Quantization

Matter Waves and Energy Quantization

In 1924 de Broglie postulated that *if* a material particle of momentum $p = mv$ has a wave-like nature, then its wavelength must be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad p = h/\lambda$$

where h is Planck's constant (h = 6.63 x 10^{-34} J s). This is called the **de Broglie wavelength.**

Photons $\quad E = hf \quad E = pc \quad p = h/\lambda$

EXAMPLE 39.6 The de Broglie wavelength of an electron

QUESTION:

EXAMPLE 39.6 **The de Broglie wavelength of an electron**

What is the de Broglie wavelength of a 1.0 eV electron?

$$p = h / \lambda \quad \longrightarrow \quad \lambda = h / p$$

$$E = \frac{mv^2}{2} \quad \longrightarrow \quad v = \sqrt{\frac{2E}{m}}$$

$$p = mv$$

EXAMPLE 39.6 The de Broglie wavelength of an electron

SOLVE An electron with 1.0 eV $= 1.6 \times 10^{-19}$ J of kinetic energy has speed

$$v = \sqrt{\frac{2K}{m}} = 5.9 \times 10^5 \text{ m/s}$$

Although fast by macroscopic standards, this is a slow electron because it gains this speed by accelerating through a potential difference of a mere 1 V. Its de Broglie wavelength is

$$\lambda = \frac{h}{mv} = 1.2 \times 10^{-9} \text{ m} = 1.2 \text{ nm}$$

Bohr's Model of Atomic Quantization

1. An atom consists of negative electrons orbiting a very small positive nucleus.
2. Atoms can exist only in certain **stationary states.** Each stationary state corresponds to a particular set of electron orbits around the nucleus. These states can be numbered 1, 2, 3, 4, . . . , where n is the *quantum number*.
3. Each stationary state has an energy E_n. The stationary states of an atom are numbered in order of increasing energy: $E_1 < E_2 < E_3 < \dots$
4. The lowest energy state of the atom E_1 is *stable* and can persist indefinitely. It is called the **ground state** of the atom. Other stationary states with energies E_2, E_3, E_4,. . . are called **excited states** of the atom.

Bohr's Model of Atomic Quantization

5. An atom can "jump" from one stationary state to another by emitting or absorbing a photon of frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}$$

where h is Planck's constant and and $\Delta E_{\text{atom}} = |E_f - E_i|$.

E_f and E_i are the energies of the initial and final states. Such a jump is called a **transition** or, sometimes, a **quantum jump.**

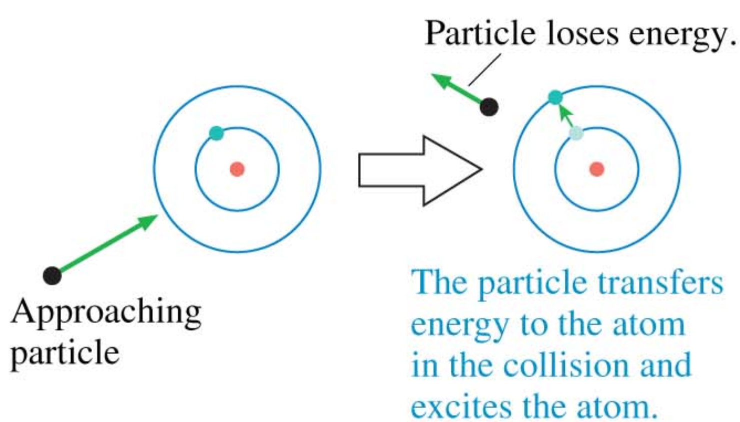
Bohr's Model of Atomic Quantization

6. An atom can move from a lower energy state to a higher energy state by absorbing energy $\Delta E_{\text{atom}} = E_{\text{f}} - E_{\text{i}}$ in an inelastic collision with an electron or another atom.

This process, called **collisional excitation,** is shown.

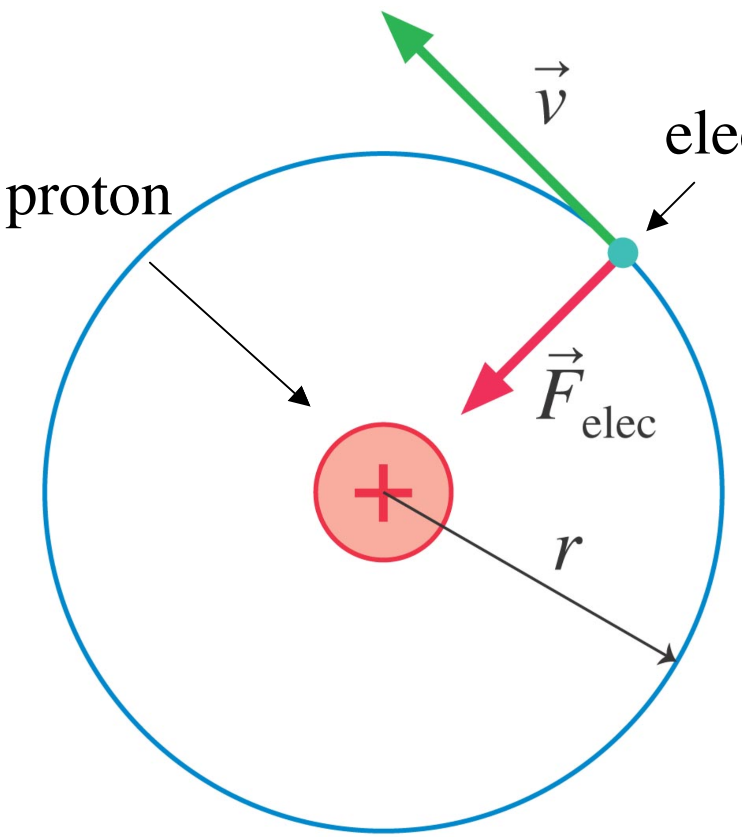
FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(b) Collisional excitation



Bohr Model of the Hydrogen Atom
(Approximate QM treatment)

Classical Picture



$$m\vec{\mathbf{a}} = \vec{\mathbf{F}}$$

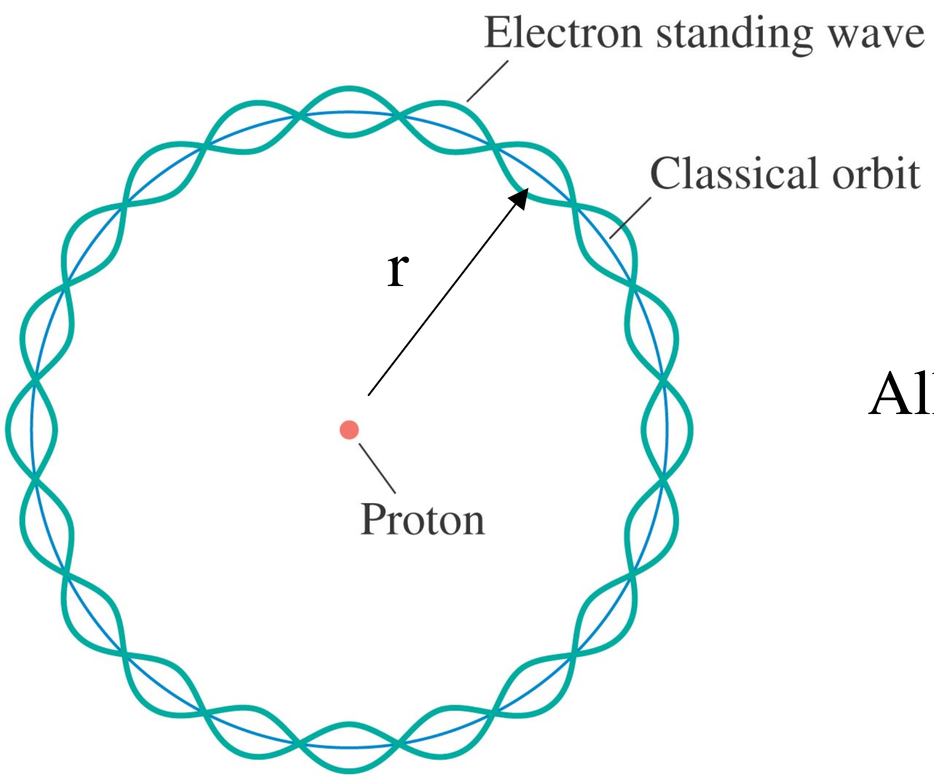
$$m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

Centripetal acceleration

Coulomb force

Classically, any value of v and r are allowed so long as F=ma above. Quantum mechanics says only certain values of r and v are allowed.

