic components

Charges in the ionosphere and solar wind

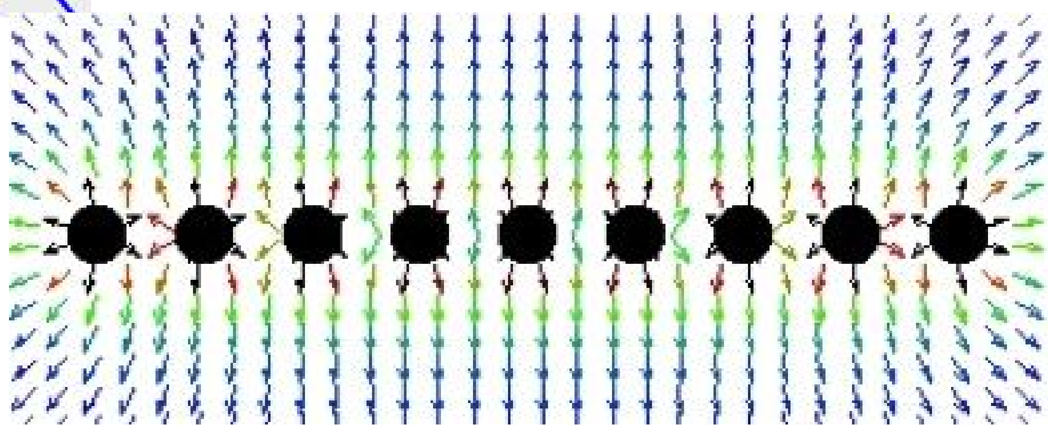
Charges separated by membranes in cells

Charges in ionic solutions



Line Charge:

A very long row of charges placed close together.



What kind of things in life are similar to line charges? Electric wires?

Yes

// How are line charges relevant to biological systems? //

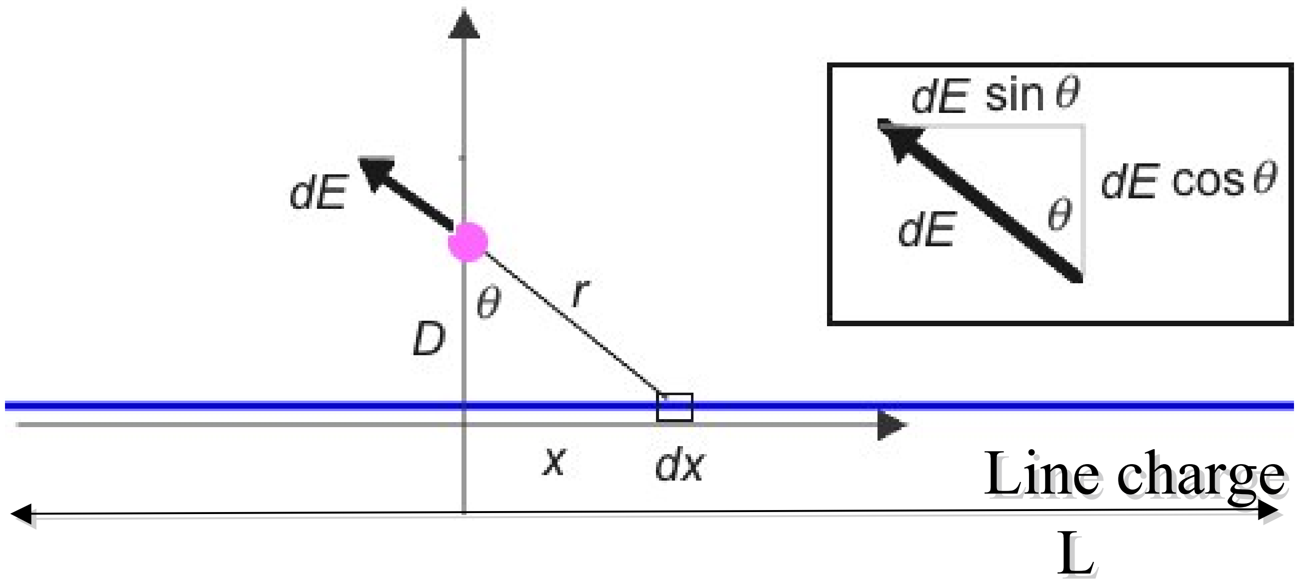
Good question,

Sheets of charge describe distributions near membranes
or on conductors in solutions

I don't understand the math done for the section on "How does it depend on stuff?" and am confused as to what it means;

Electric field due to a line charge

$$E = \frac{2k_c\lambda}{D}$$

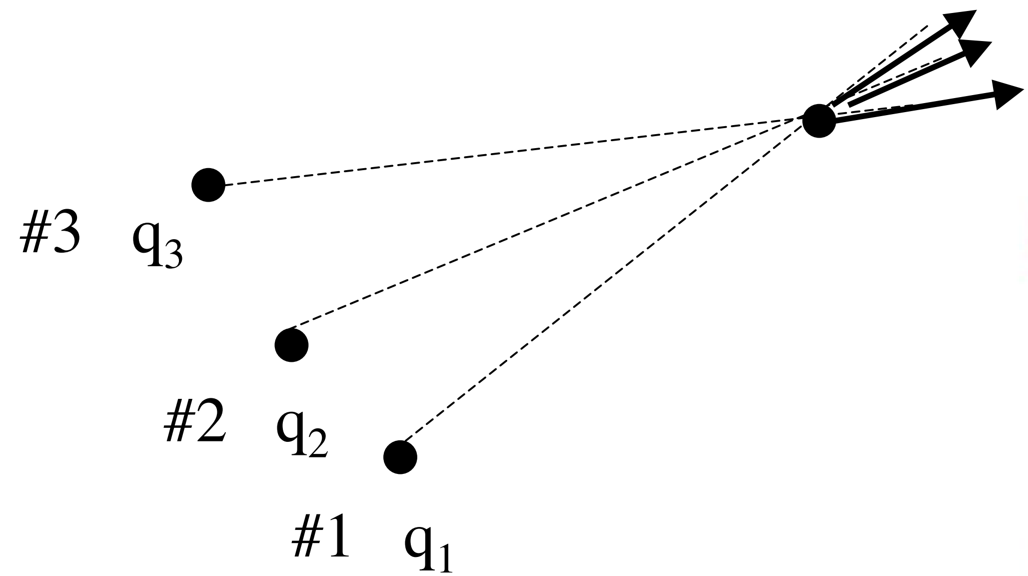


Suppose you are a distance D from a row of charges of length L. Suppose there are λ coulombs of charge (each of charge q) per meter.

In terms of L, D, q, and λ, when can we consider this an infinite line charge?

Whiteboard, TA & LA

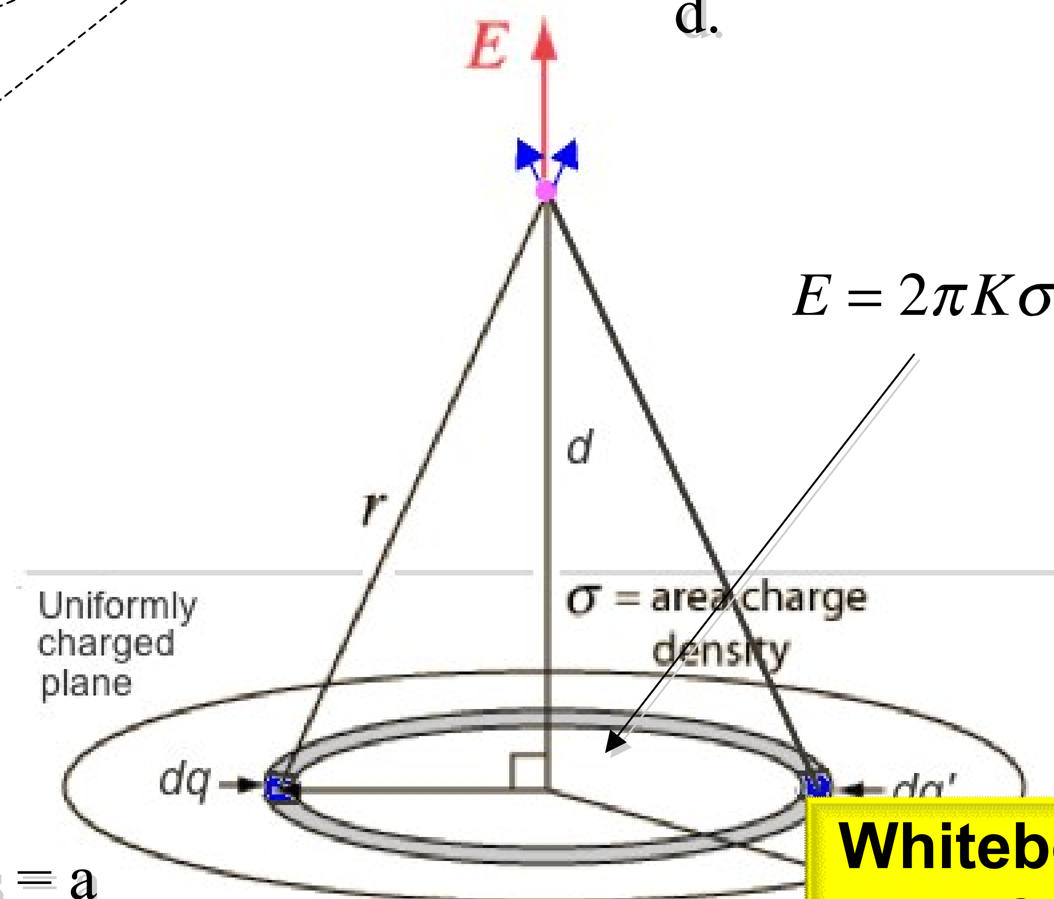
Field due to a sheet of charge



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \sum_{q_j} \frac{K q_j}{r_j^2} \hat{\mathbf{r}}_{\mathbf{j}}$$

Draw a graph-sketch of how E depends on d.

