Refraction and Lenses

Snell's Law

Total internal reflection

Dispersion

Absorption

Scattering

Two things happen when a light ray is incident on a smooth boundary between two transparent materials:

1. Part of the light *reflects* from the boundary, obeying the law of reflection.
2. Part of the light continues into the second medium. The transmission of light from one medium to another, but with a change in direction, is called **refraction**.

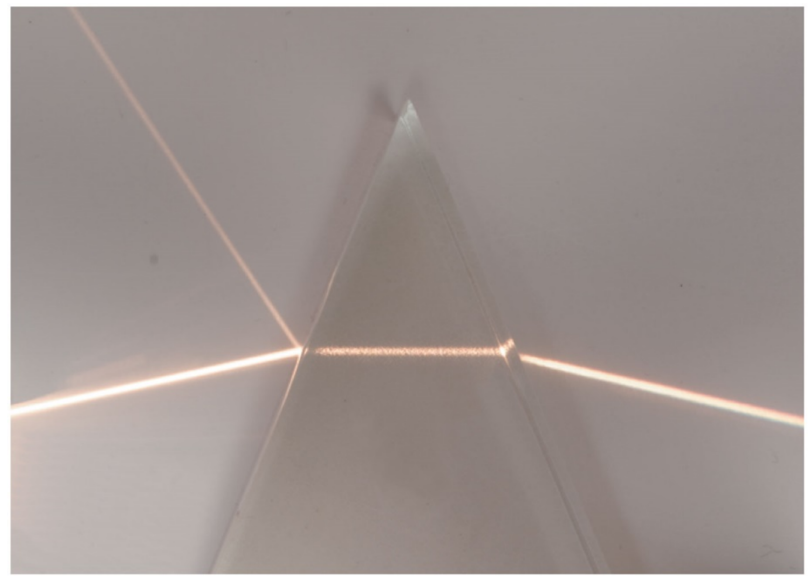
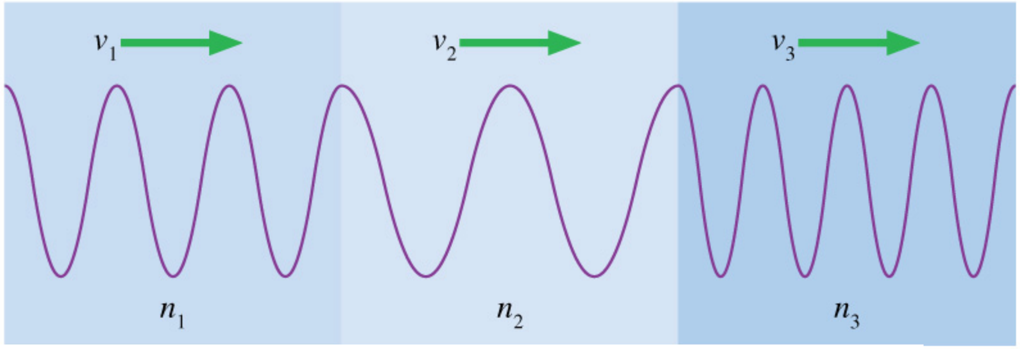


TABLE 23.1 Indices of refraction

Medium	***n***
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

$$n = \frac{c}{v_{\text{medium}}}$$

A light wave travels through three transparent materials of equal thickness. Rank in order, from the largest to smallest, the indices of refraction n_1, n_2, and n_3.