Quantum mechanics: Orbit must be an integer # of de Broglie wavelengths



Only certain r's are allowed.

$$r_n = n^2 a_0 \qquad a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

$$a_0 = 5.3 \times 10^{-11}\,\text{m}$$

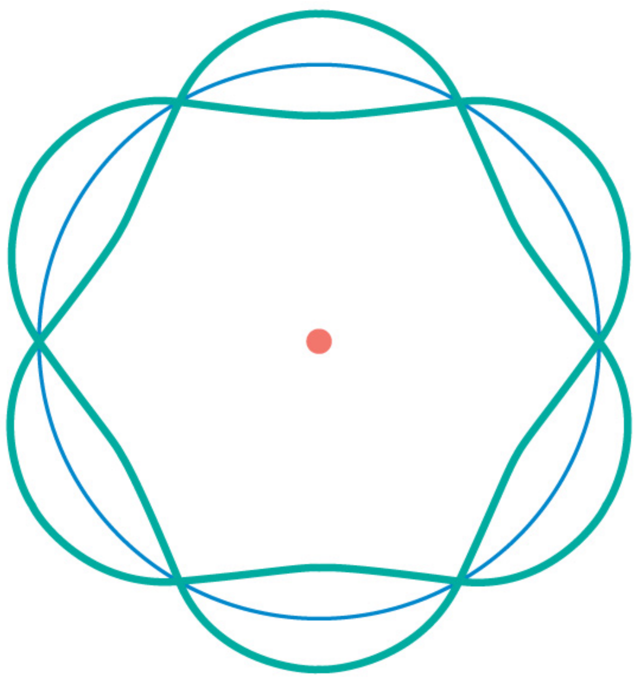
Bohr radius

Allowed energies of Hydrogen

$$E_n = -\frac{2.18 \times 10^{-18}}{n^2}\,\text{J}$$

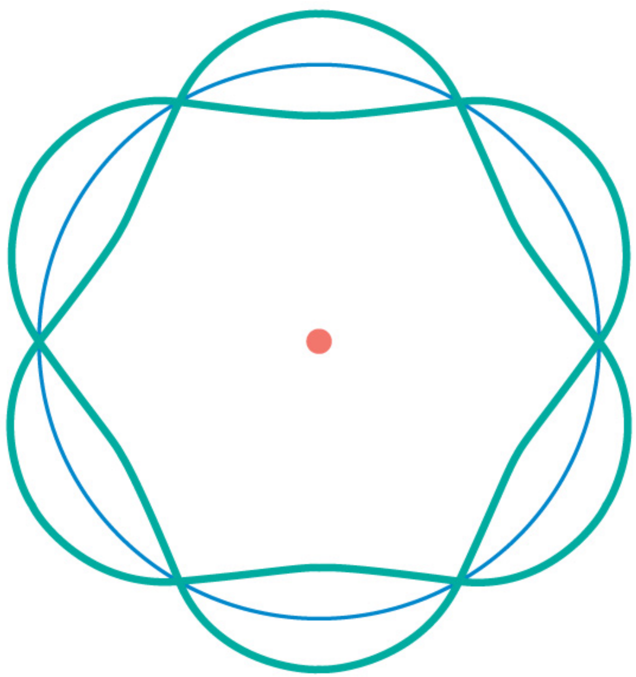
$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

What is the quantum number of this hydrogen atom?



A. $n = 5$
B. $n = 4$
C. $n = 3$
D. $n = 2$
E. $n = 1$

What is the quantum number of this hydrogen atom?



A. $n = 5$
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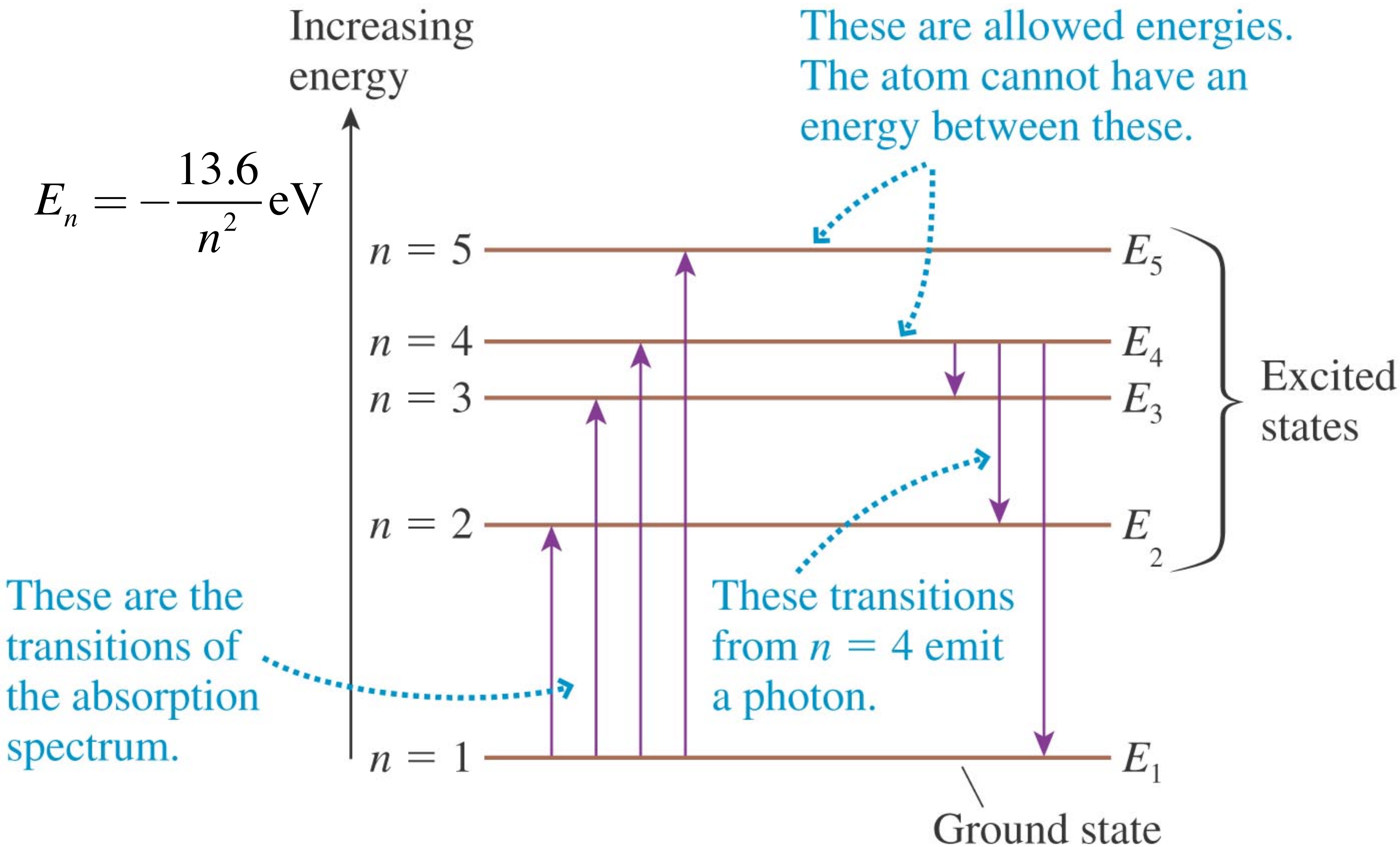
$$E_n = -\frac{13.6}{n^2}\text{eV}$$



Foothold Ideas: Light interacting with Matter

- Atoms and molecules naturally exist in states having specified energies. EM radiation can be absorbed or emitted by these atoms and molecules.
- When light interacts with matter, both energy and momentum are conserved.
- The energy of radiation either emitted or absorbed therefore corresponds to the <u>difference</u> of the energies of states.