$$E = 2\pi K \sigma$$

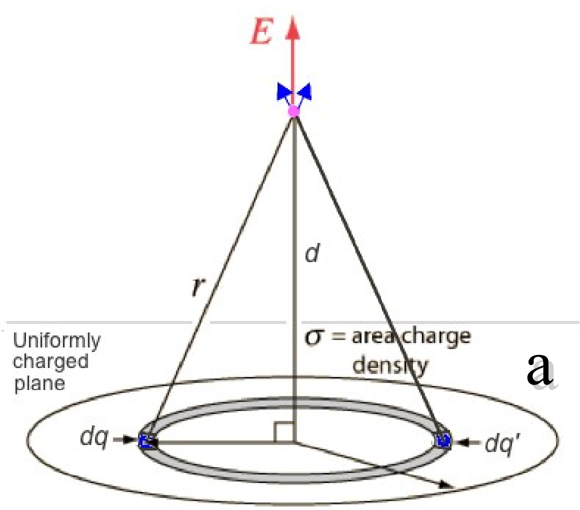


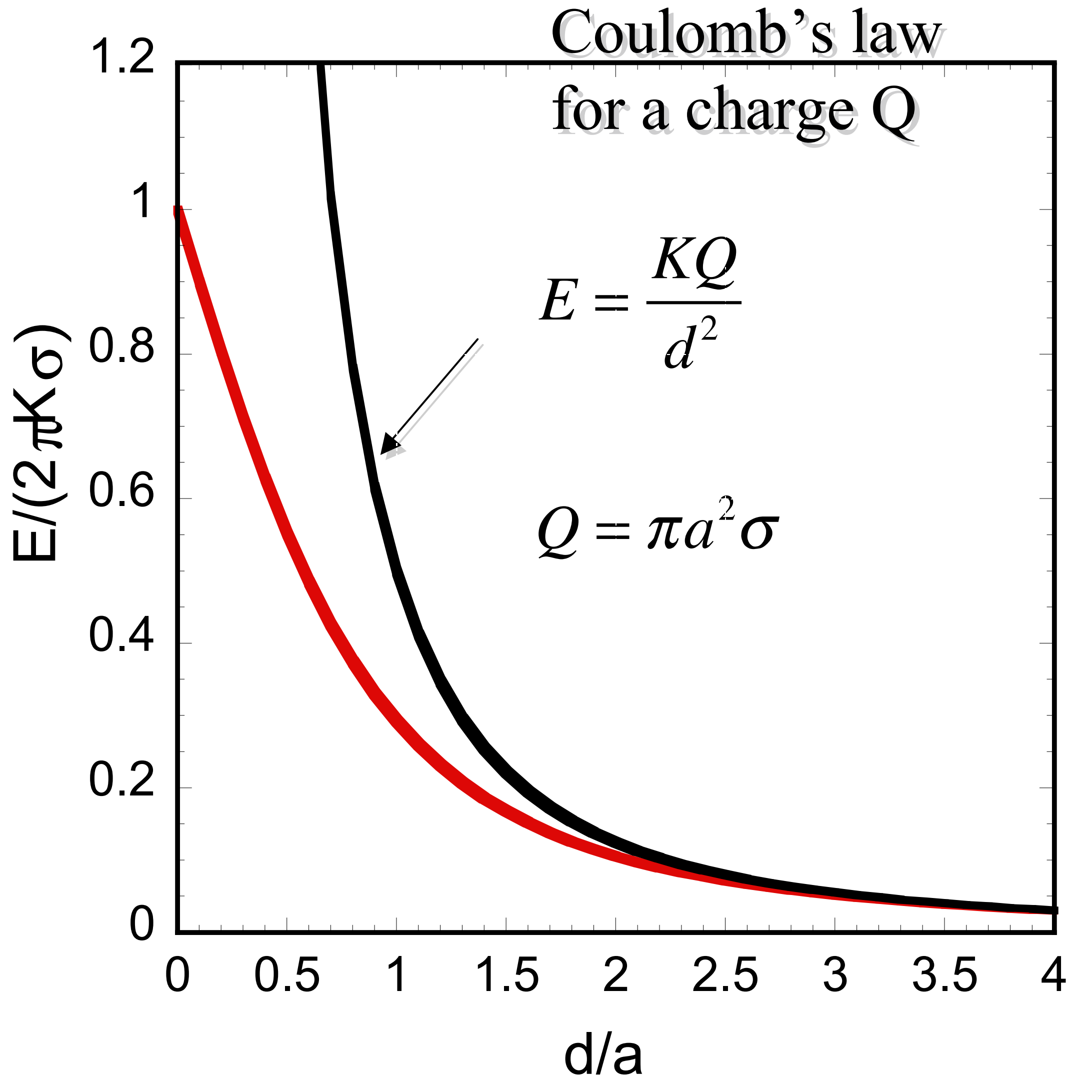
Suppose radius = a

Whiteboard, TA & LA

Field above a disc of Charge: this is a doable problem

$$E = 2\pi K\sigma\left[1 - \frac{d}{\sqrt{d^2 + a^2}}\right]$$

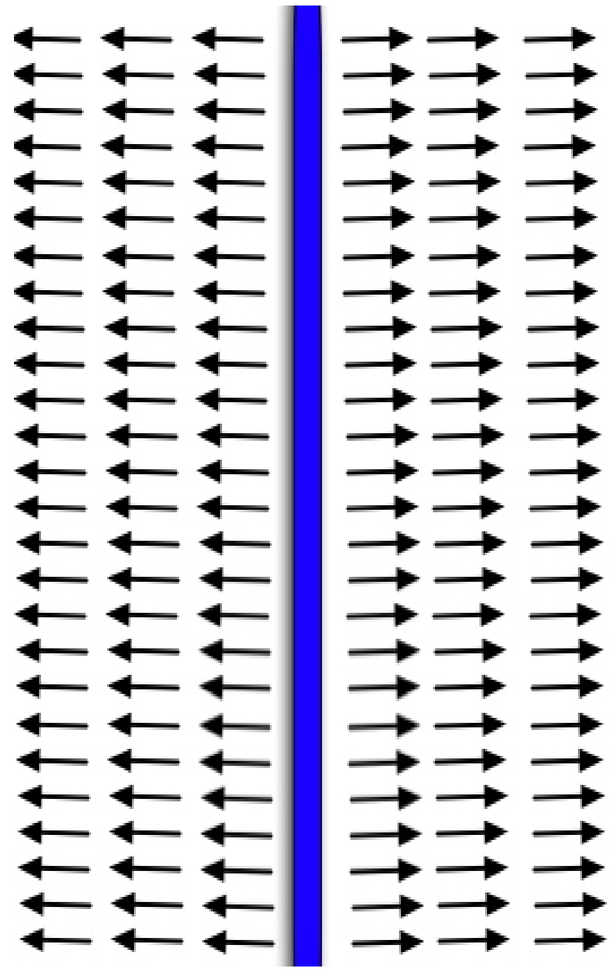




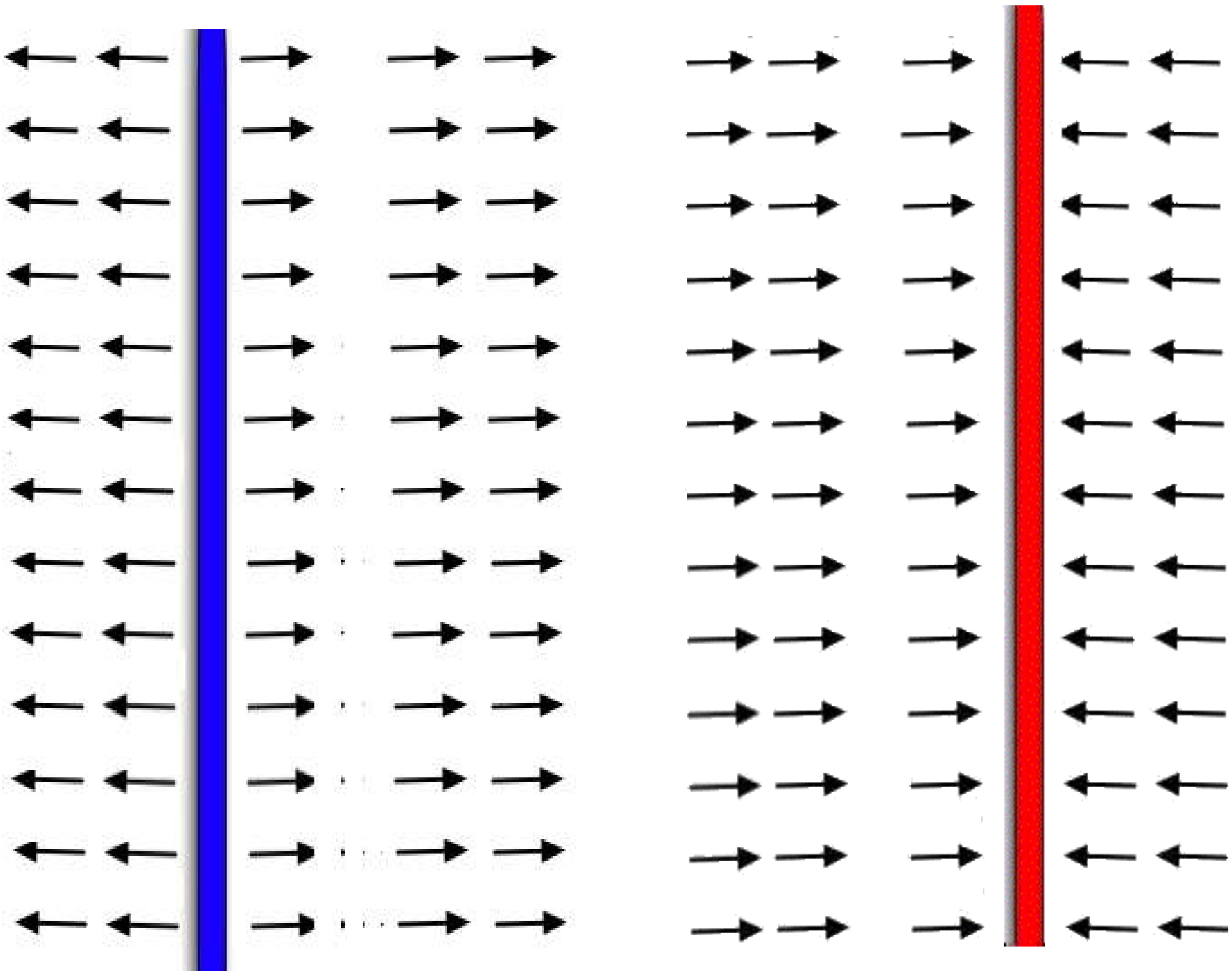
When would we model the field as a sheet instead of a line? //
How do you know if it is a sheet charge or line charge?

The sheet of charge

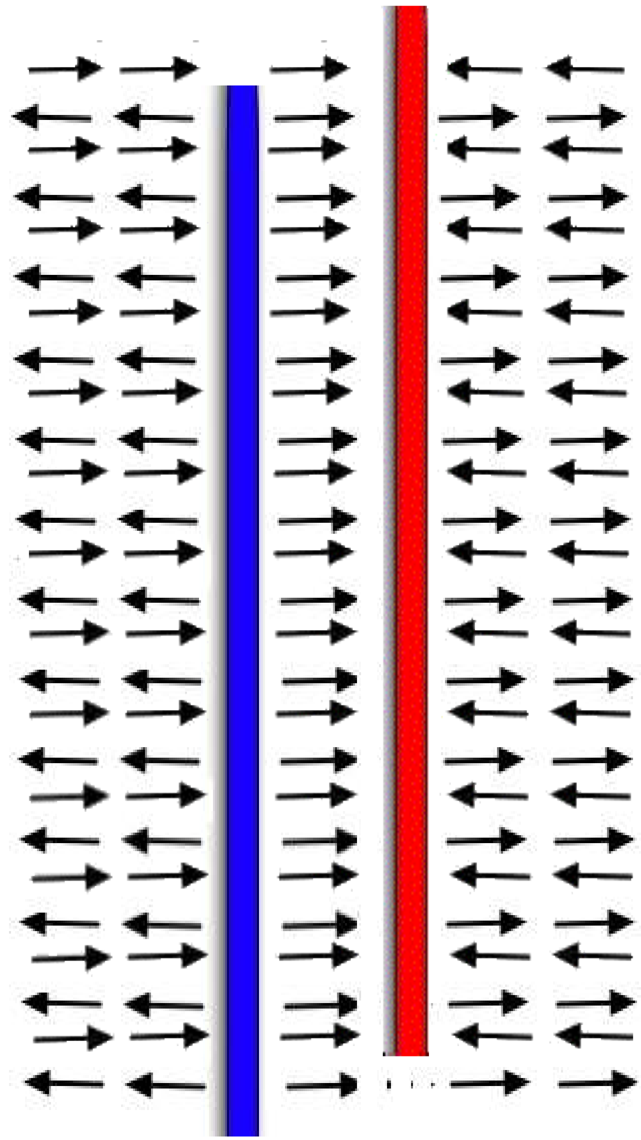
- Field is constant, pointing away from positive sheet, towards negative sheet.
- Constant!!? How can that be?



Two sheets of charge



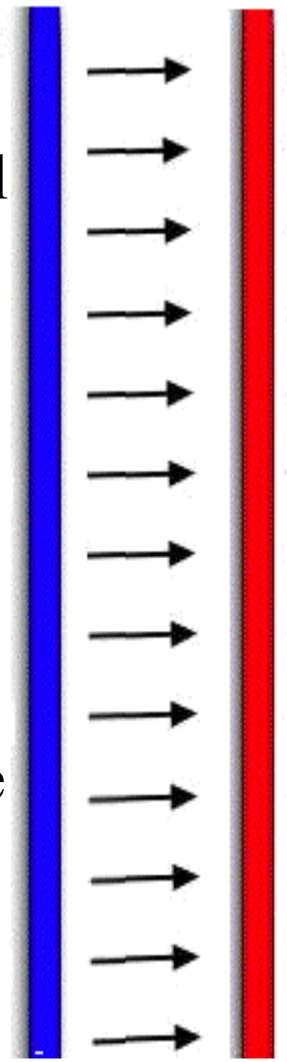
Two sheets of charge



Result

The fields of the two plates cancel each other on the outside.

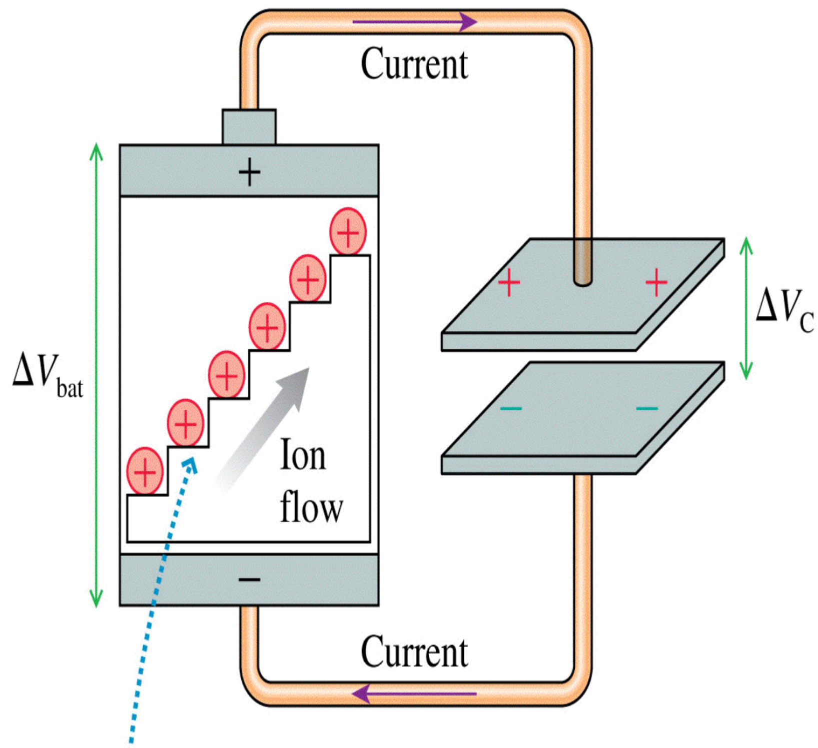
The fields of the two plates add on the inside, producing double the field of a single plate.



The fields of the two plates cancel each other on the outside.

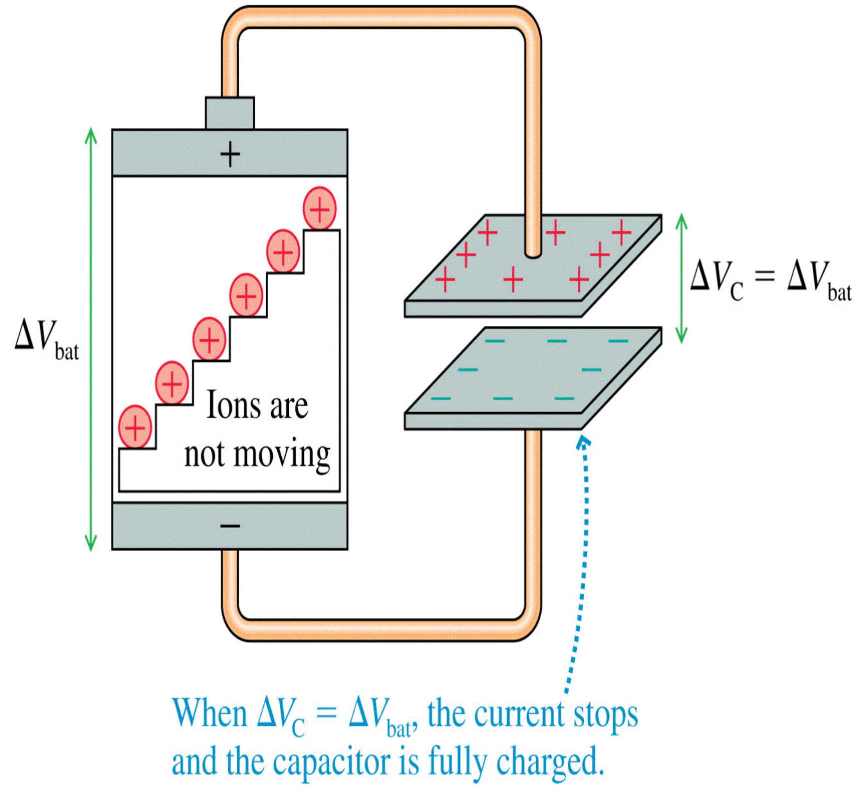
Capacitors

- The figure shows a capacitor *just after* it has been connected to a battery.
- Current will flow in this manner for a nanosecond or so until the capacitor is fully charged.