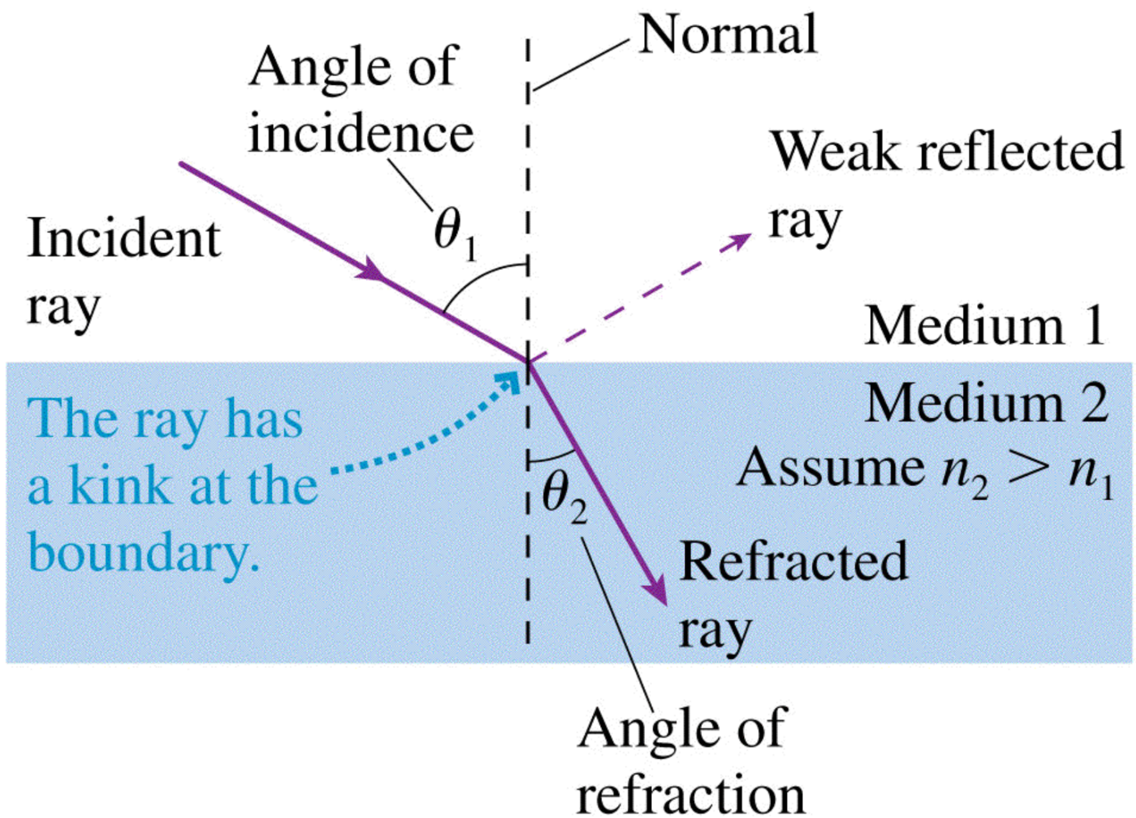
A. $n_1 > n_2 > n_3$

B. $n_2 > n_1 > n_3$

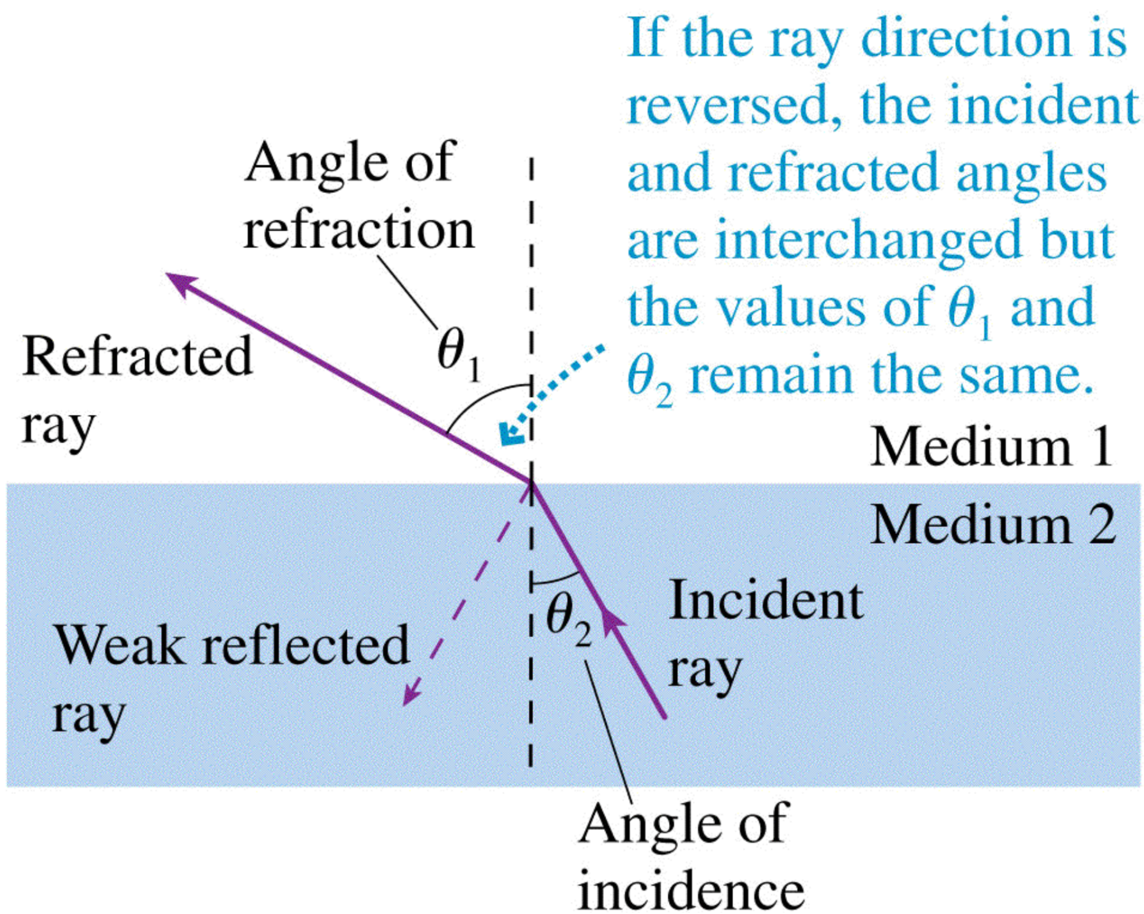
✔ **C. $n_3 > n_1 > n_2$**

D. $n_3 > n_2 > n_1$

E. $n_1 = n_2 = n_3$