Line Spectra

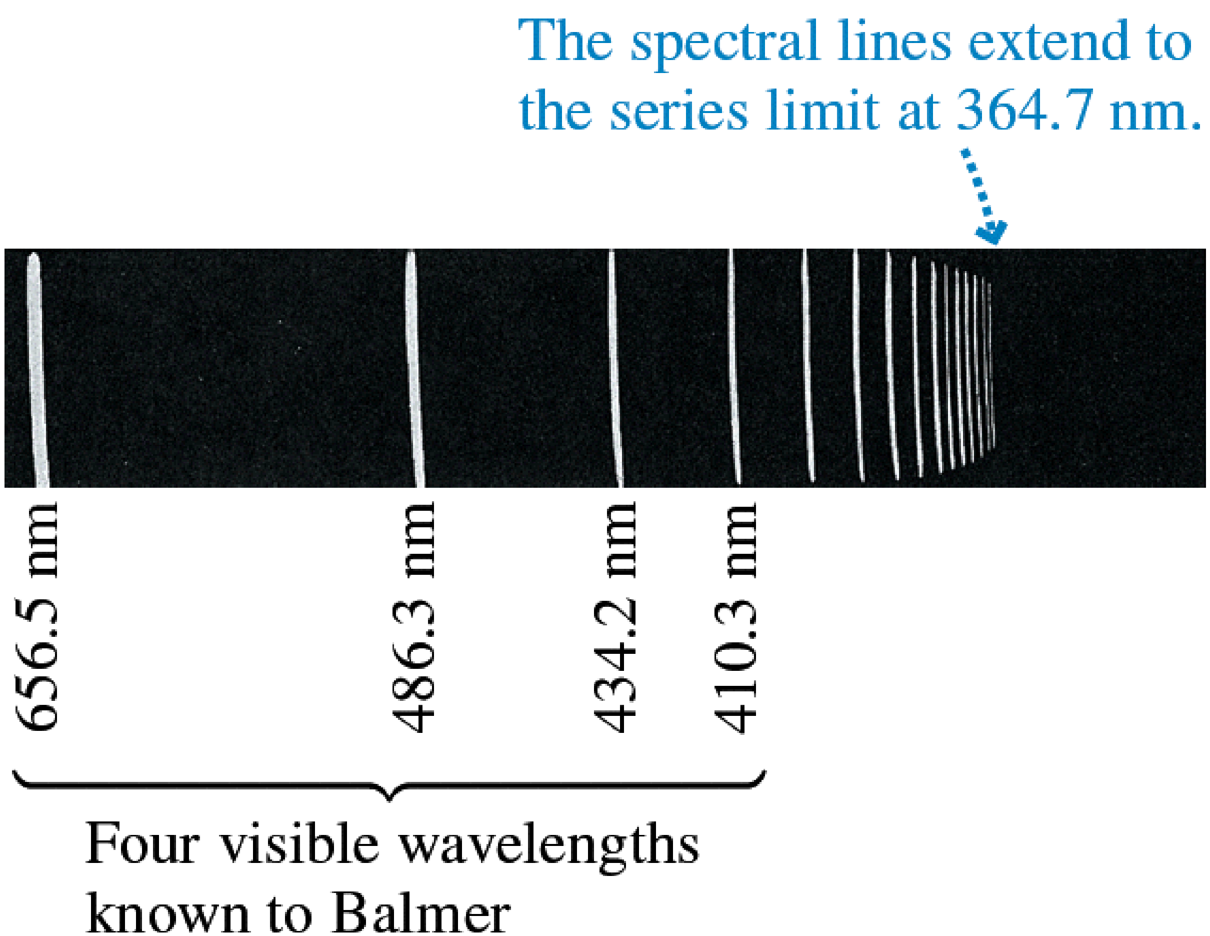
- When energy is added to gases of pure atoms or molecules by a spark, they give off light, but not a continuous spectrum.
- They emit light of a number of specific colors — *line spectra*.
- The positions of the lines are characteristic of the particular atoms or molecules.

- In 1885 a Swiss schoolteacher named Johann Balmer discovered a formula which accurately describes every wavelength in the emission spectrum of hydrogen:

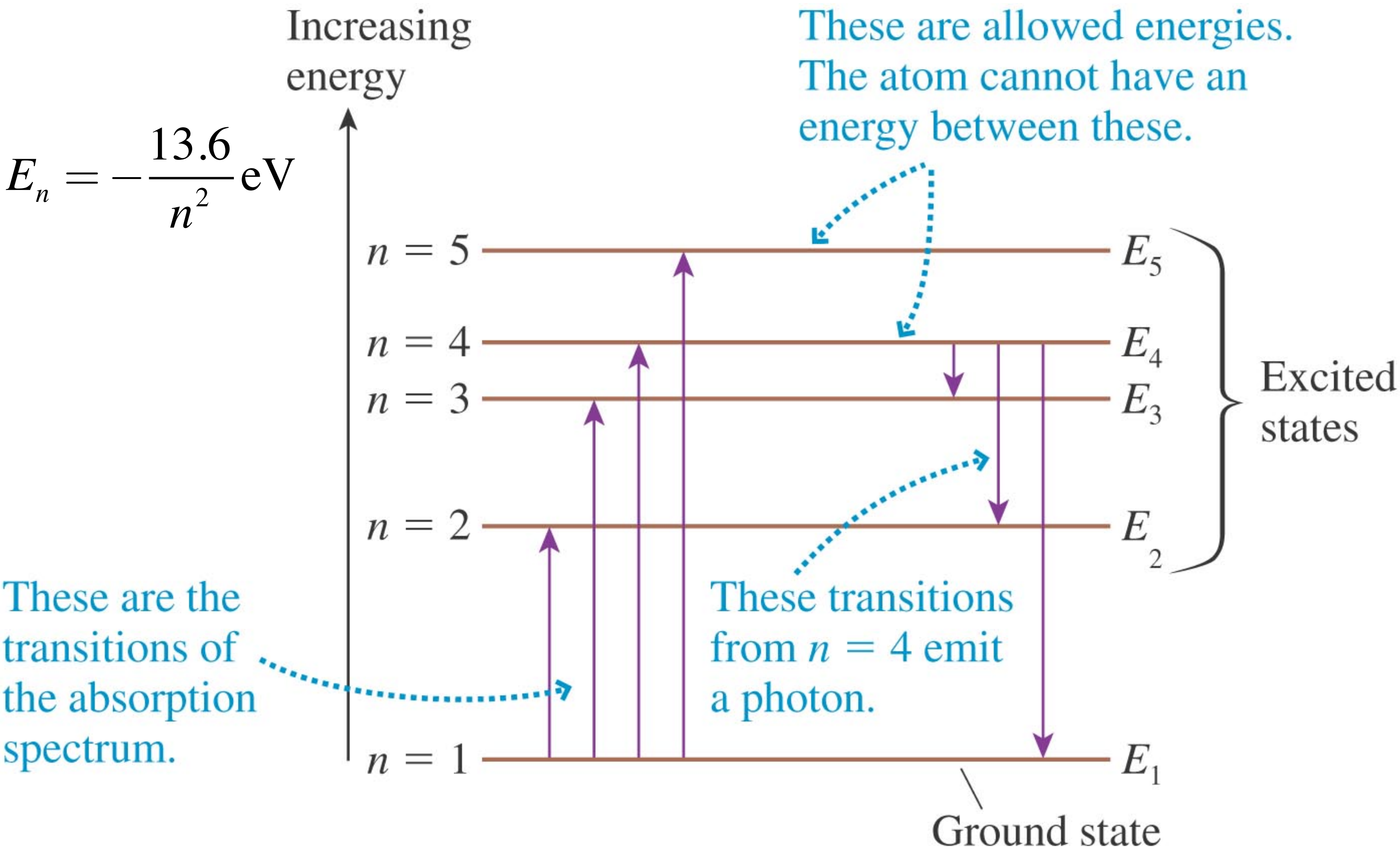
$$\lambda = \frac{91.18 \text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad m = 1, 2, 3, \ldots \qquad n = m + 1, m + 2, \ldots$$

- This result is called the **Balmer formula**.
- Balmer's original version only included $m = 2$.
- When first discovered, the Balmer formula was *empirical knowledge*; it did not rest on any physical principles or physical laws.