The charge escalator moves charge from one plate to the other. ΔV_C increases as the charge separation increases.

- The figure shows a *fully charged* capacitor.
- Now the system is in electrostatic equilibrium.



- Capacitance always refers to the charge per voltage on a *fully charged* capacitor.

- The ratio of the charge Q to the potential difference ΔV_C is called the **capacitance** C:

$$C \equiv \frac{Q}{\Delta V_C} = \frac{A/4\pi}{k\downarrow C\, d} \qquad \text{(parallel-plate capacitor)}$$

- Capacitance is a purely *geometric* property of two electrodes because it depends only on their surface area and spacing.
- The SI unit of capacitance is the **farad**:

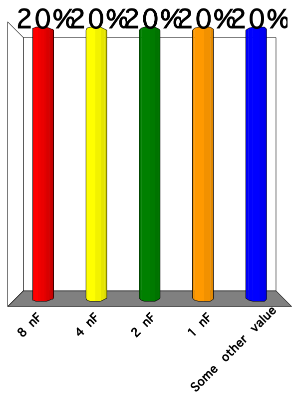
$$1 \text{ farad} = 1 \text{ F} \equiv 1 \text{ C/V}$$

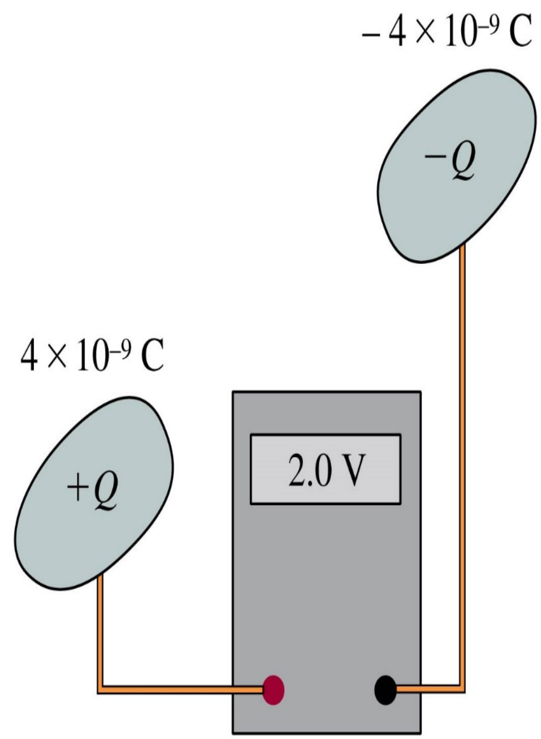
- **The charge on the capacitor plates is directly proportional to the potential difference bet**

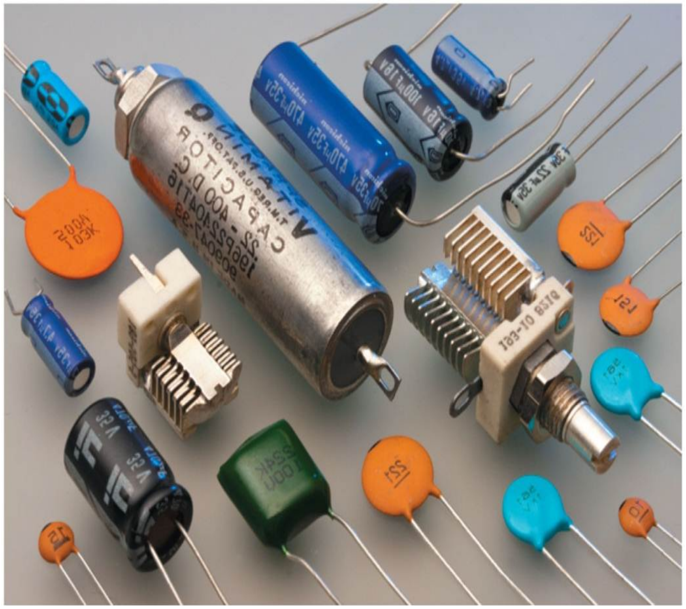
$$Q = C\,\Delta V_C \qquad \text{(charge on a capacitor)}$$

What is the capacitance of these two electrodes?

A. 8 nF
B. 4 nF
C. 2 nF
D. 1 nF
E. Some other value





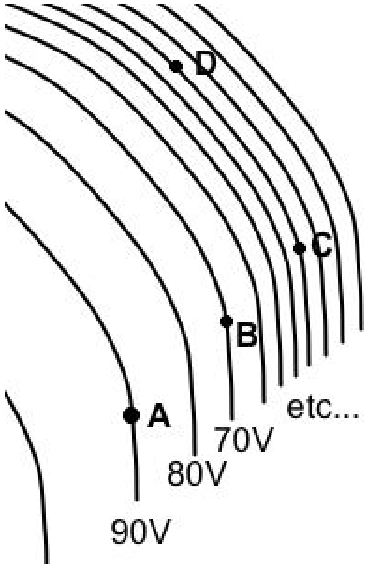


Capacitors are important elements in electric circuits. They come in a variety of sizes and shapes.

The keys on most computer keyboards are capacitor switches. Pressing the key pushes two capacitor plates closer together, increasing their capacitance.

Wed 3/12

Quiz review



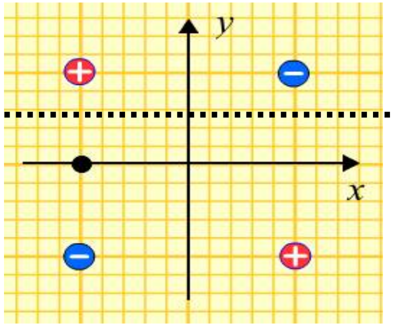
1.1 highest force?

1.2 comparing work C-A and D-A

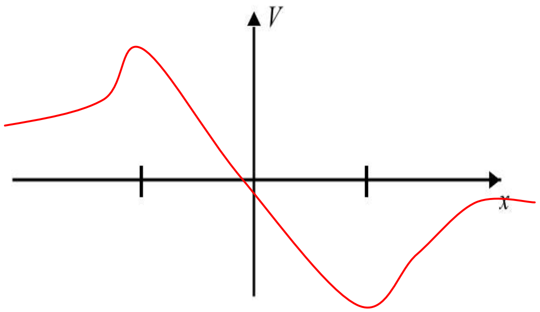
1.3 Movement of neg. charge at B

Quiz review

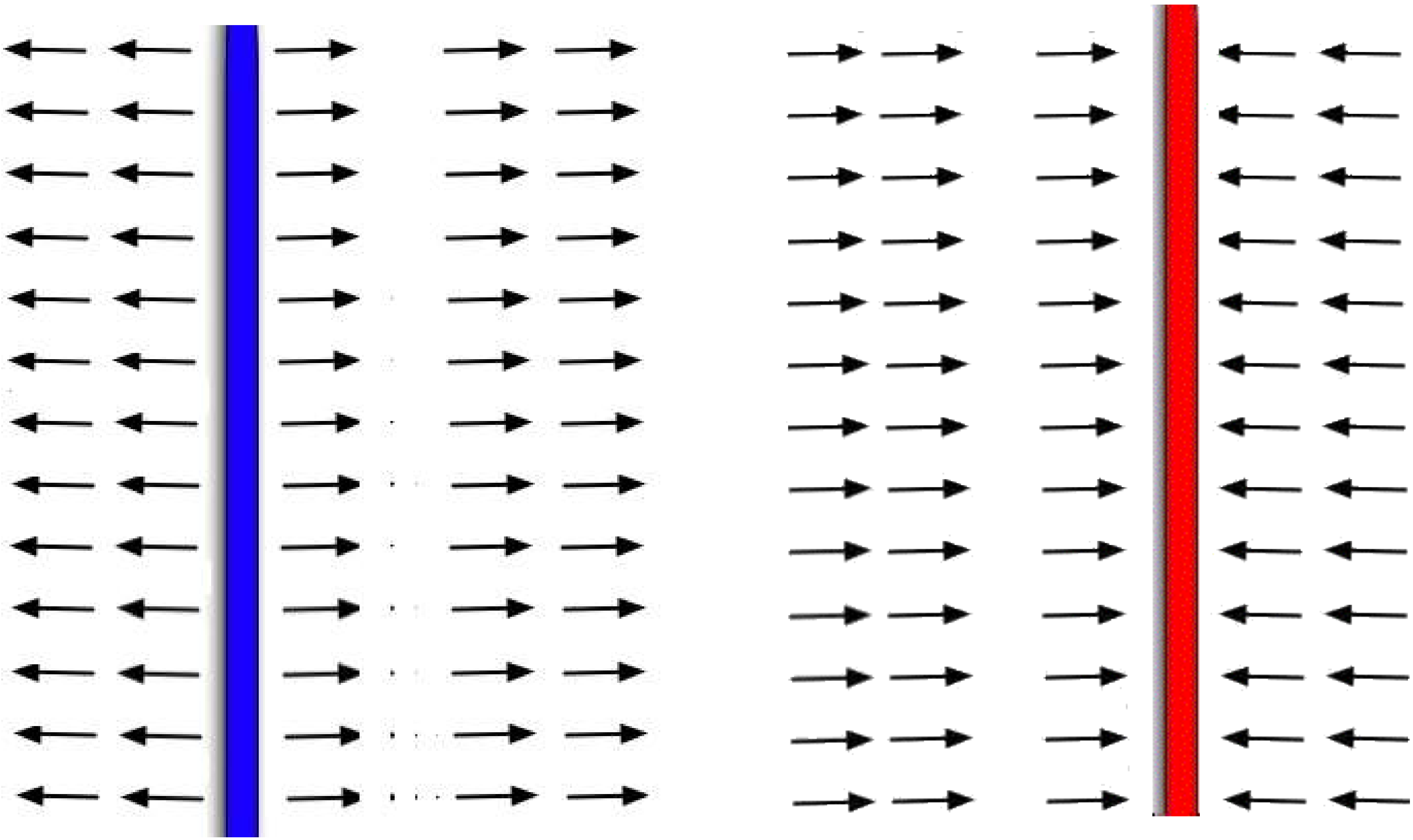
2.1 direction of field



2.2 sketch



Two sheets of charge



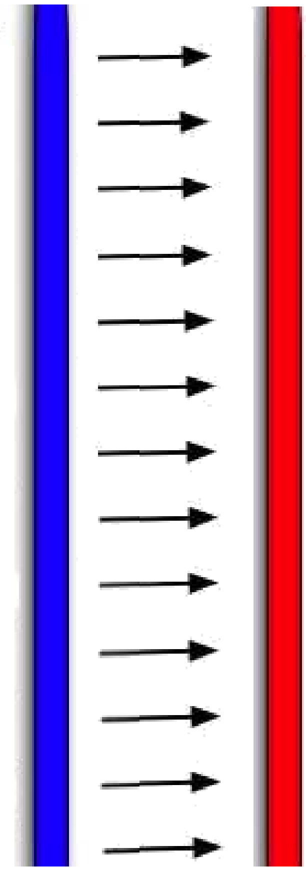
Capacitor Equations

$$\Delta V = E\Delta x = Ed$$

$$E = 4\pi k_C \sigma = 4\pi k_C \frac{Q}{A} \quad \Rightarrow \quad Q = \left(\frac{A}{4\pi k_C}\right)E$$

4πk_c is often written as "1/_0"

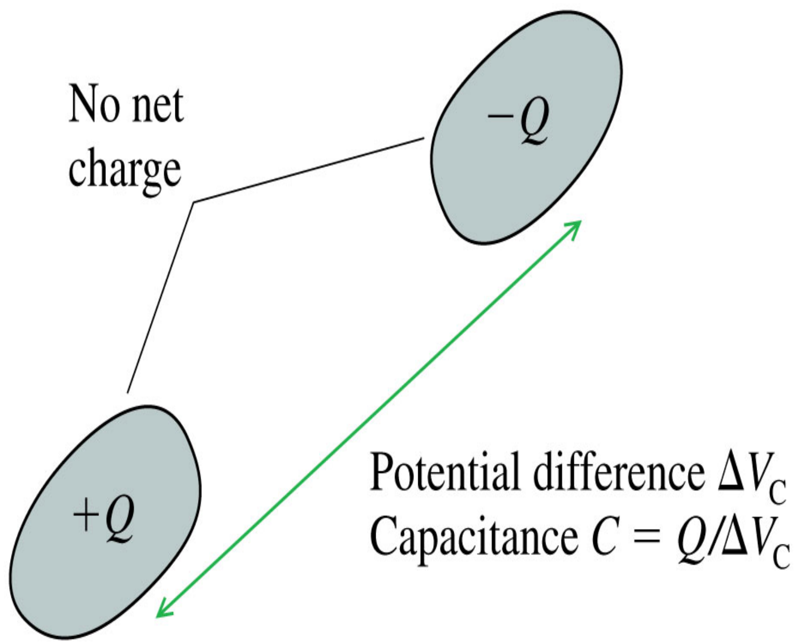
$$Q = \left(\frac{A}{4\pi k_C d}\right)\Delta V$$



$$Q = C\Delta V$$

- The figure shows two arbitrary electrodes charged to $\pm Q$.
- Since $C \equiv \frac{Q}{\Delta V_C}$

It might appear that the capacitance depends on the amount of charge, but the potential difference is proportional to Q.