$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's law of refraction)}$$



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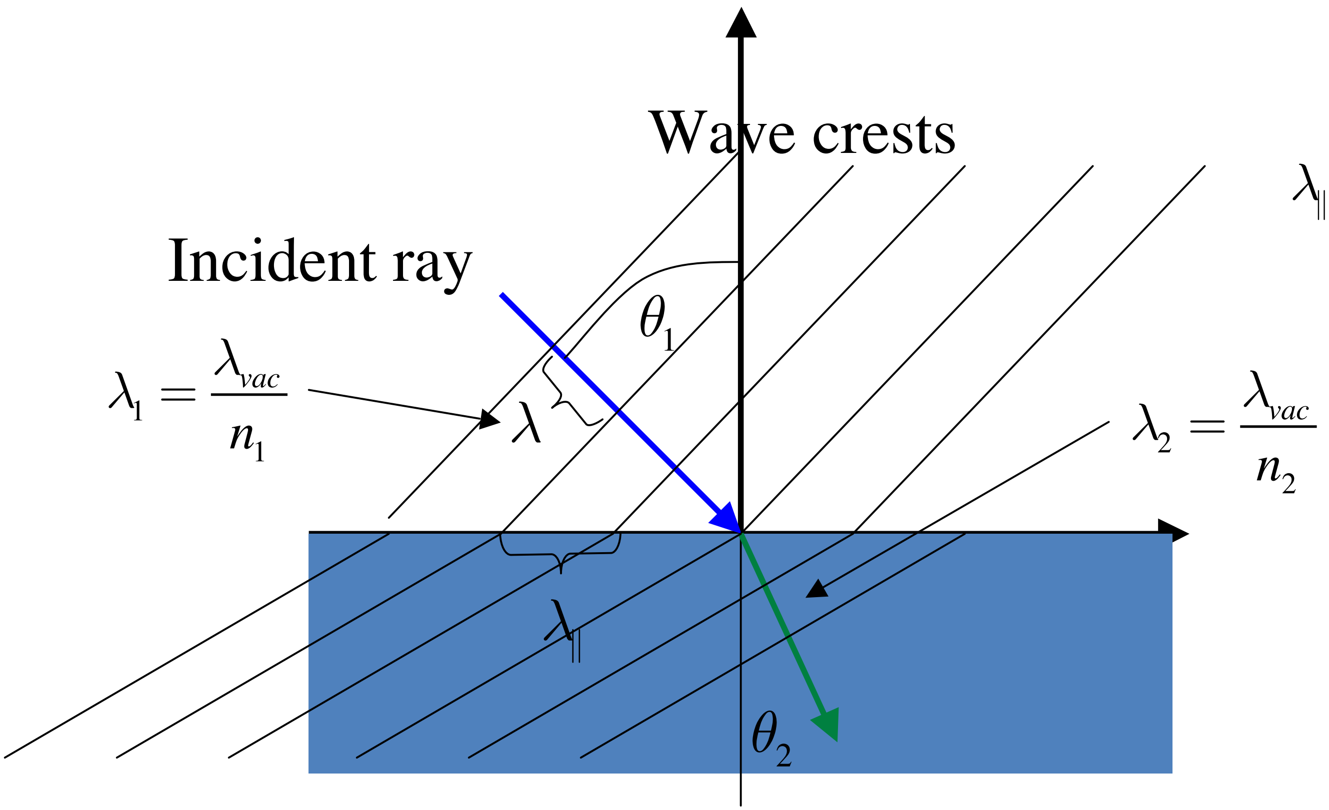
Why Snell's Law?