FIGURE 25.3 The Balmer series of hydrogen as seen on the photographic plate of a spectrometer.



$$E_n = -\frac{13.6}{n^2}\,\mathrm{eV}$$



The Spectrum of Hydrogen

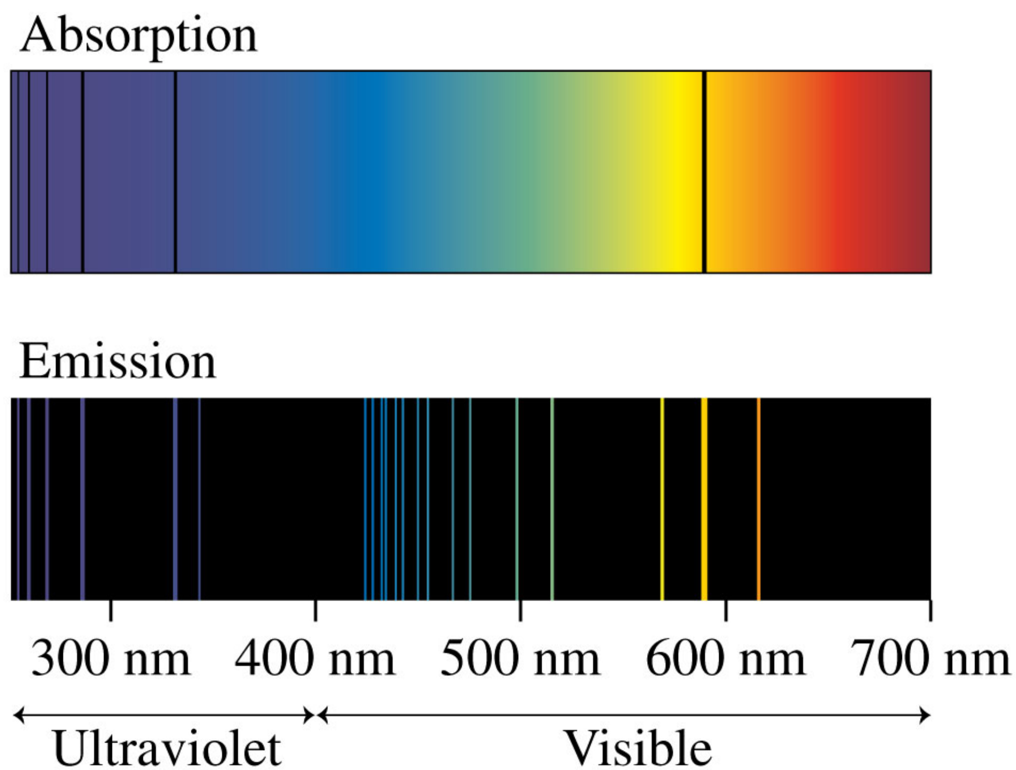
- Hydrogen is the simplest atom, with one electron orbiting a proton, and it also has the simplest atomic spectrum.
- The emission lines have wavelengths which correspond to two integers, *m* and *n*.
- Every line in the hydrogen spectrum has a wavelength given by

$$\Delta E = \frac{hc}{\lambda} = |E_n - E_m| = 13.6eV \left| \frac{1}{n^2} - \frac{1}{m^2} \right|$$

$$\lambda = \frac{91.18\text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \vdots & \end{cases}$$

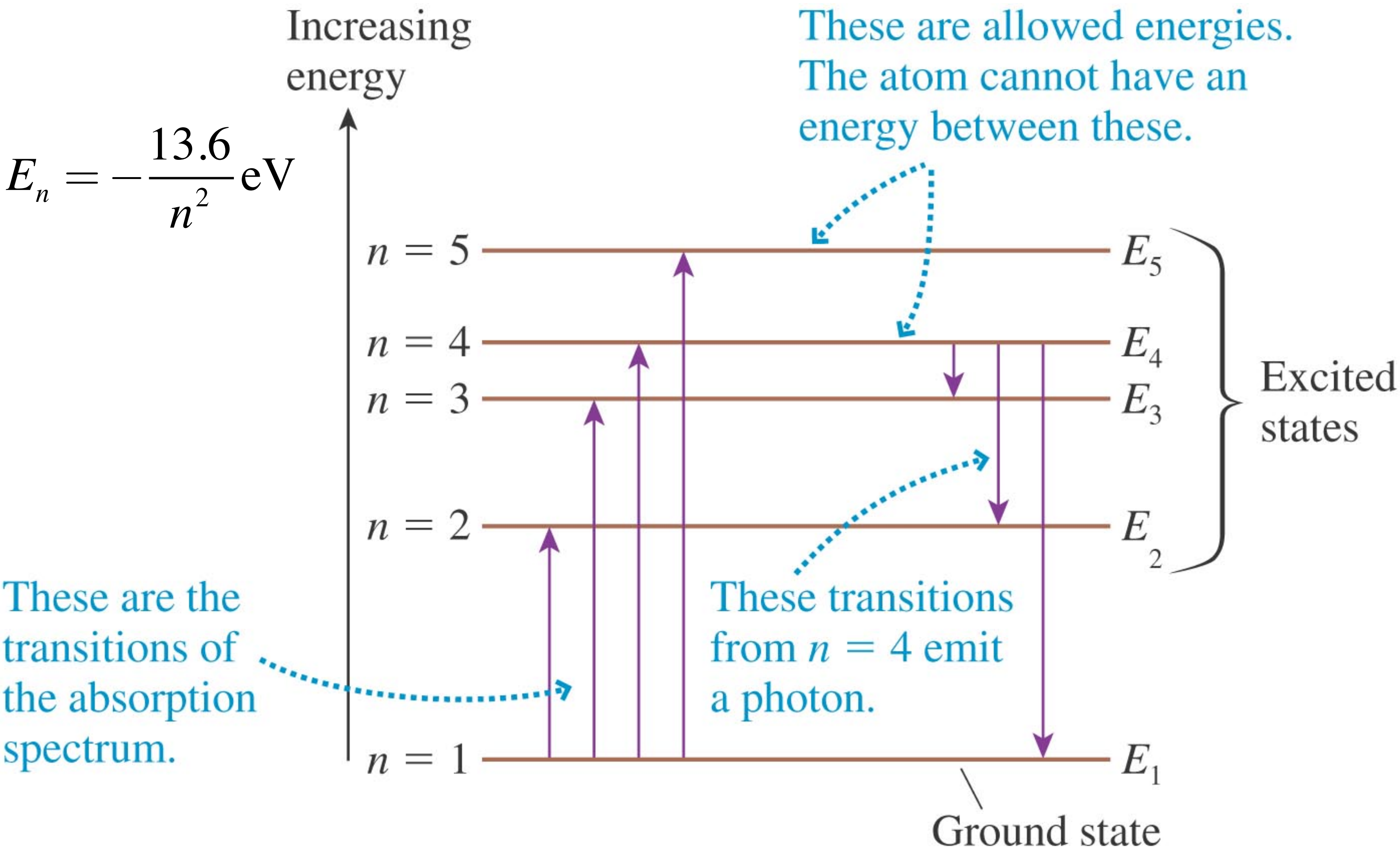
$$n = m + 1, m + 2, \ldots$$

- Not only do gases emit discrete wavelengths, they also absorb discrete wavelengths.
- The top figure shows the spectrum when white light passes through a sample of gas.
- Any wavelengths absorbed by the gas are missing, and the film is dark at that wavelength.

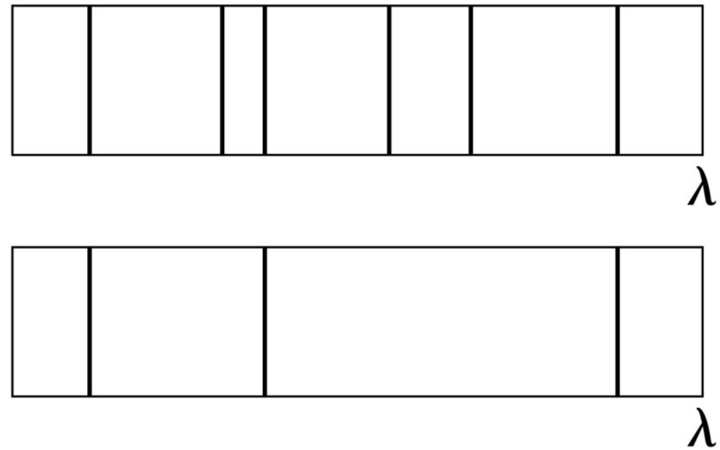


- Although the emission and absorption spectra of a gas are both discrete, there is an important difference.
- **Every wavelength absorbed by the gas is also emitted, but *not* every emitted wavelength is absorbed.**
- The wavelengths in the absorption spectrum are a subset of those in the emission spectrum.
- All the absorption wavelengths are prominent in the emission spectrum, but there are many emission wavelengths for which no absorption occurs.