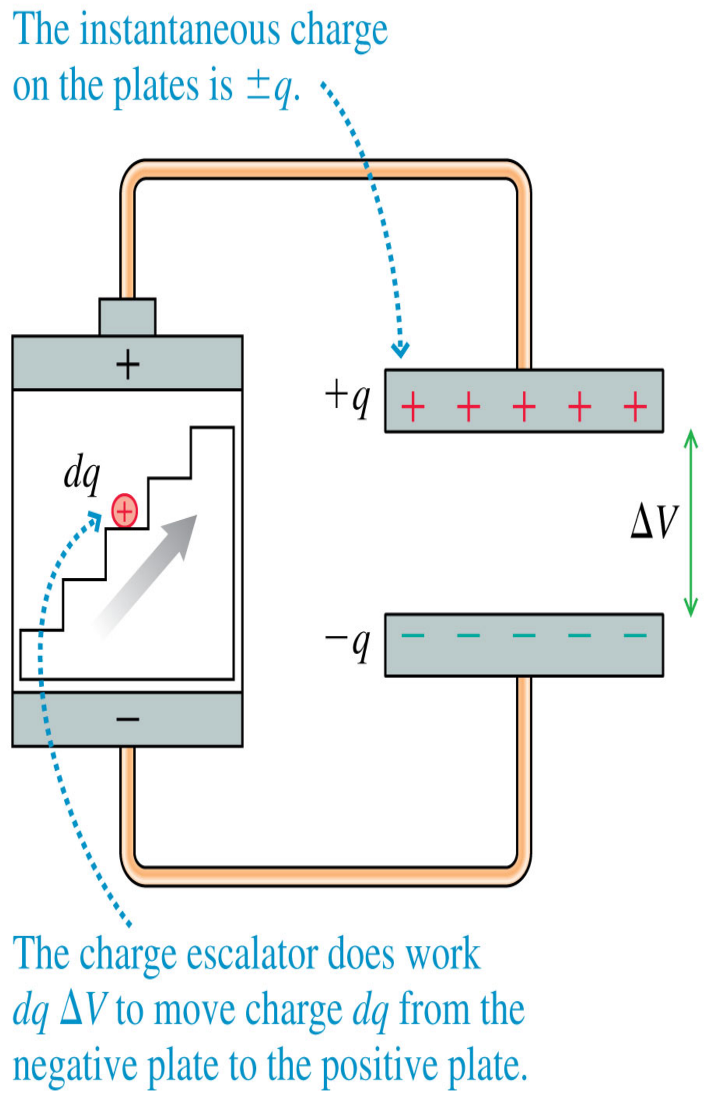
- Consequently, **the capacitance depends only on the geometry of the electrodes.**

- The figure shows a capacitor being charged.
- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by:

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

- The total energy transferred from the battery to the capacitor is:

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



- Capacitors are important elements in electric circuits because of their ability to store energy.
- The charge on the two plates is $\pm q$ and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.
- In terms of the capacitor's potential difference, the potential energy stored in a capacitor is:

$$U_C = \frac{Q^2}{2C} = \frac{1}{2} C (\Delta V_C)^2$$

- A capacitor can be charged slowly but then can release the energy very quickly.
- An important medical application of capacitors is the *defibrillator.*

- A heart attack or a serious injury can cause the heart to enter a state known as *fibrillation* in which the heart muscles twitch randomly and cannot pump blood.
- A strong electric shock through the chest completely stops the heart, giving the cells that control the heart's rhythm a chance to restore the proper heartbeat.

Charging a capacitor

The spacing between the plates of a 1.0 μF capacitor is 0.050 mm.

a. What is the surface area of the plates?

b. How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

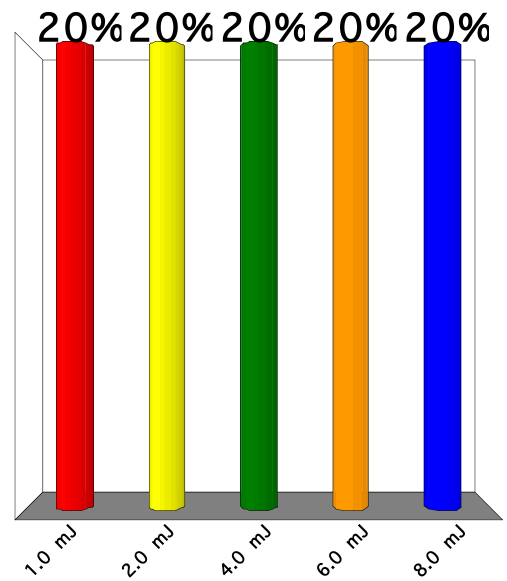
MODEL Assume the battery is ideal and the capacitor is a parallel-plate capacitor.

SOLVE a. From the definition of capacitance,

Whiteboard, TA & LA

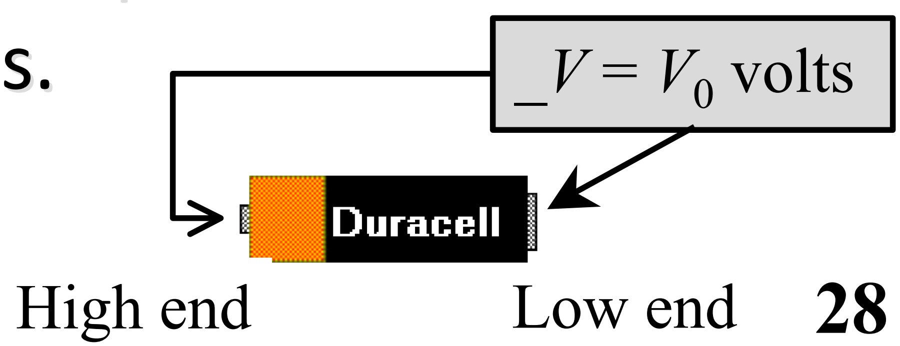
A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V, it will store

A. 1.0 mJ
B. 2.0 mJ
C. 4.0 mJ
D. 6.0 mJ
E. 8.0 mJ

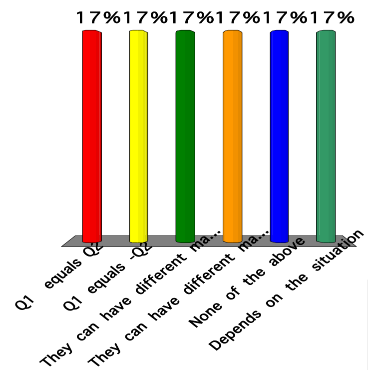


Some basic electrical ideas

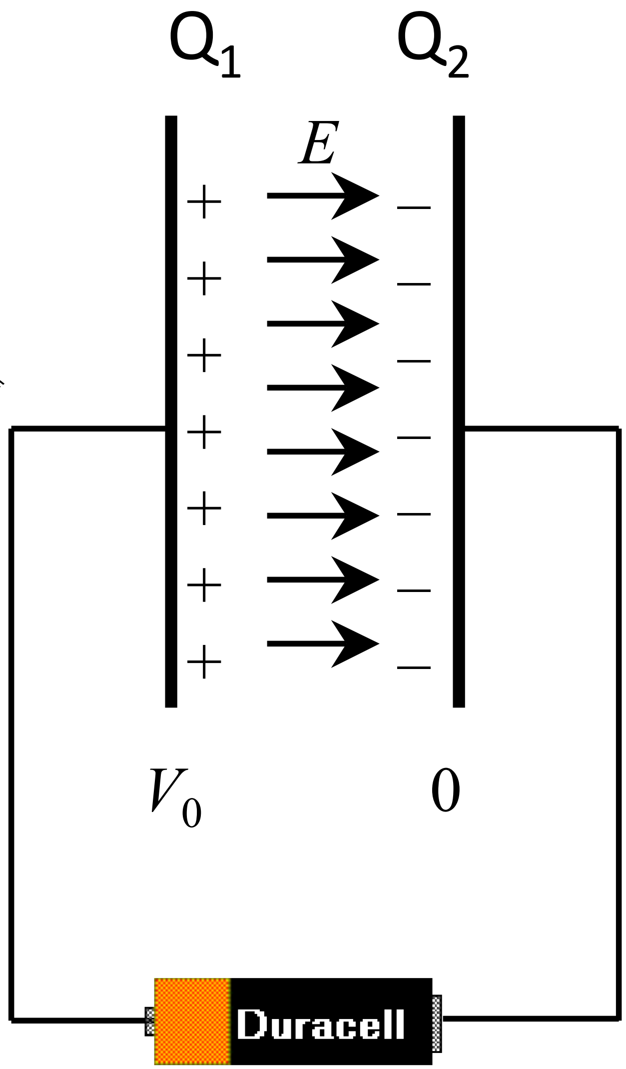
- ***Conductor*** – a material that permits some of its charges to move freely within it.
- ***Insulator*** – a material that permits some of its charges to move a little, but not freely.
- ***Battery*** – a device that creates and maintains a constant potential difference across its terminals.



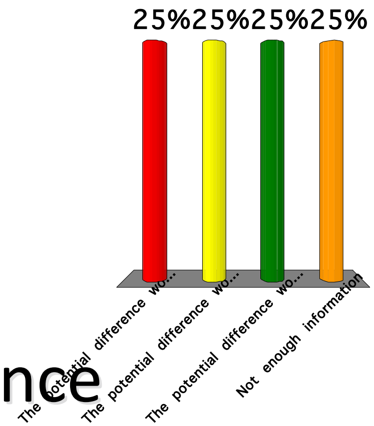
What can we determine about the charges on the capacitor plates are Q_1 and Q_2?



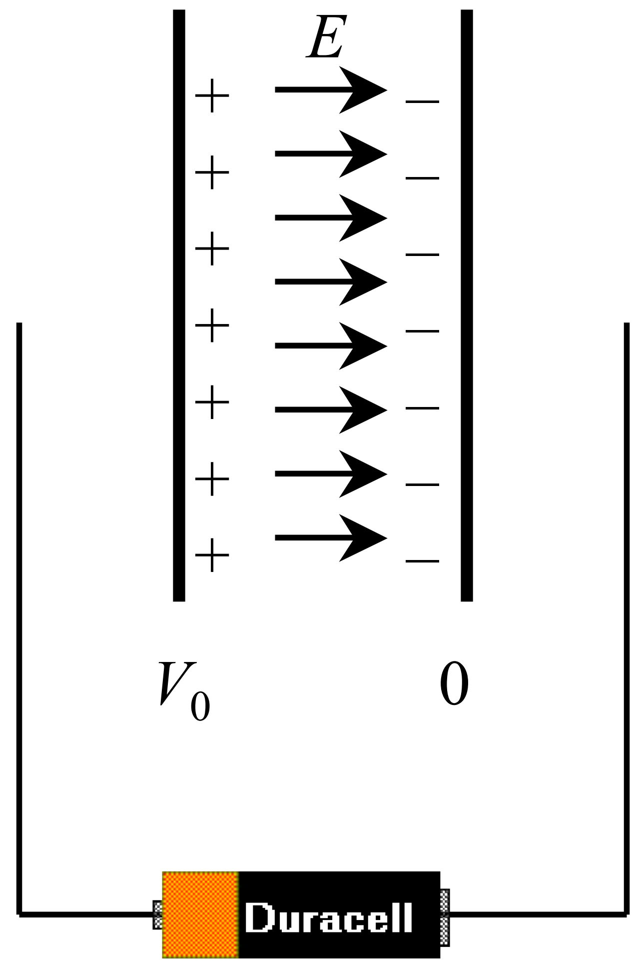
A. Q_1 equals Q_2
B. Q_1 equals $-Q_2$
C. They can have different magnitude but need to have opposite signs
D. They can have different magnitude but need to have the same signs
E. None of the above
F. Depends on the situation



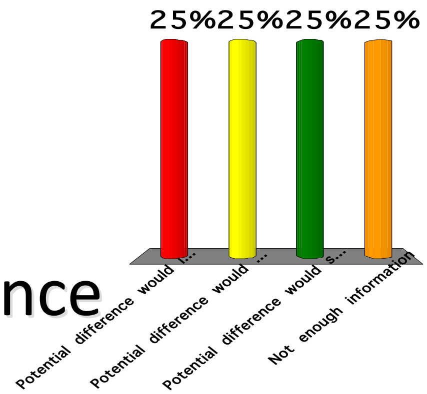
What would happen to the voltage if you first disconnected the battery and pulled the plates further apart?



A. The potential difference would increase
B. The potential difference would decrease
C. The potential difference would stay the same
D. Not enough information



What would happen to the voltage if I keep it connected to the battery and pulled the plates further apart?