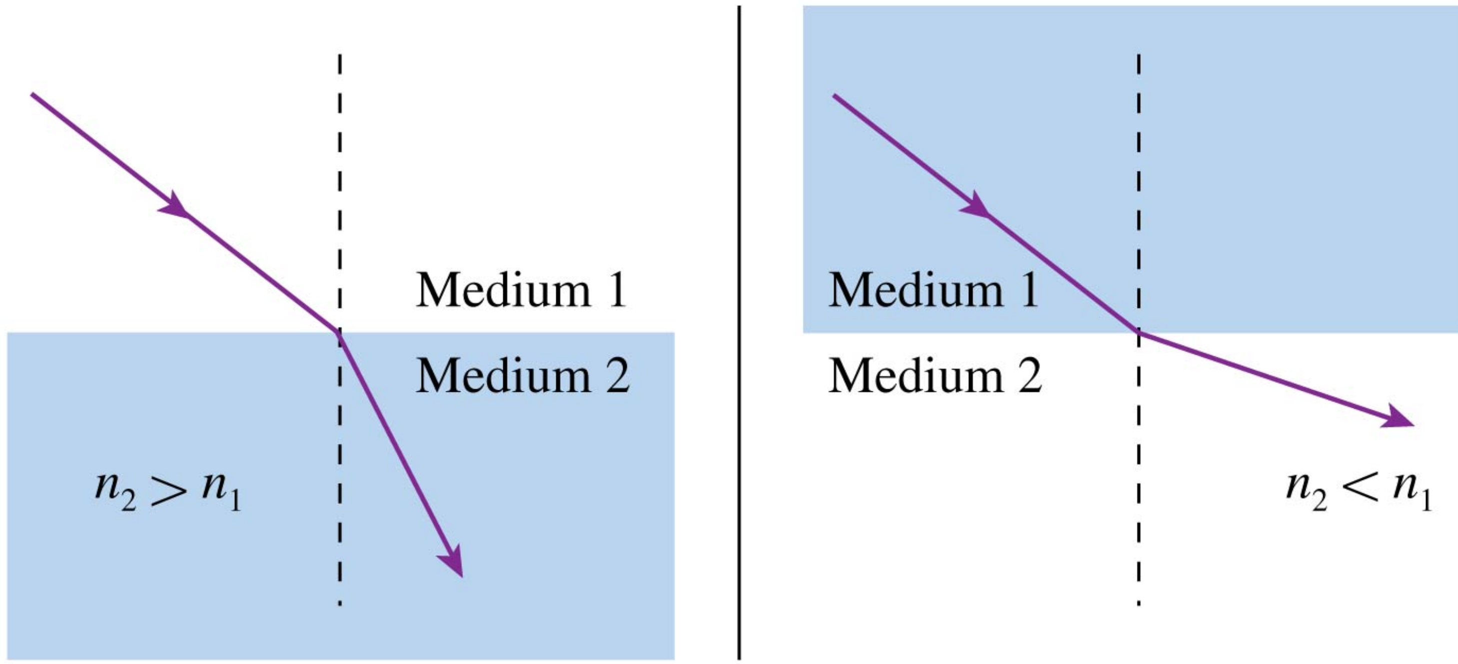


$$\lambda_\parallel = \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

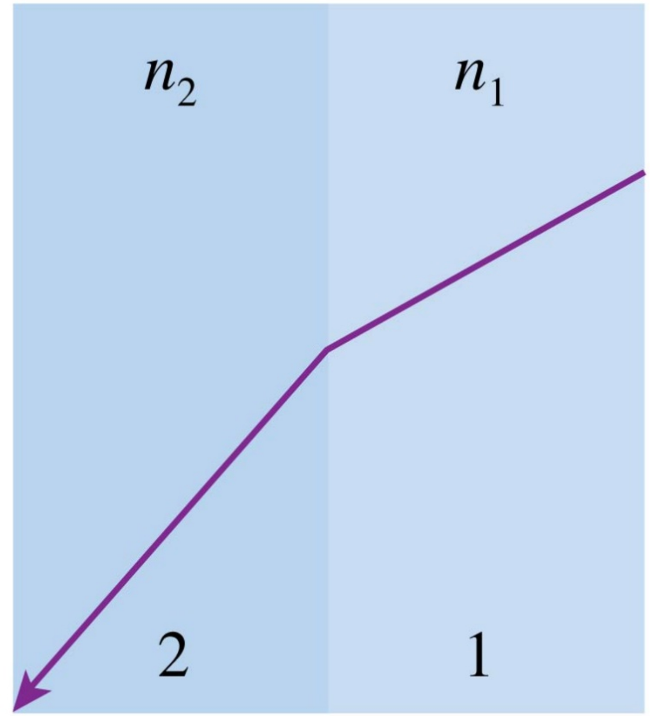
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Incident and Transmitted wave crests must match up along surface

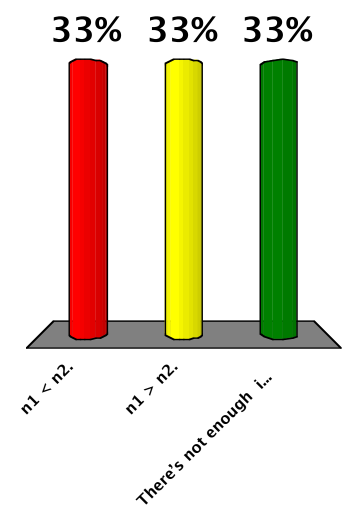
- When a ray is transmitted into a material with a higher index of refraction, it bends *toward* the normal.
- When a ray is transmitted into a material with a lower index of refraction, it bends *away from* the normal.



A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?