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$



These spectra are from the same element. Which is an emission spectrum, which an absorption spectrum?

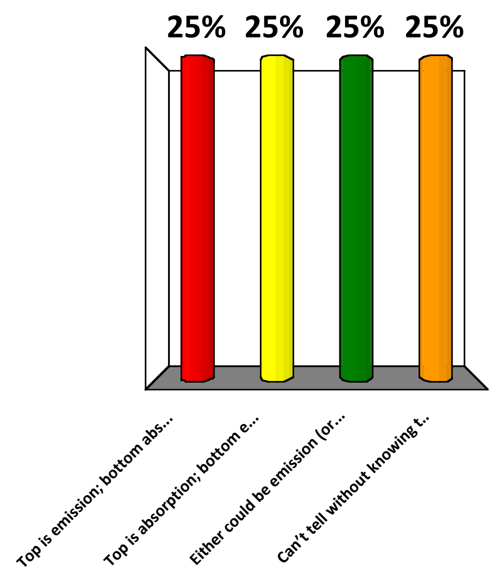


A. Top is emission; bottom absorption.

B. Top is absorption; bottom emission.

C. Either could be emission (or absorption), depending on the conditions with which they were made.

D. Can’t tell without knowing the element.



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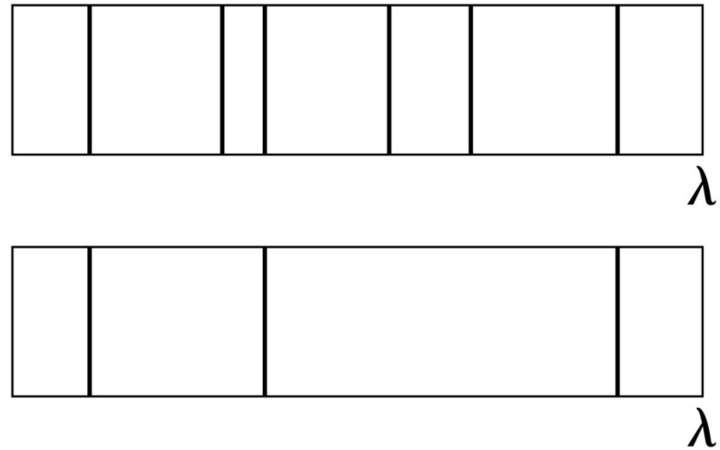

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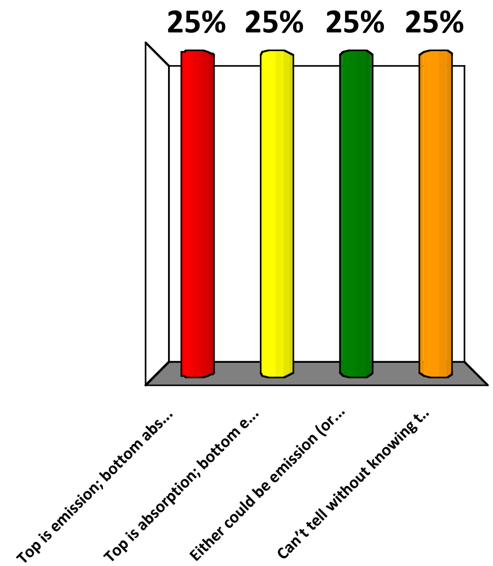
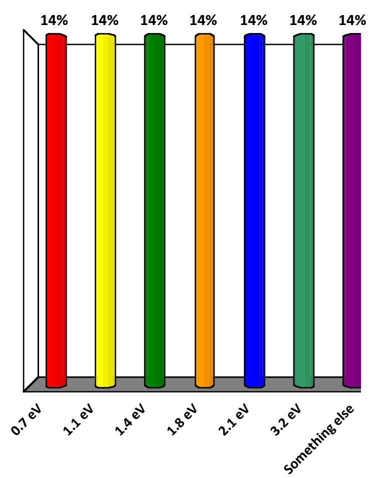
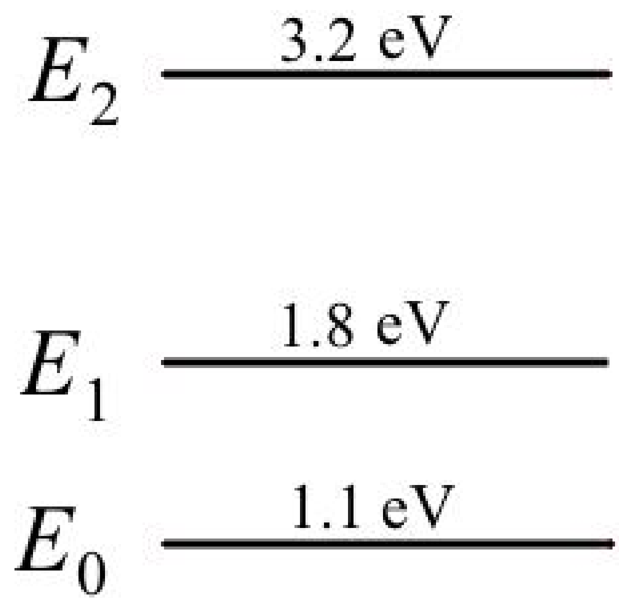


A molecule has the energy levels shown in the diagram at the right. We begin with a large number of these molecules in their ground states. We want to raise a lot of these molecules to the state labeled E_2 by shining light on it. What energy photon should we use?

A. 0.7 eV
B. 1.1 eV
C. 1.4 eV
D. 1.8 eV
E. 2.1 eV
F. 3.2 eV
G. Something else





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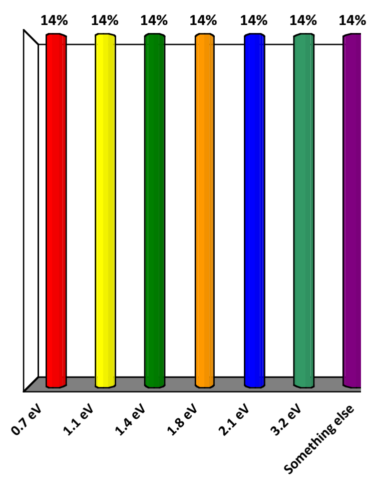
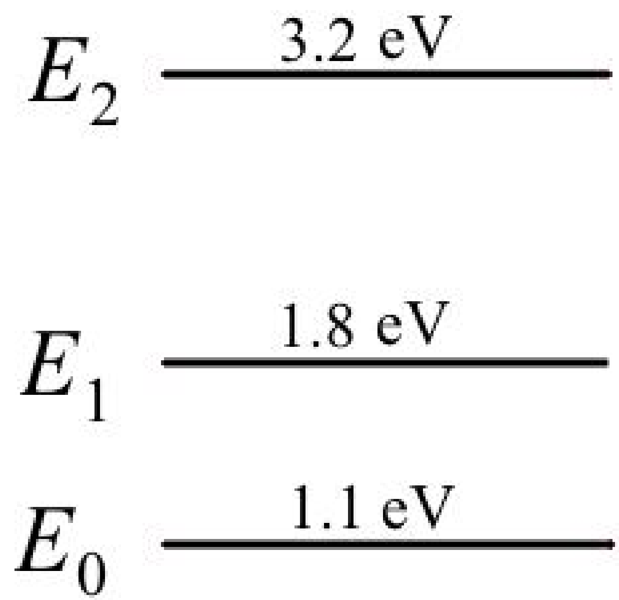
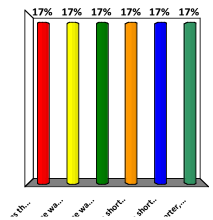
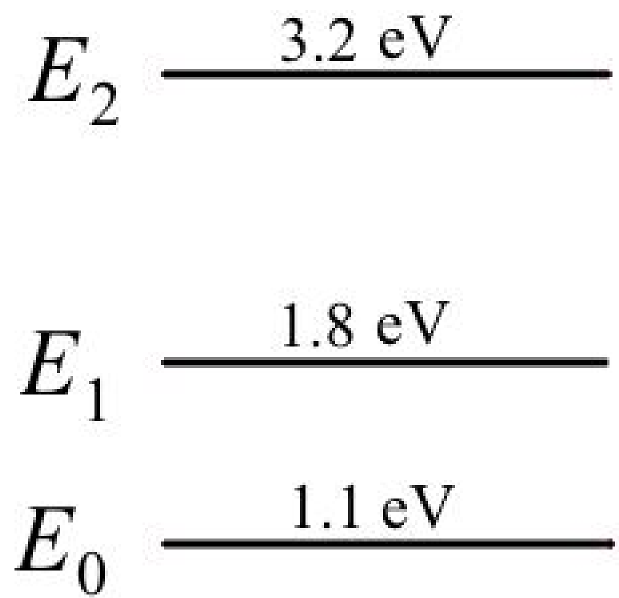




A molecule has the energy levels shown in the diagram at the right. We have a large number of these molecules in the state E_2. The state decays by emitting photons. What might we expect about the wavelength of the emitted photons?