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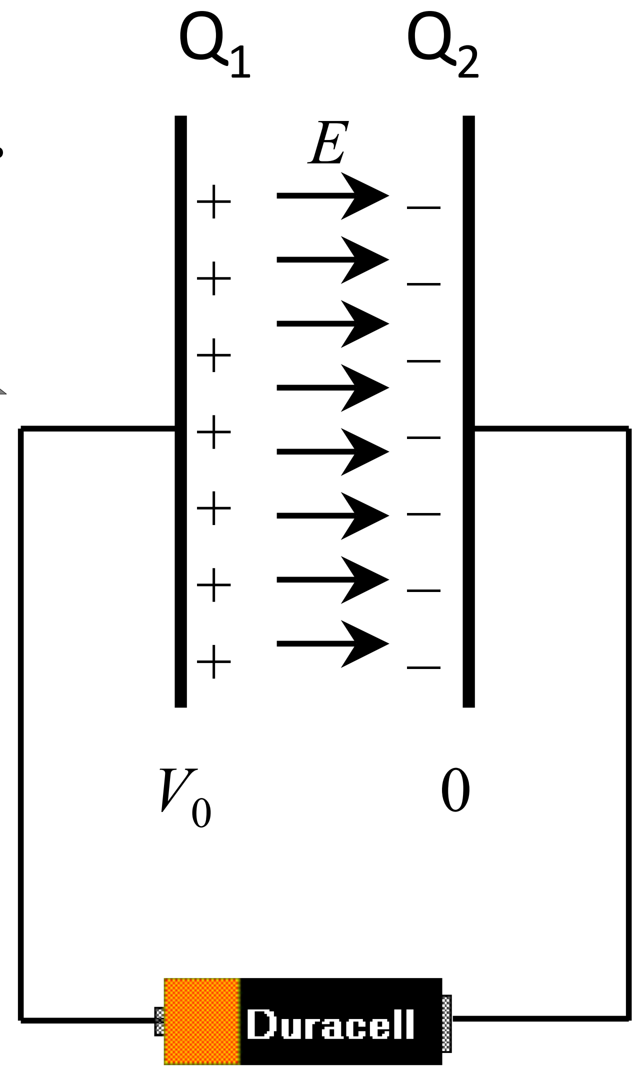
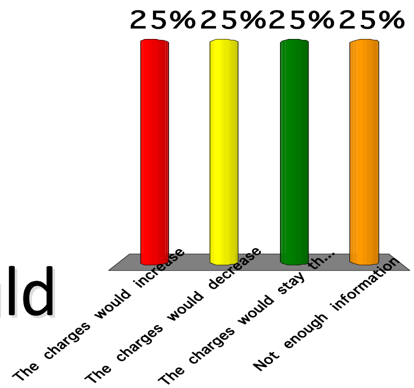
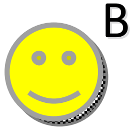


What would happen to the charge on either plate if I keep it connected to the battery and pulled the plates further apart?



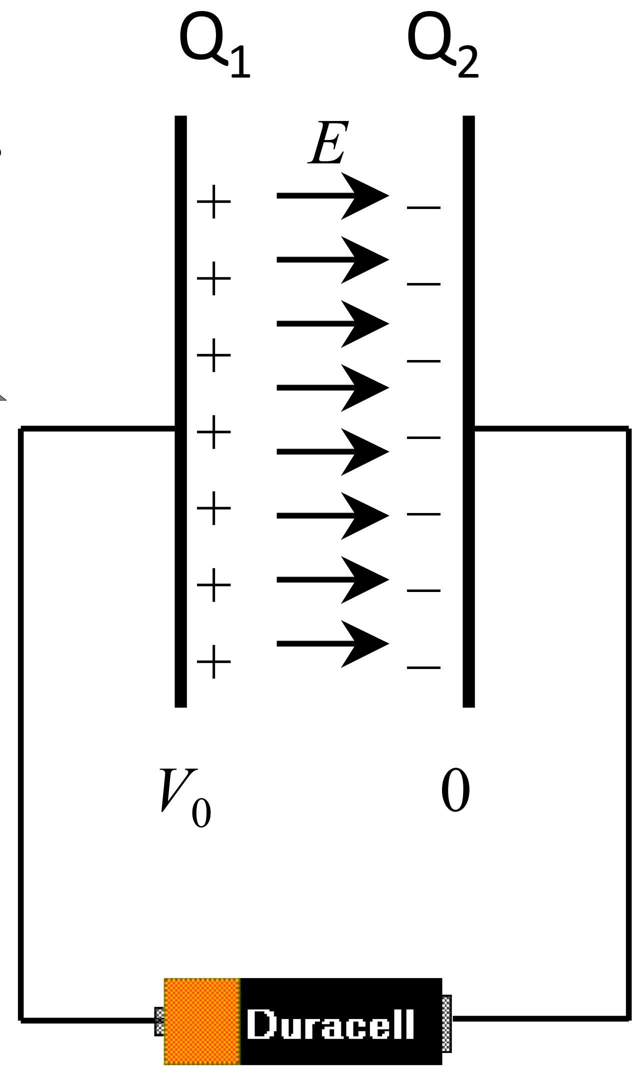
A. The charges would increase



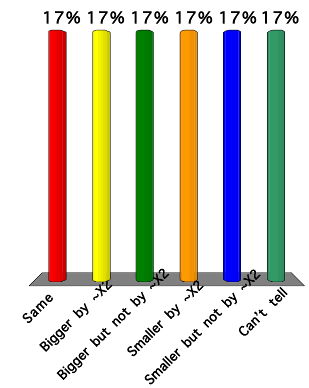
B. The charges would decrease

C. The charges would stay the same

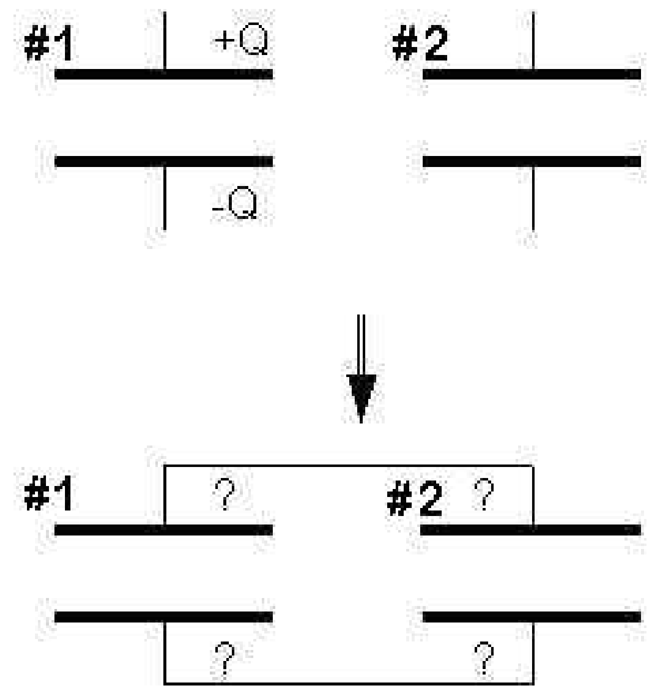
D. Not enough information



Cap #1 is charged by connecting it to a battery.
Cap #2 is not charged but otherwise is the same as C#1.
C#1 is disconnected from the battery and connected to C#2.
How does the magnitude of the E field in C#1 change?

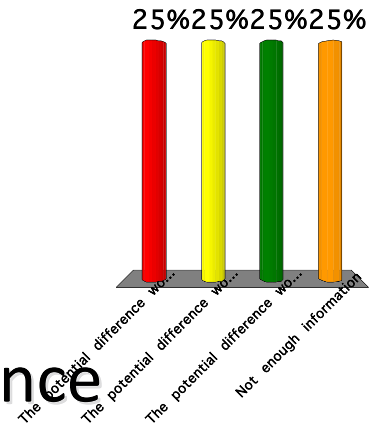


A. Same
B. Bigger by ~X2
C. Bigger but not by ~X2
D. Smaller by ~X2
E. Smaller but not by ~X2
F. Can’t tell

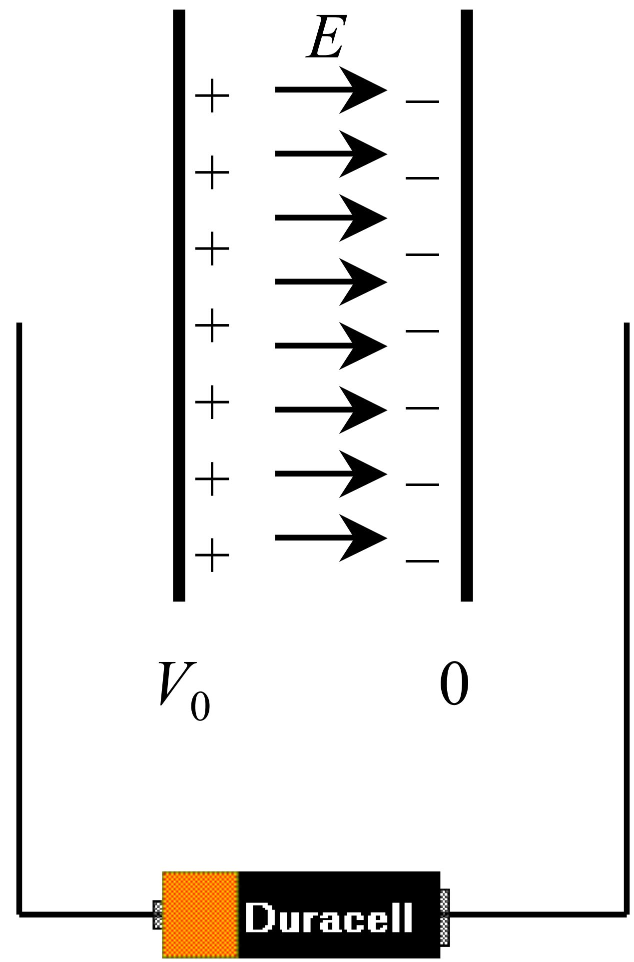


Whiteboard, TA & LA

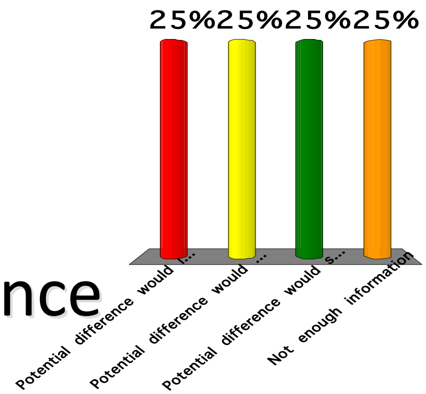
What would happen to the voltage if you first disconnected the battery and pulled the plates further apart?



A. The potential difference would increase
B. The potential difference would decrease
C. The potential difference would stay the same
D. Not enough information



What would happen to the voltage if I keep it connected to the battery and pulled the plates further apart?



A. Potential difference would increase
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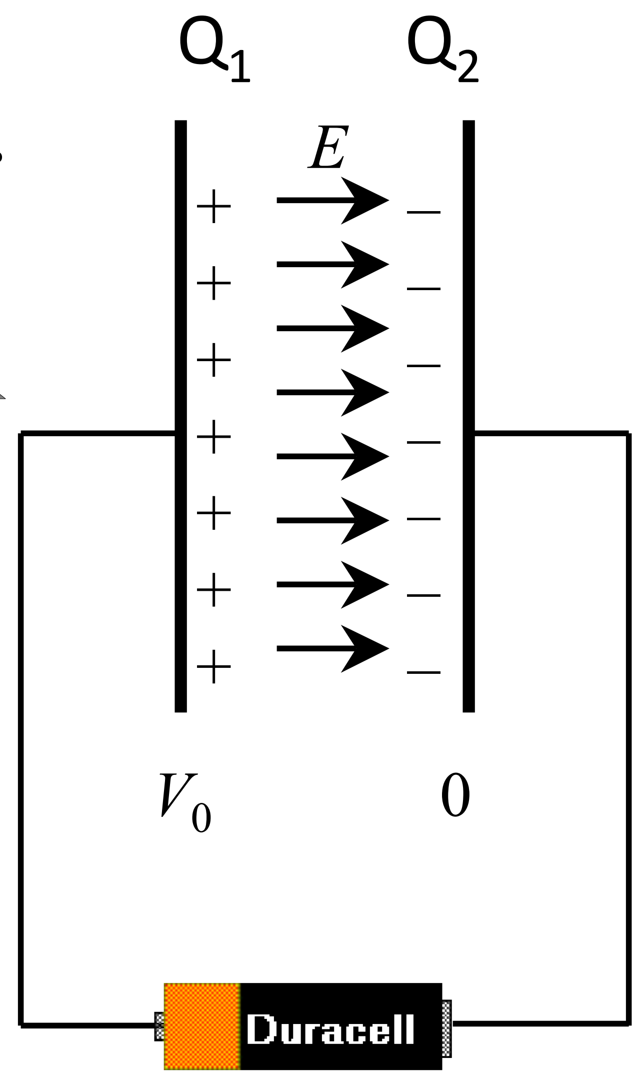
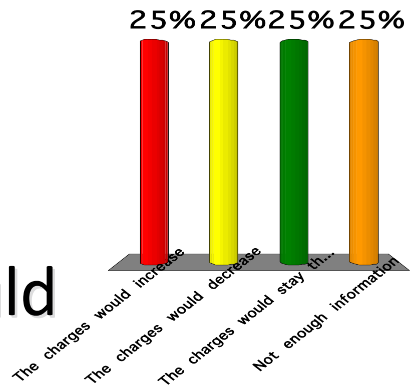
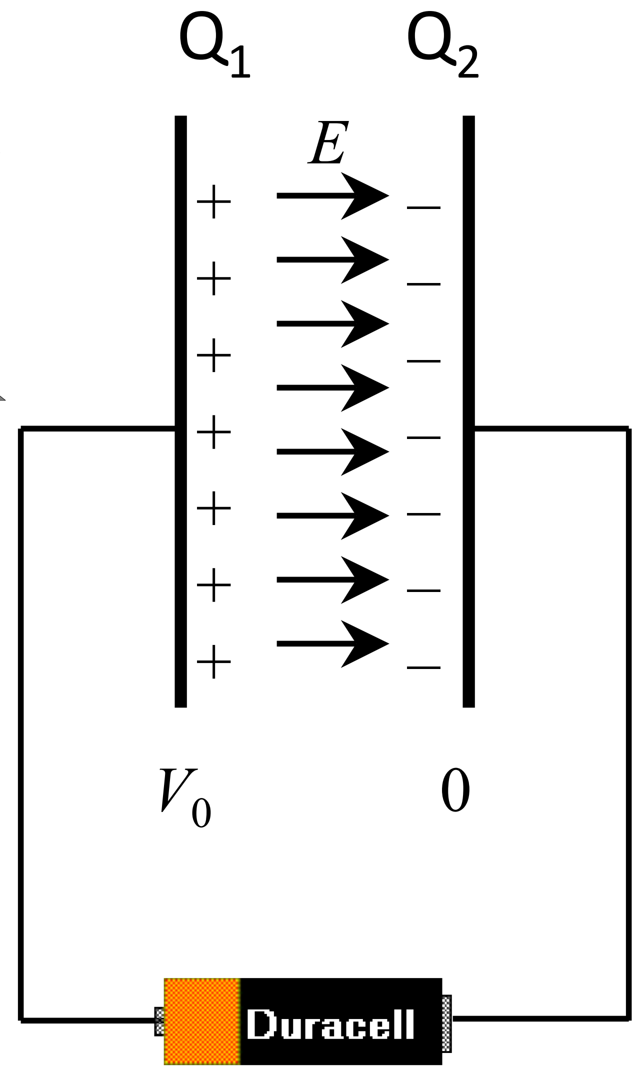


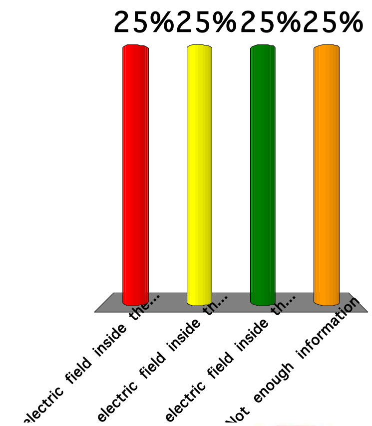
What would happen to the charge on either plate if I keep it connected to the battery and pulled the plates further apart?