A. $n_1 < n_2$.
B. $n_1 > n_2$.
C. There's not enough information to compare n_1 and n_2.

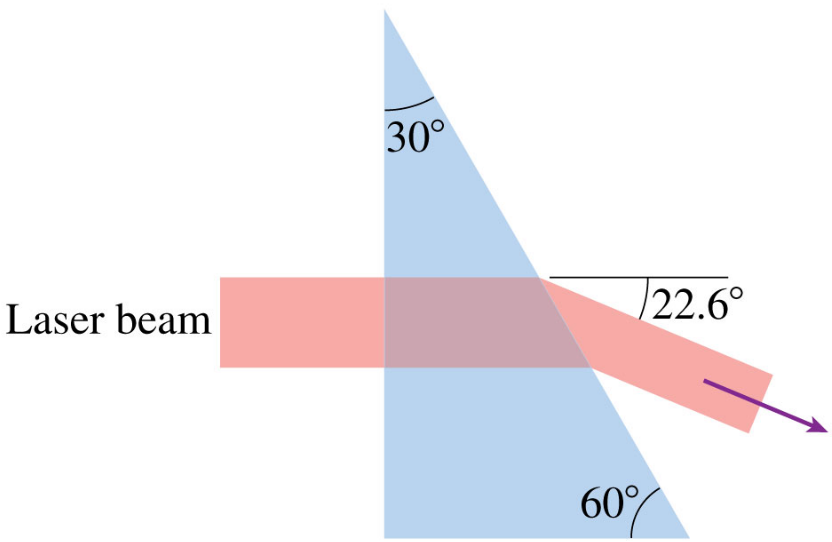


Whiteboard, TA & LA

Measuring the index of refraction

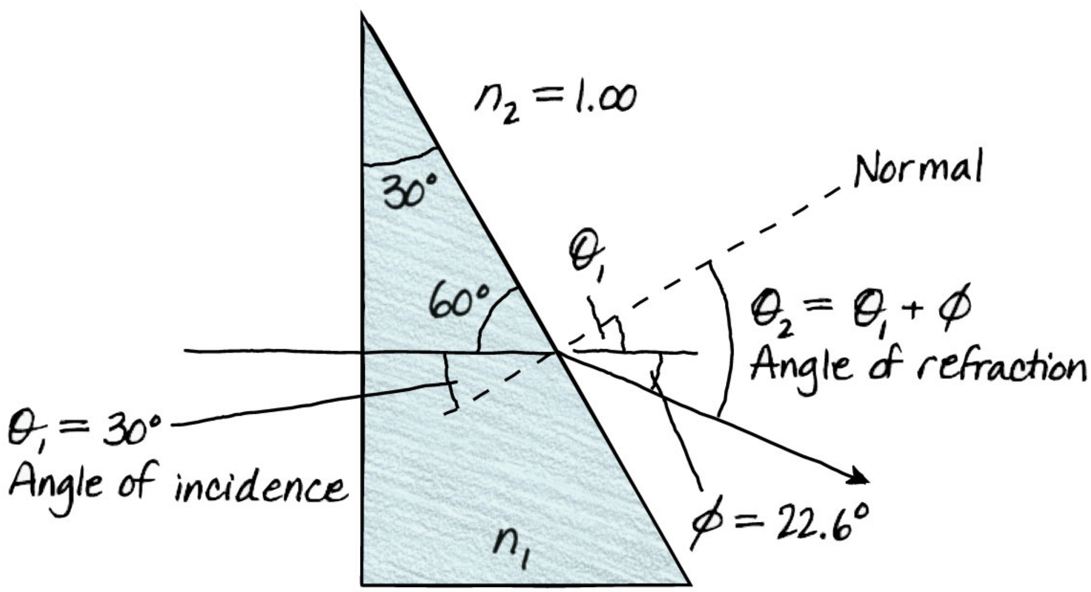
The figure below shows a laser beam deflected by a 30°-60°-90° prism. What is the prism's index of refraction?

MODEL Represent the laser beam with a single ray and use the ray model of light.



What is the angle of incidence?

What is the angle of refraction ?

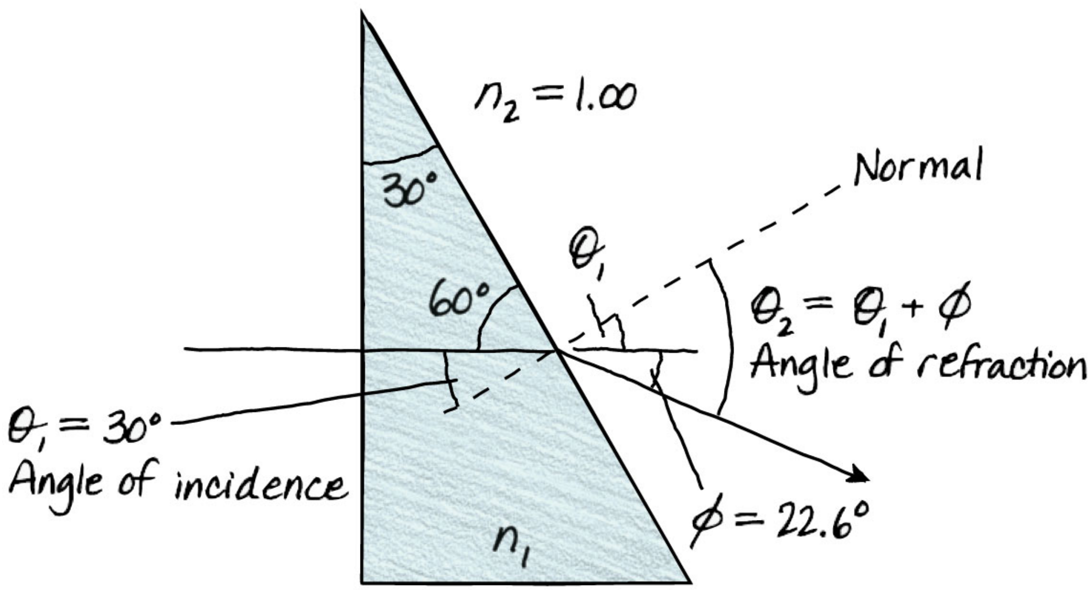


θ_1 and θ_2 are measured from the normal.