A. They will be the same as the wavelength of the photons that were used to pump the molecules up to state E_2.
B. Some might be the same wavelength, bu some might be shorter.
C. Some might be the same wavelength, bu some might be longer.
D. You only expect to see shorter wavelengths 2.1 eV
E. You only expect to see shorter wavelengths
F. You will see longer, shorter, and the same wavelengths.



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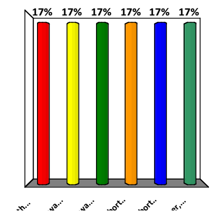

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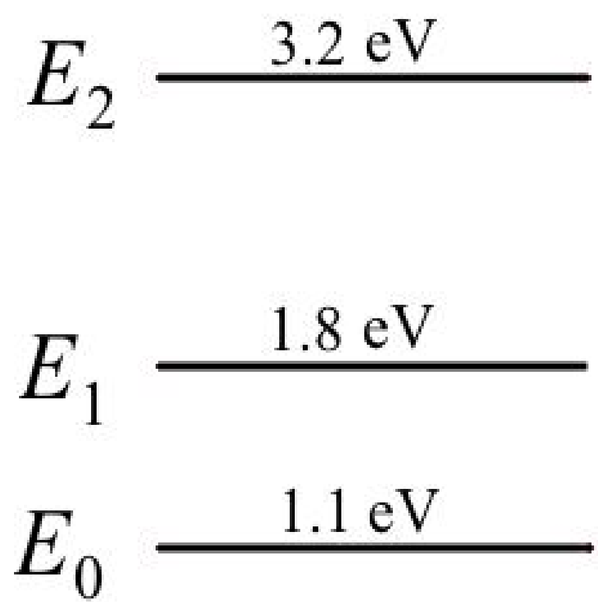

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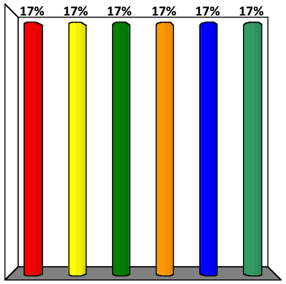
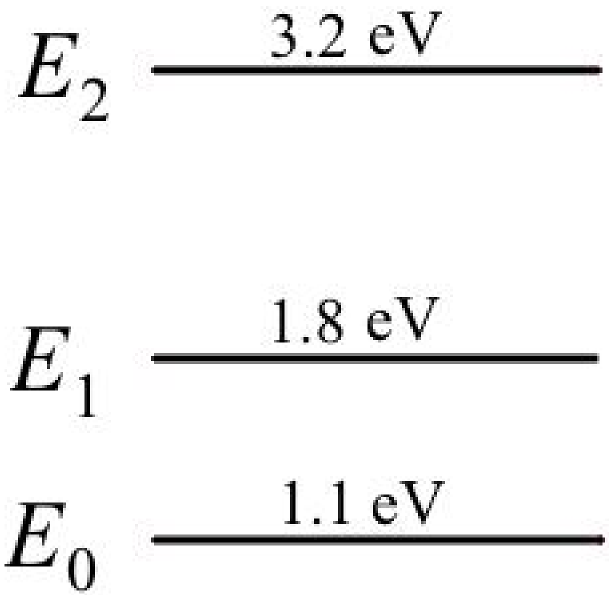


A. 0.7 eV
B. 1.1 eV
C. 1.4 eV
D. 1.8 eV
E. 2.1 eV
F. 3.2 eV

1. B D F
2. B D
3. C
4. C E
5. A C E
6. Some other set



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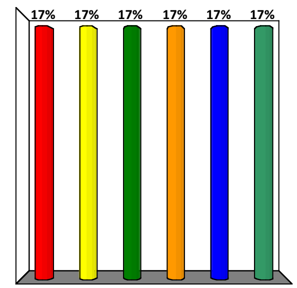

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1. B D F
2. B D
3. C
4. C E
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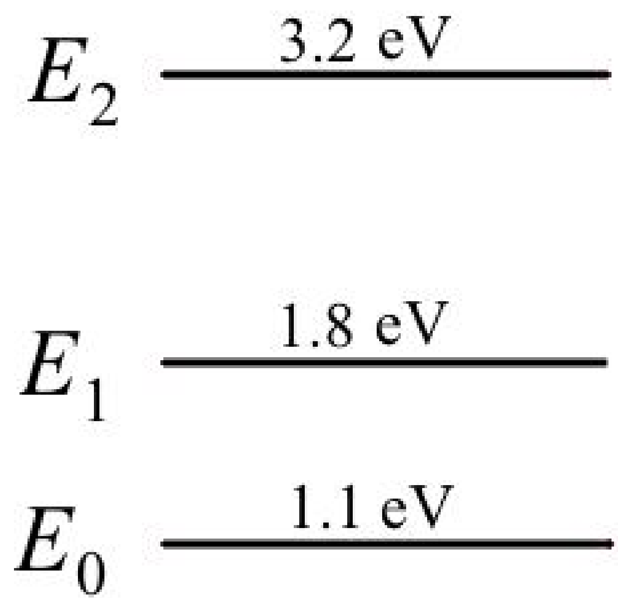

A photon with a wavelength of 414 nm has energy $E_{photon} = 3.0$ eV. Do you expect to see a spectral line with $\lambda = 414$ nm in the absorption spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will absorb it?

A. Yes. 1→0

B. Yes. 2→0

C. Yes. 2→1

D. Yes. 3→1

E. No

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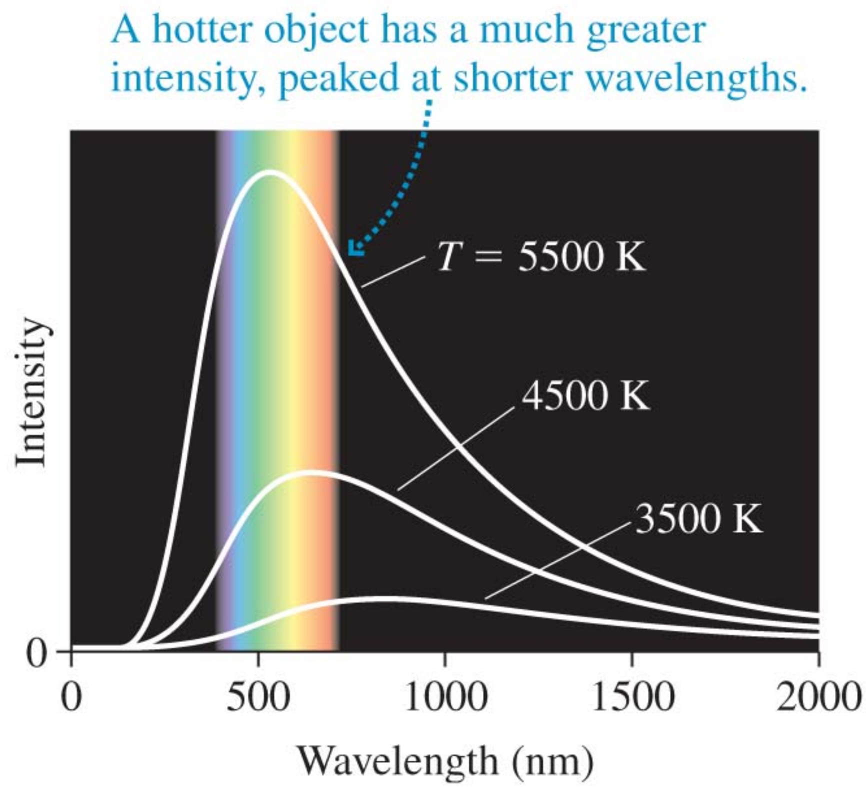
A. Yes. 1→0
B. Yes. 2→0
C. Yes. 2→1
D. Yes. 3→1
✔ **E. No**