A. The charges would increase
B. The charges would decrease
C. The charges would stay the same
D. Not enough information



What happens if we fill half the gap between plates with a conductor?

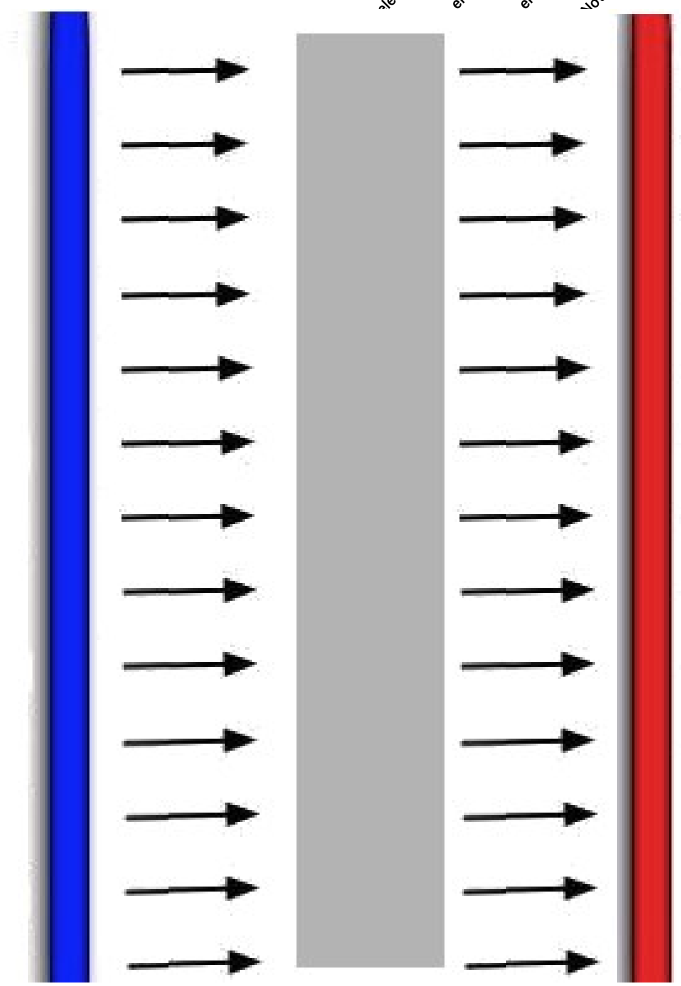


A. The electric field inside the conductor is the same as outside

B. The electric field inside the conductor is opposite to the field outside

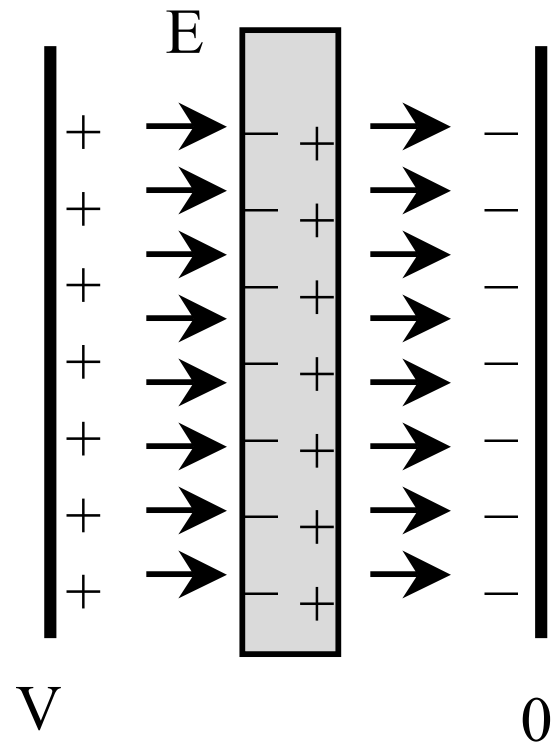
C. The electric field inside the conductor is zero

D. Not enough information



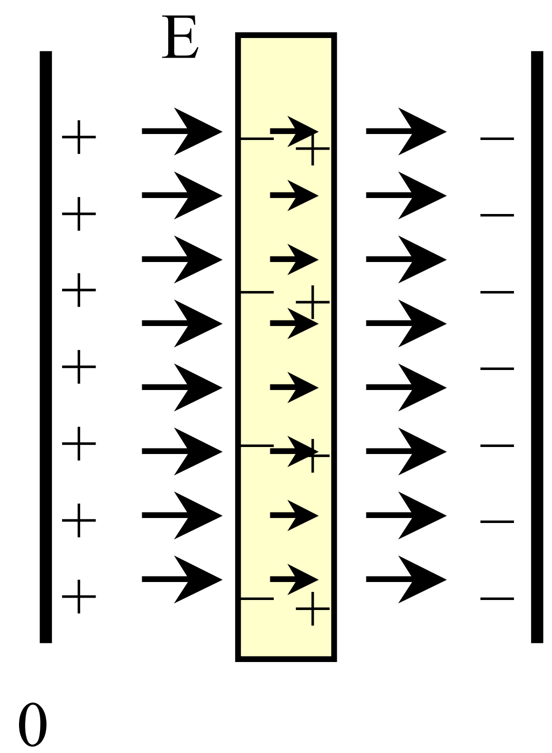
Conductors

- Putting a conductor inside a capacitor eliminates the electric field inside the conductor.
- The distance, d', used to calculate the ΔV is only the place where there is an E field, so putting the conductor in reduces the ΔV for a given charge. $C = \frac{1}{4\pi k_C}\frac{A}{d'}$



Consider what happens with an insulator

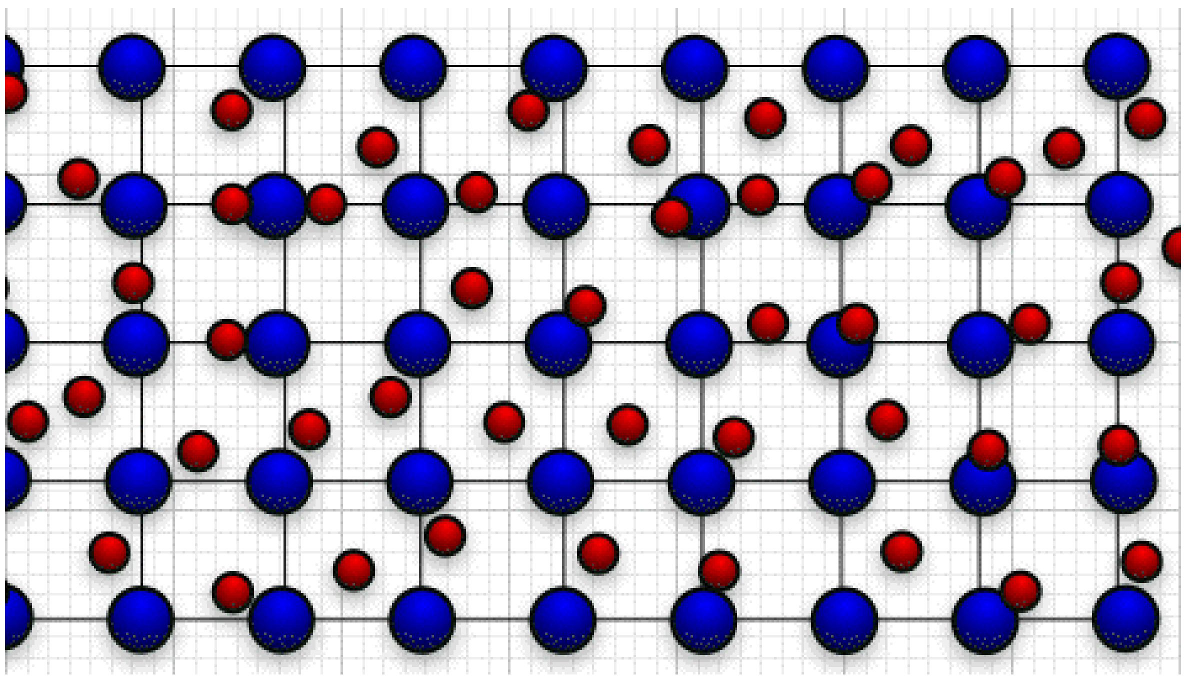
- We know that charges separate even with an insulator.
- This reduces the field inside the material, just not to 0.
- The field reduction factor is defined to be κ .



$$E_{\text{inside material}} = \frac{1}{\kappa} E_{\text{if no material were there}}$$

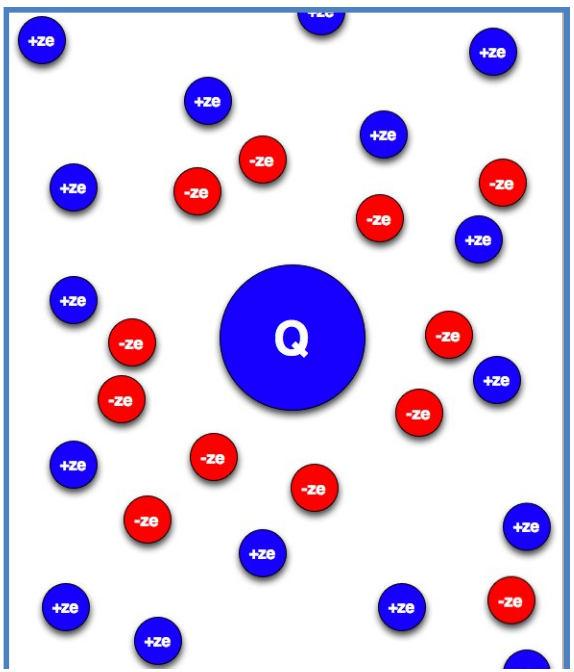
Charged objects in Conducting Solids

- What happens if place a charged object into a neutral conductor?
  - Positive ions are fixed in the solid
  - Some negative charges (shared electrons) are free to move



Charged objects in Conducting Fluids

- What happens if place a charged object into a neutral fluid?
  - Opposite charged ions are attracted to object
  - Like charged ions are repelled
  - Thermal energy keeps ions moving



Friday

Electrical Current

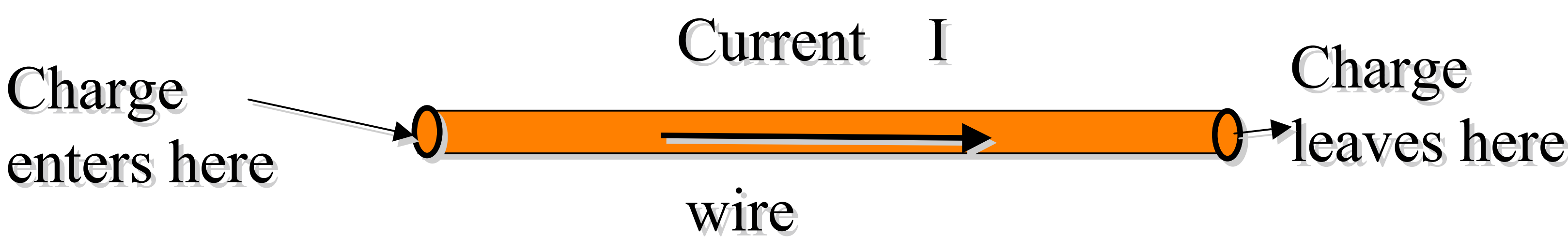
Fluid Flow Analogy (model)

Conductors

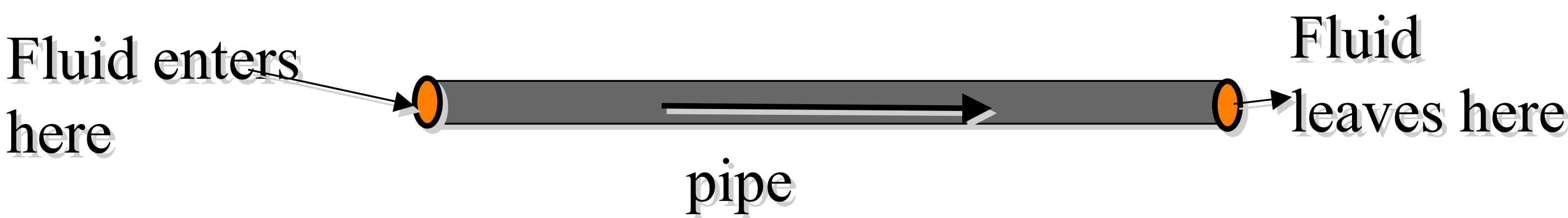
Ohms's Law $V = IR$

Electric Current

When charge flows, we say there is an electrical current.