Measuring the index of refraction

SOLVE From the geometry of the triangle you can find that the laser's angle of incidence on the hypotenuse of the prism is $\theta_1 = 30°$, the same as the apex angle of the prism. The ray exits the prism at angle θ_2 such that the deflection is $\phi = \theta_2 - \theta_1 = 22.6°$. Thus $\theta_2 = 52.6°$. Knowing both angles and $n_2 = 1.00$ for air, we can use Snell's law to find n_1:

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{1.00 \sin 52.6°}{\sin 30°} = 1.59$$



θ_1 and θ_2 are measured from the normal.

$$n_1 = 1.59$$

ASSESS Referring to the indices of refraction in Table 23.1, we see that the prism is made of plastic.

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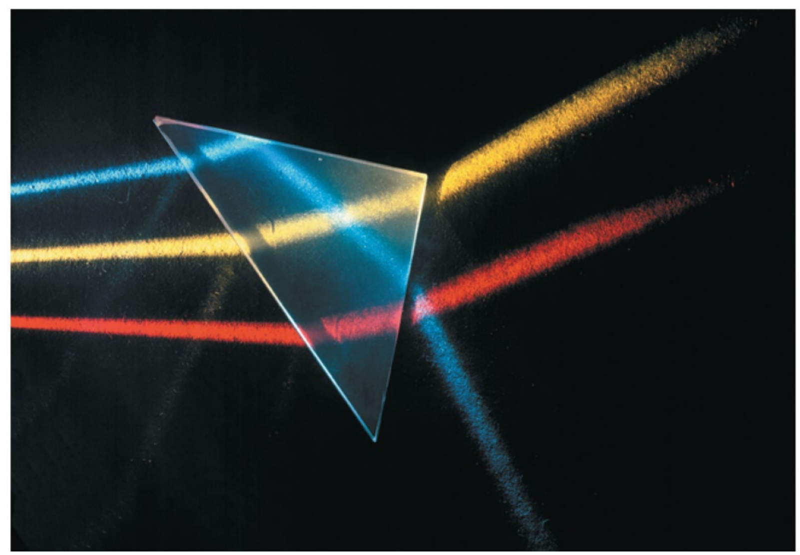
- When a ray crosses a boundary into a material with a lower index of refraction, it bends *away* from the normal.
- As the angle θ_1 increases, the refraction angle θ_2 approaches 90°, and the fraction of the light energy transmitted decreases while the fraction reflected increases.
- The critical angle of incidence occurs when $\theta_2 = 90°$:

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$

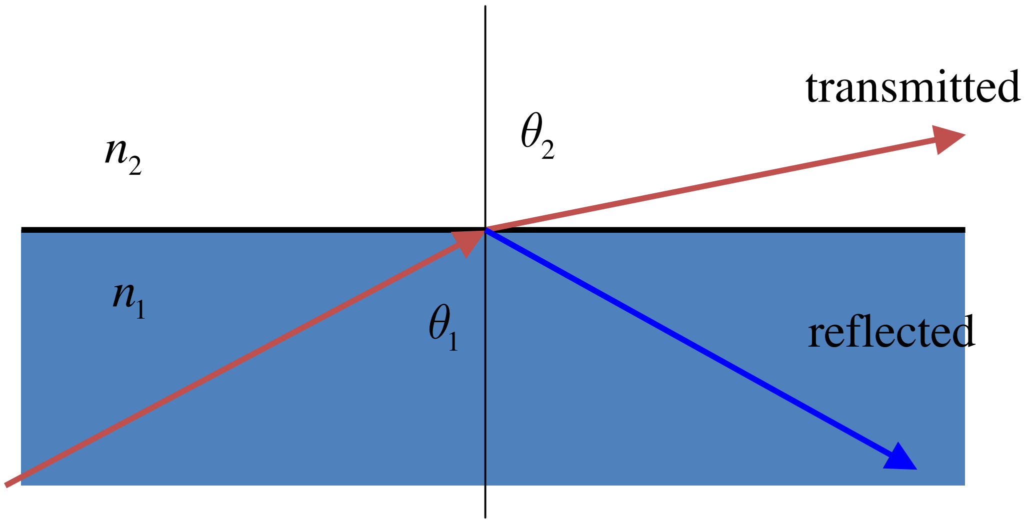


- The refracted light vanishes at the critical angle and the reflection becomes 100% for any angle $\theta_1 > \theta_c$.