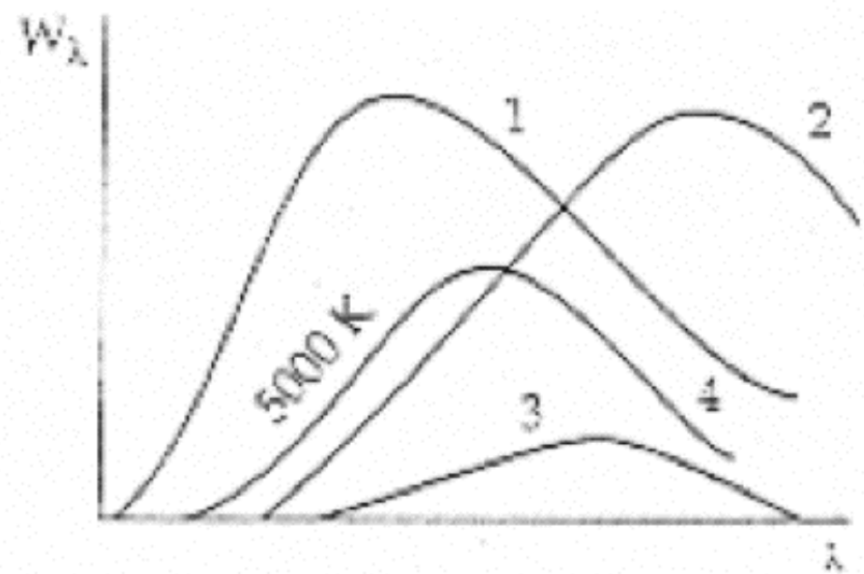
The wavelength of the peak in the intensity graph is given by Wien's law (T must be in kelvin):

$$\lambda_{\text{peak}}(\text{in nm}) = \frac{2.90 \times 10^6 \text{ nm K}}{T}$$

Wien's Displacement Law

The curve 4 is a blackbody radiation curve for a body at a temperature of 5000 K. The number of the curve that might be the radiation curve for a body at 7000 K is

a. 1
b. 2
c. 3
d. 4
e. none of these



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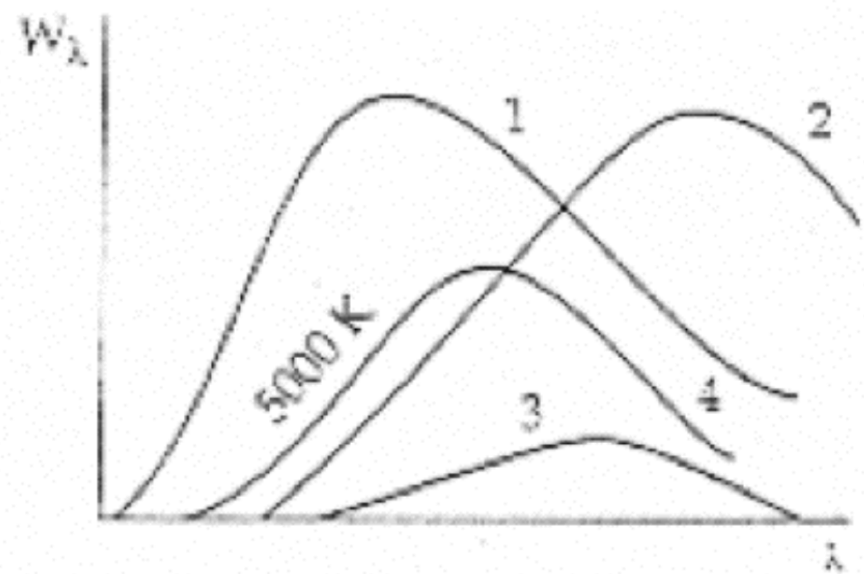
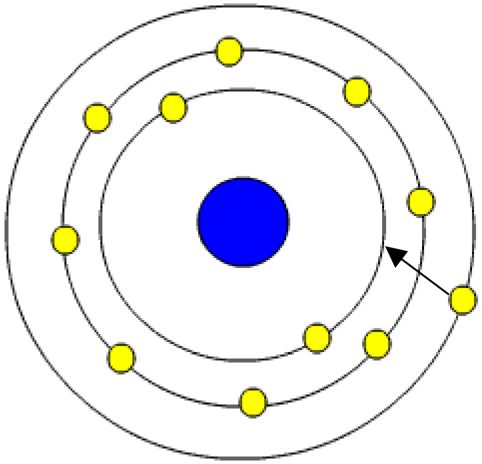
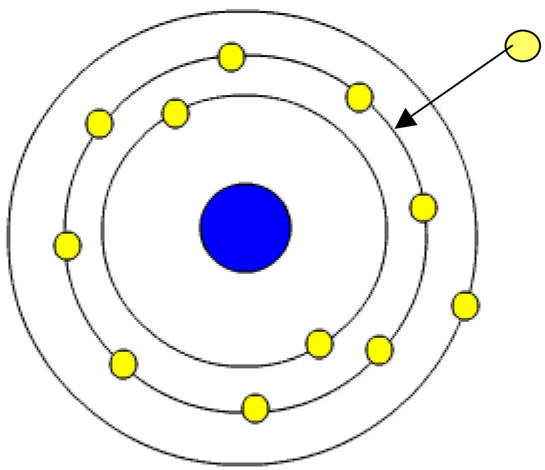




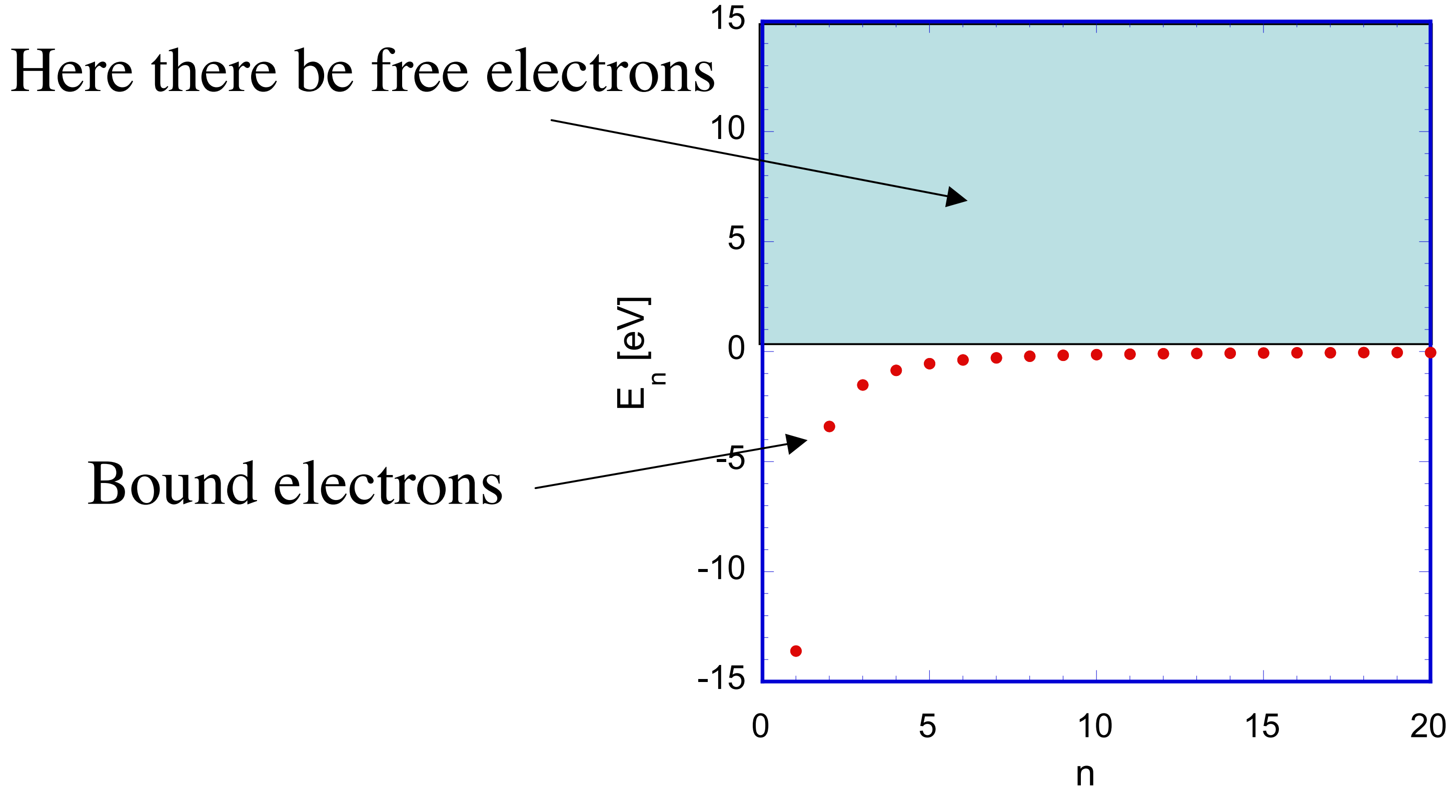
Energy levels of an isolated atom are quantized. When an electron makes a transition from one state to another it gives up a specific amount of energy that creates a photon with a specific wavelength.