Just like fluid flow in a pipe



Your questions about models

<u>Water Flow</u>

In the water-flow model how is pressure analogous to voltage? //
Could you further explain the water model? //
We'll get to that. It's a good model for understanding circuits.

<u>Air Flow</u>

Same as water flow model //

<u>Rope Model</u>

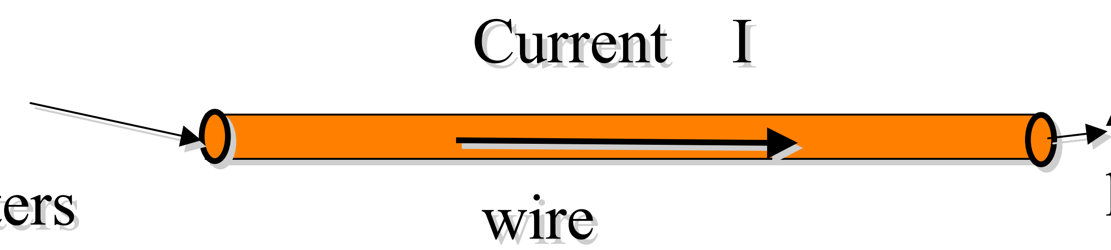
Never heard of it. Perhaps it emphasizes that a path for current to complete a <u>circuit</u> must be present.

How do we quantify flow?

	Fluid	Electricity
Amount of quantity	Liters	Coulombs
Flow rate	Liters/sec	Coulombs/sec = Amperes

$$I = \Delta Q/\Delta t$$

Charge ΔQ enters here during time Δt



ΔQ leaves here during time Δt

Generally not the same charge, but the same <u>amount</u> of charge

Electrical Conductors

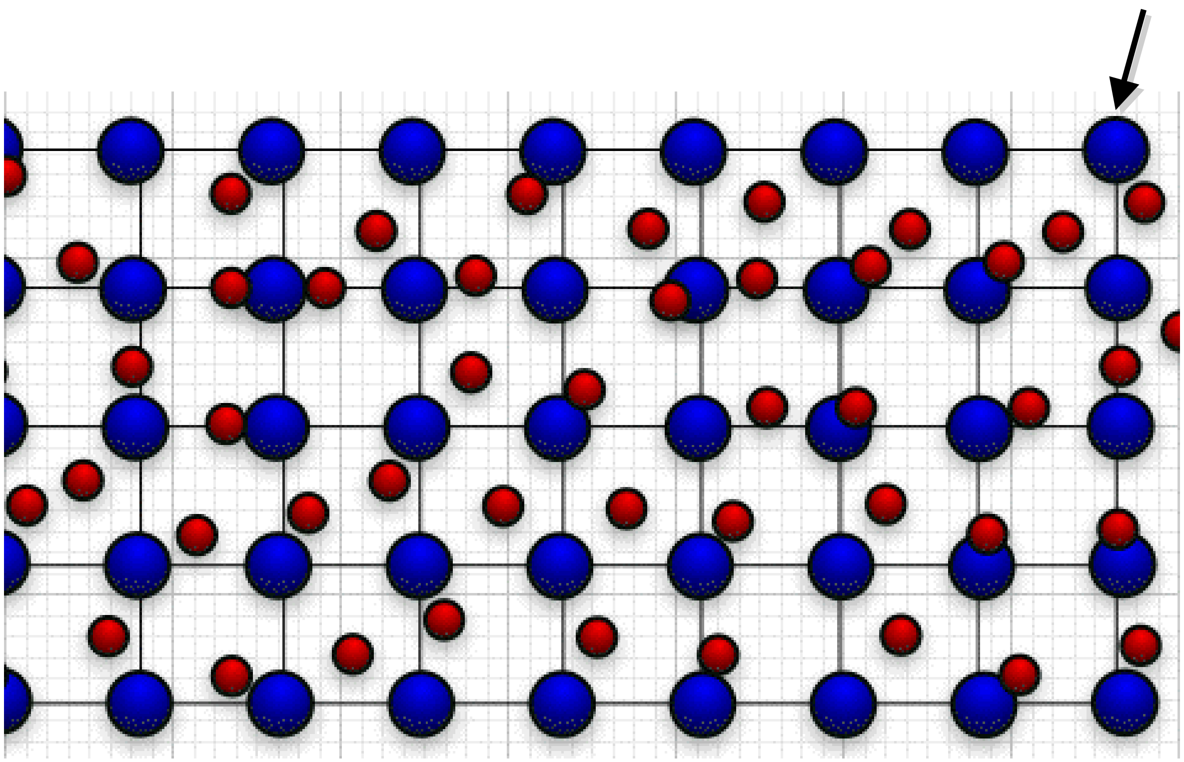
Materials that readily allow current to flow when an electric field is applied are called conductors. (Others called insulators)

Examples:

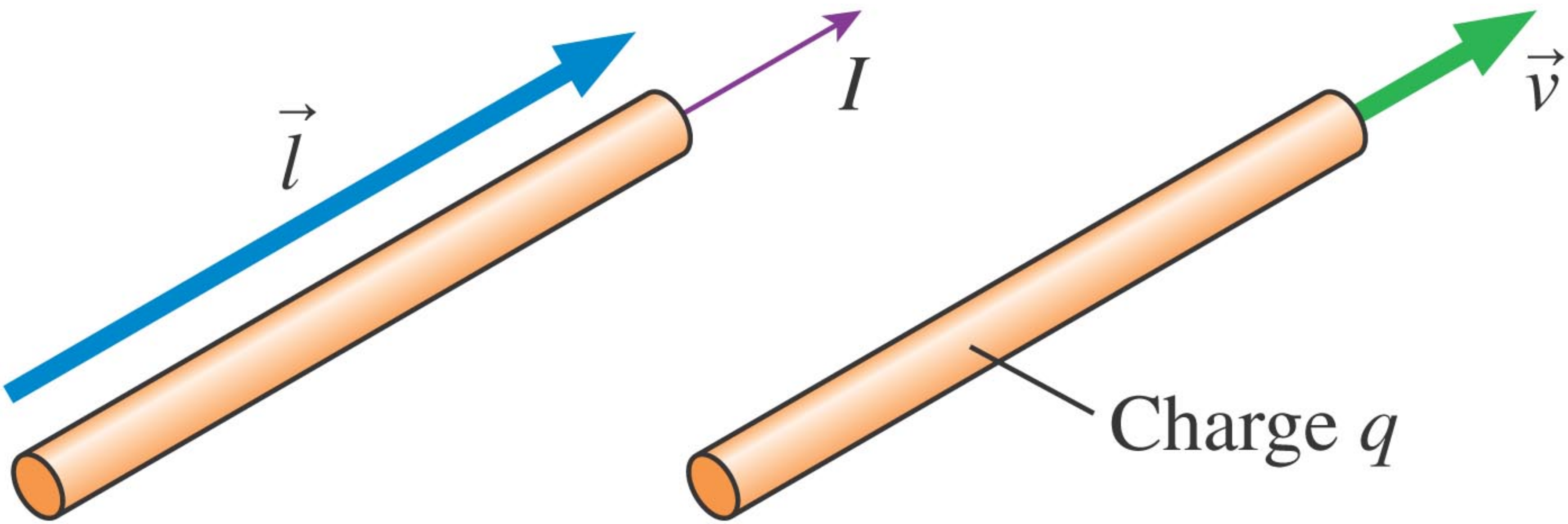
- Metals
- Ionic solutions
  - People (under their skin)
  - Sea water (salt)
- Doped semiconductors (Basis for solid state electronics)
- Plasmas (most of the universe, e.g. sun, starts space between, ionosphere)

What is it about metals that make them good conductors?

Metals are crystals. Lattice of positive ions

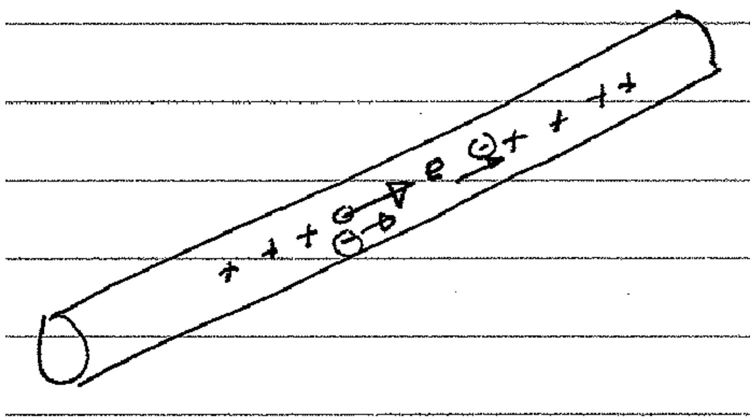


Some electrons are bound to particular ions, the rest are free to roam. In a perfect lattice they would be have as free.



A current consists of charge carriers q moving with velocity $\vec{v}$.

Current carrying wire



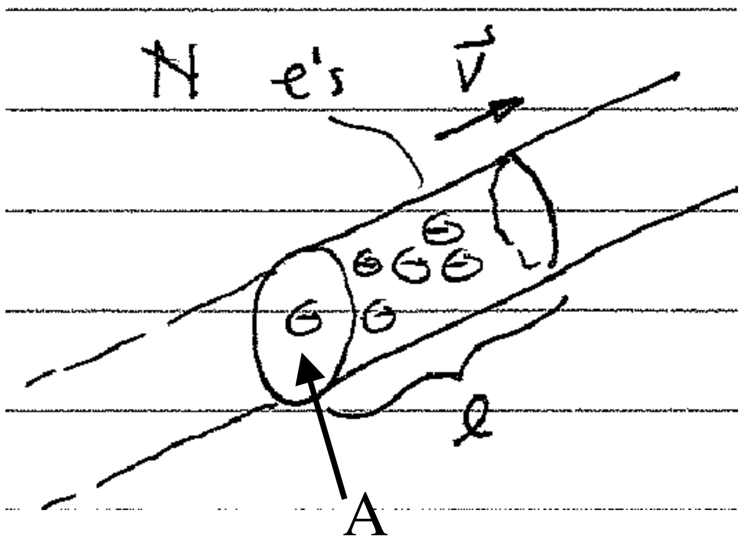
Inside the wire there are stationary charges and moving charges.

Metallic conductor
Moving - free electrons
Stationary - positive ions and bound electrons

In any length of wire the number of positive and negative charges is essentially the same (unless the wire is charged).

The wire can carry a current because the free electrons move.

Current, number density and velocity



Number density, n, of free electrons is a property of the material.

For copper $n \simeq 1.1 \times 10^{29} \quad m^{-3}$

Consider a segment of length l, cross sectional area A

The segment has N free electrons $N = n\left(lA\right)$ ← volume

Suppose the free electrons have speed v. How long will it take all the electrons to leave the segment?

$$\Delta t = l / v$$

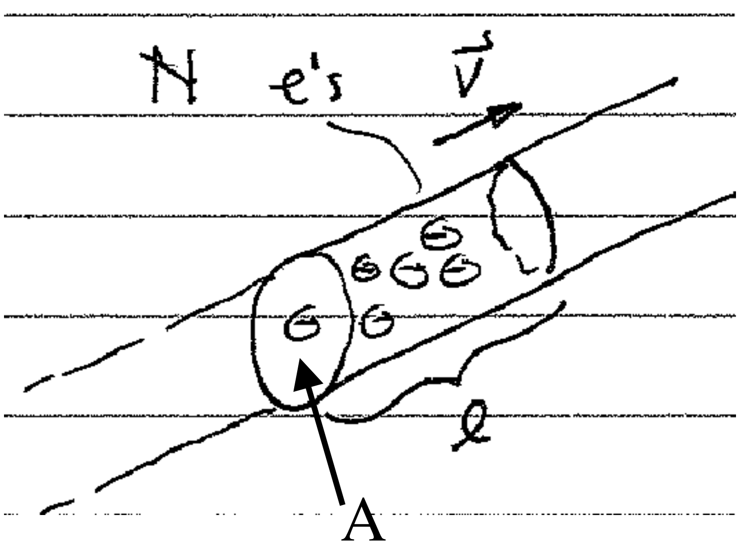
Whiteboard, TA & LA

Current

During a time interval $\Delta t = l/v$, a net charge Q = -eN flows through any cross section of the wire.

The wire is thus carrying I

$$I = \frac{Q}{\Delta t} = -Aenv$$



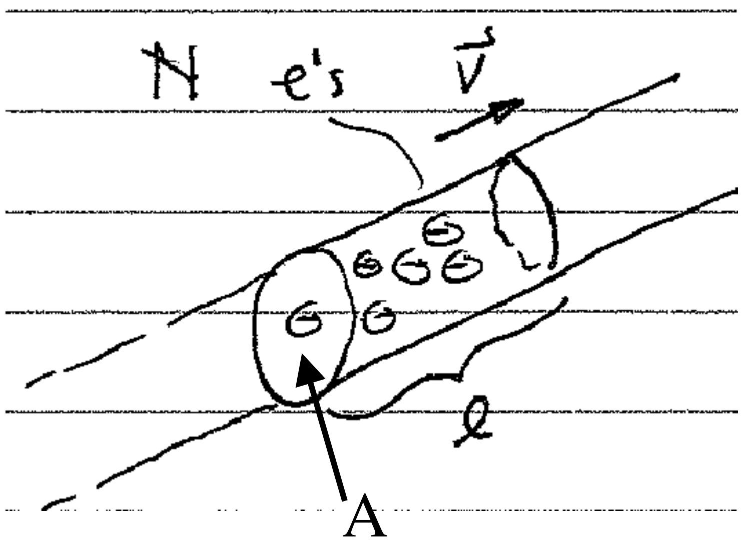
The current is flowing in a direction opposite to that of v.

$$I = -Aenv = Aj$$

$$j = -env$$

j - current density

How big is v for typical currents in copper?