Total Internal Reflection

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Based on picture, which is bigger? n_1 or n_2 ?

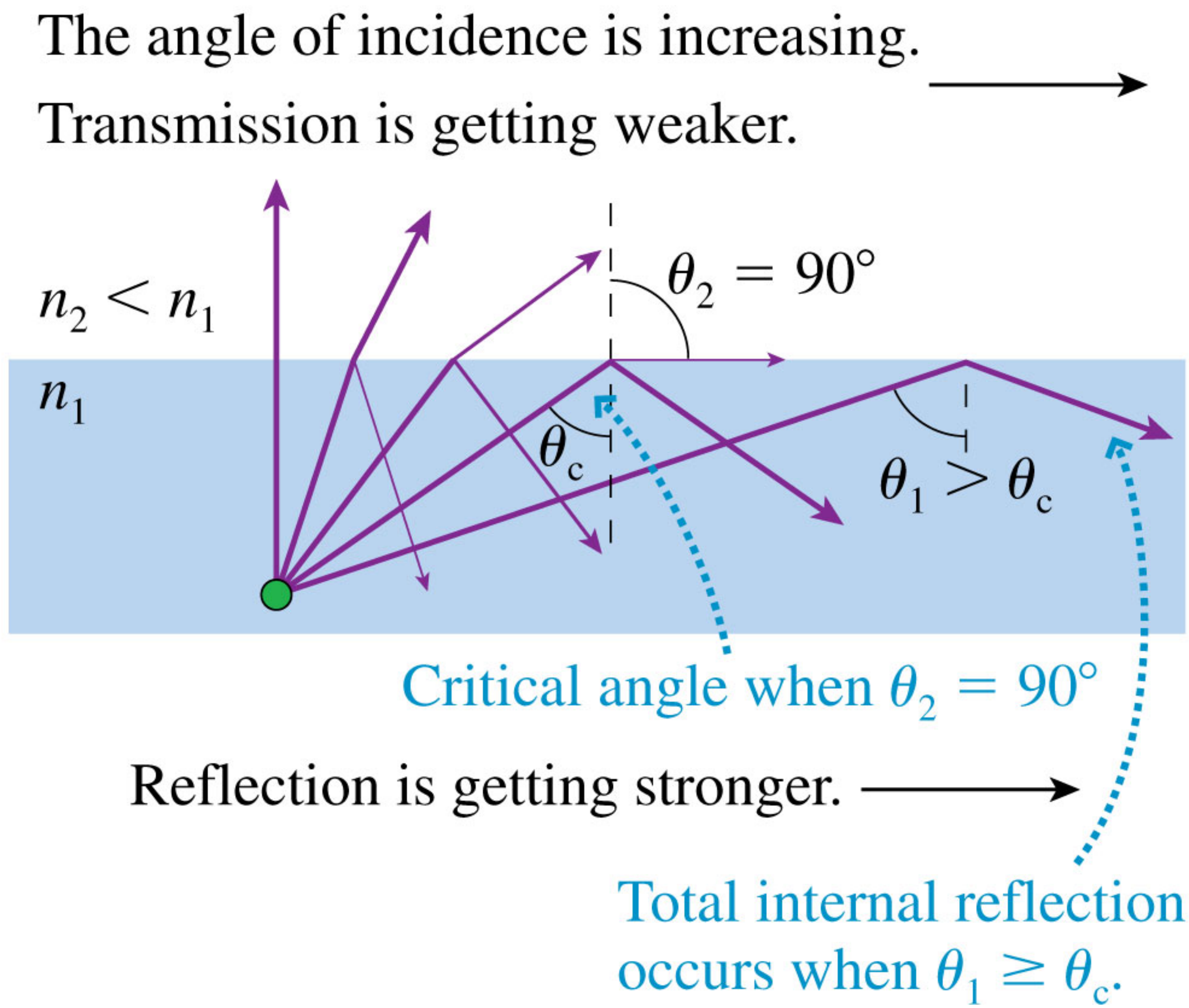
Solve for θ_2

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

What if $\frac{n_1}{n_2}\sin\theta_1 > 1$?

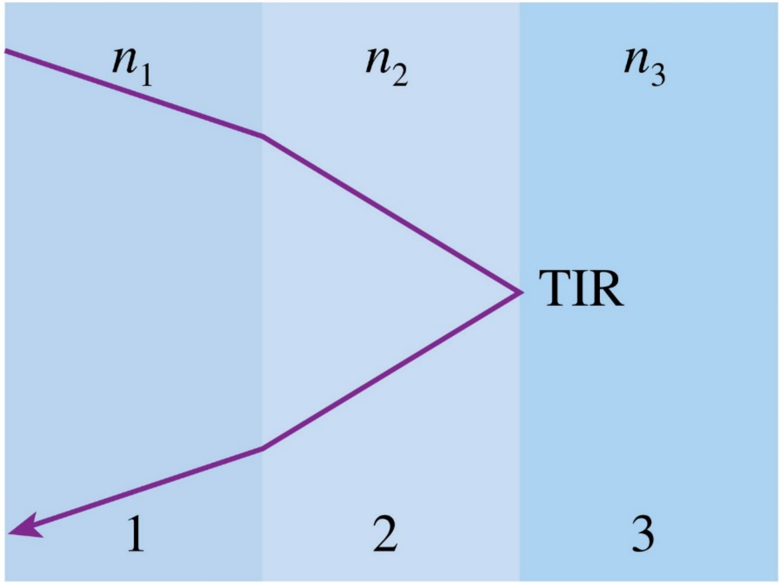
Then there is no θ_2 satisfying SL - no transmission - total reflection