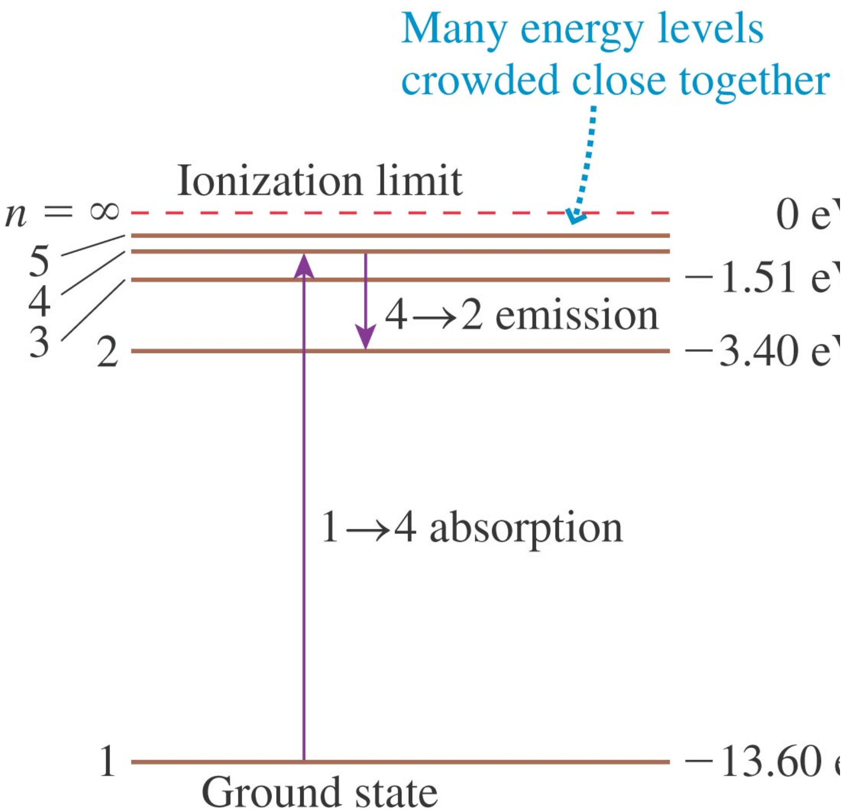


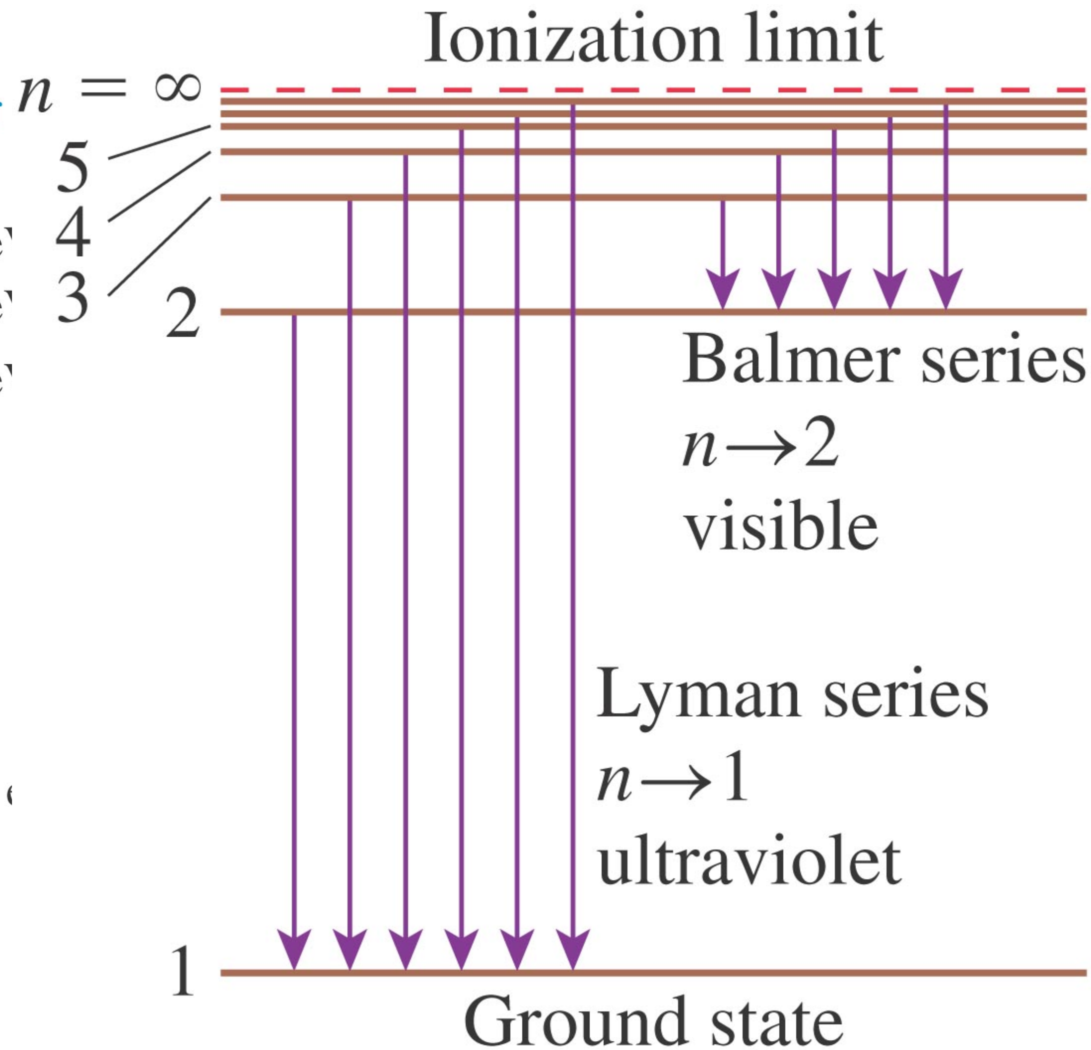
A free electron can have any energy. When it is captured by an ion the amount of energy going to make a photon is not some specific value but falls in a range of values.

Bound electron energy

$$E_n = -\frac{1}{n^2}\frac{e^2}{8\pi\varepsilon_0 a_0} = -\frac{2.18\times 10^{-18}}{n^2}\,\mathrm{J} = -\frac{13.6}{n^2}\,eV$$




Many energy levels
crowded close together
Ionization limit
n = ∞
0 e
5
4
3
2
−1.51 e
4→2 emission
−3.40 e
1→4 absorption
1
−13.60
Ground state


Ionization limit
n = ∞
5
4
3
2
Balmer series
n→2
visible
Lyman series
n→1
ultraviolet
1
Ground state

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