A. Close to the speed of light
B. Supersonic
C. About as fast as a Prius
D. Glacial

$$I = -Aenv$$

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For copper $n \simeq 1.1\times 10^{29} \quad m^{-3}$

Whiteboard, TA & LA

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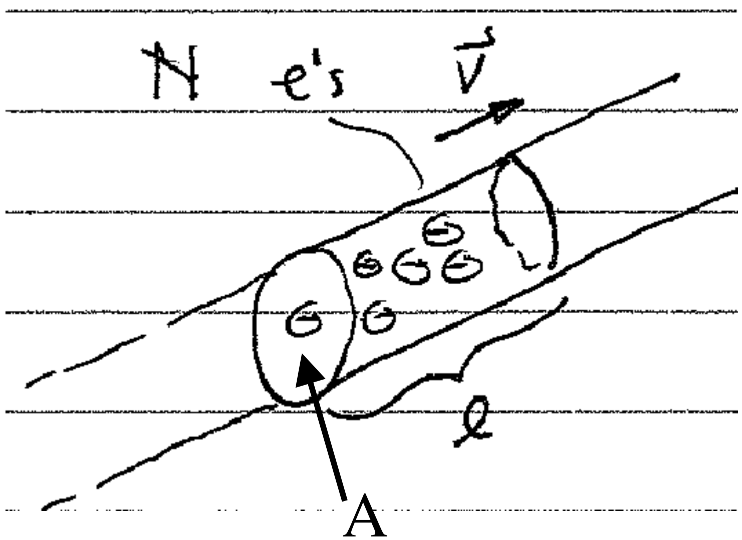

A. Close to the speed of light
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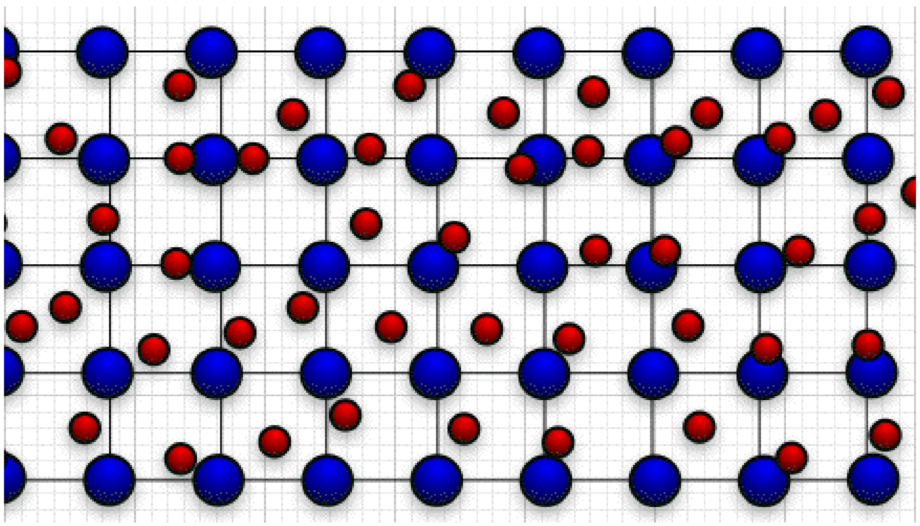
For a 1mm radius copper wire carrying 1 A of current

$$|v| = \frac{I}{Aen} = 1.8 \times 10^{-5} m / s$$

But lattices are not perfect



Free electrons feel a friction with the lattice that gives rise to “resistivity”

Materials have impurities.

Lattices have dislocations

Can you think of any thing else? Hint: its effect depends on temperature.

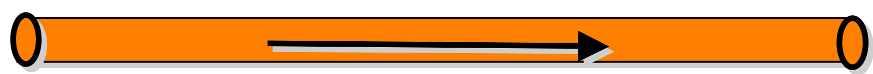
Whiteboard, TA & LA

Resistance of a wire - V=IR

Recall relation between current and velocity of charges in a wire

$$I = -Aenv = Aj$$

$$j = -env$$



But what determines v ? Answer:Newton's laws

Force on an electron:

$$F_e = -eE - mv / \tau$$

Friction is to a good approximation proportional to velocity. τ is the time between collisions

electrons acquire a velocity

$$v = -\mu E = -(e\tau / m)E$$

Ohm's Law

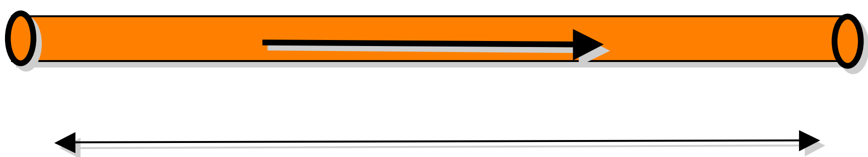
$$I = -Aenv = Aj$$

$$v = -\mu E = -(e\tau / m)E$$

mobility

Resistivity: a property of the material

$$j = e\mu nE = E / \rho$$



l

$$l\,E = V$$ Electric potential

When the dust settles

$$I = V / R$$

$$R = l\rho / A$$

Current and Fluid Flow

$$V = RI = \frac{l\rho}{A} I$$

The volumetric flow rate of the fluid, I = v*A, will be governed by the Hagen-Poiseuille (H-P) equation:

$$\Delta P \propto \frac{l\mu_{visc}}{A^2} I$$

How are they the same? different? Why?

Whiteboard, TA & LA

A better analogy

When charge flows, we say there is an electrical current.

Just like fluid flow in a pipe

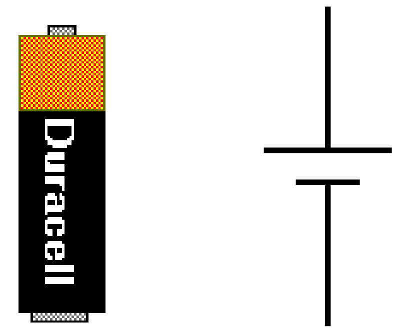
pipe clogged with porous material

Units

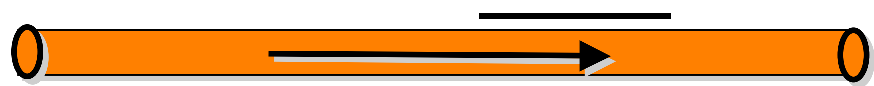
- Current (*I*) — **Ampere** = Coulomb/sec
- Voltage (*V*) — **Volt** = Joule/Coulomb
- E-Field (*E*) — Newton/Coulomb = Volt/meter
- Resistance (*R*) — **Ohm** = Volt/Ampere
- Capacitance (*C*) — **Farad** = Volt/Coulomb
- Power (*P*) — **Watt** = Joule/sec

Electric circuit elements

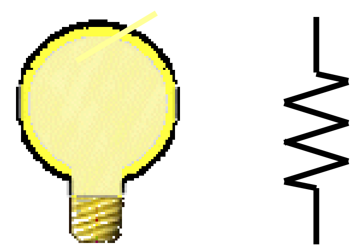
- *Batteries* —devices that maintain a constant electrical potential difference across their terminals



- *Wires* — charges flow quickly need very little forces to move

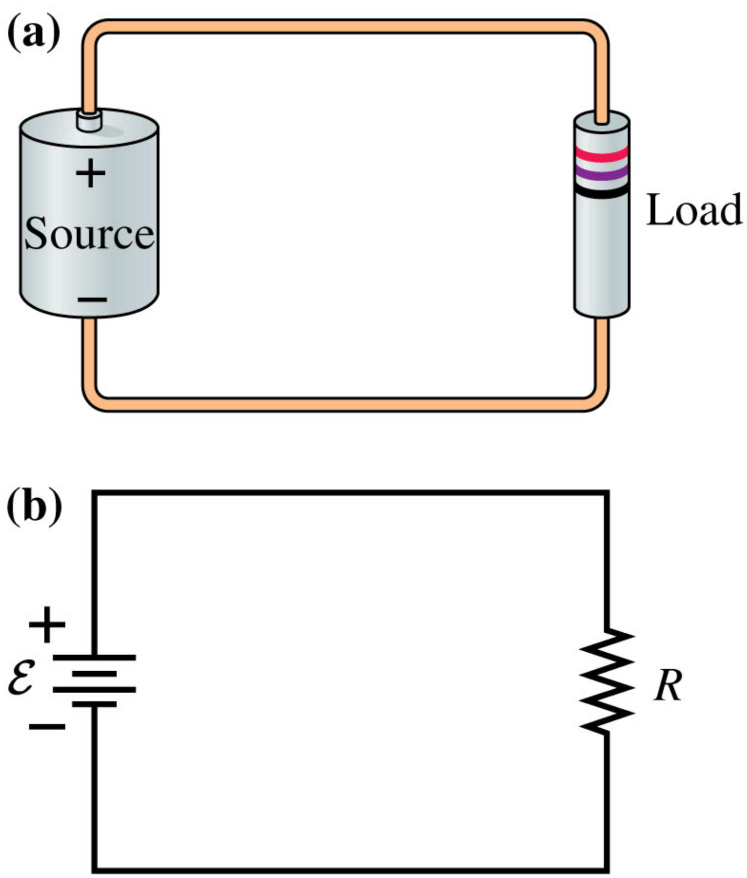


- *Resistors* — charges need a larger force to move. Examples are Resistors and Lightbulbs



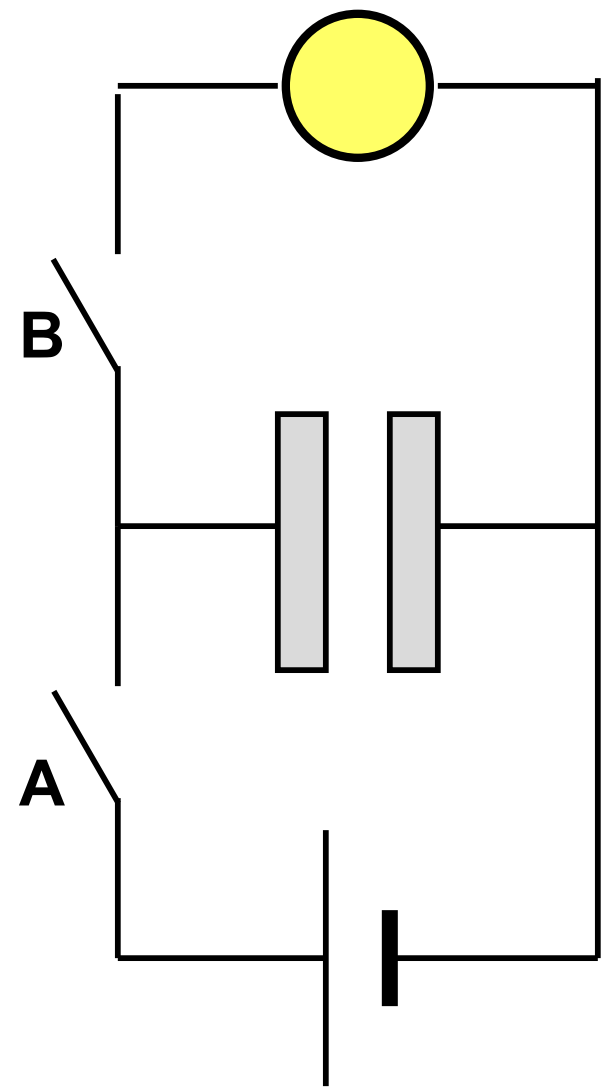
- *Capacitors* - You know about these!

Why do we call them circuits?



- The most basic electric circuit is a single resistor connected to the two terminals of a battery.
- Figure (a) shows a literal picture of the circuit elements and the connecting wires.
- Figure (b) is the circuit diagram.
- This is a **complete circuit**, forming a continuous path between the battery terminals.

- Suppose we:
  - Close A for a few seconds
  - Open A
  - Close B
- What happens to the bulb?
  - 1. It stays off.
  - 2. It stays on after you close A
  - 3. It stays on after you close B
  - 4. It flashes when you close A
  - 5. It flashes when you open A
  - 6. It flashes when you close B



As the lightbulb flashes which of the following is true

1. **Positive** charges move across the lightbulb, they move at roughly constant speed
2. **Positive** charges move across the lightbulb, they move slowest at the lightbulb
3. **Negative** charges move across the lightbulb, they move at roughly constant speed
4. **Negative** charges move across the lightbulb, they move slowest at the lightbulb
5. None of the above

Foothold ideas: Currents

- Charge is moving: How much?

$$I = \frac{\Delta q}{\Delta t}$$

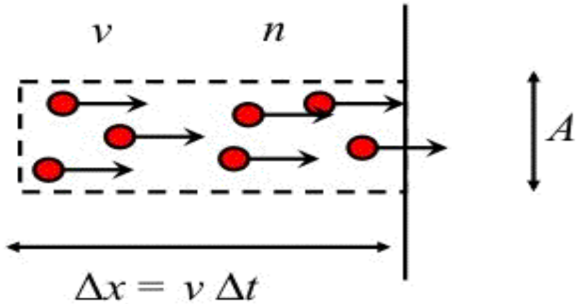
- How does this relate to the individual charges?

$$I = q\, n\, A\, v$$

- What pushes the charges through resistance? Electric force implies a drop in *V*!

$$F_e = qE$$

$$\Delta V = -\frac{E}{L}$$



Ohm's Law

- Current proportional to change in Electrical Potential

$$\Delta V = IR$$

- Does R depend on the Area of the resistor?
- Does R depend on the length of the resistor?

1. R Increases
2. R decreases
3. R remains the same
4. Depends on material

Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part (*L/A*) times a part independent of the size and shape but dependent on the material. This coefficient is called the *resistivity* of the material (ρ).
- Its reciprocal (*g*) is called *conductivity*. *(The reciprocal of the resistance is called the conductance (G).)*

$$R = \rho \frac{L}{A} = \frac{1}{g}\frac{L}{A} = \frac{1}{G}$$

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