Can only happen if wave is incident from high index material, viz. $n_1 > n_2$.



The angle of incidence is increasing.
Transmission is getting weaker.
$n_2 < n_1$
n_1
$\theta_2 = 90°$
θ_c
$\theta_1 > \theta_c$
Critical angle when $\theta_2 = 90°$
Reflection is getting stronger.
Total internal reflection occurs when $\theta_1 \geq \theta_c$.

A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?



A. $n_1 < n_3$.

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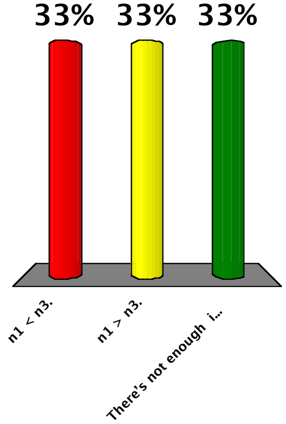
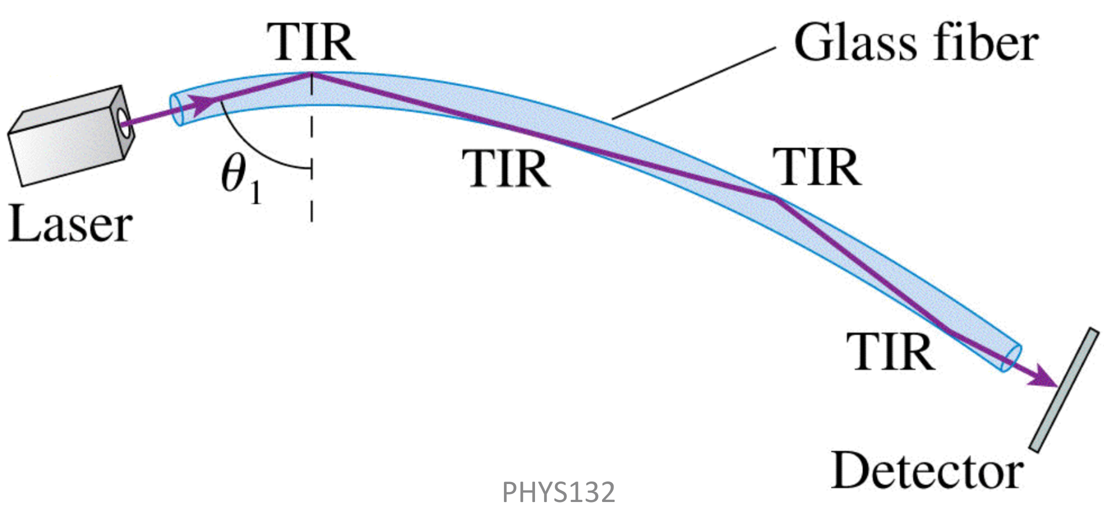
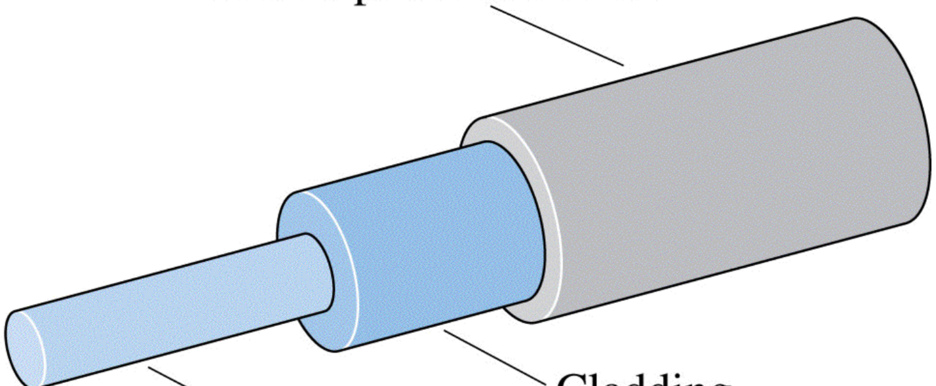


- The most important modern application of **total internal reflection** (TIR) is optical fibers.
- Light rays enter the glass fiber, then impinge on the inside wall of the glass at an angle above the critical angle, so they undergo TIR and remain inside the glass.
- The light continues to “bounce” its way down the tube as if it were inside a pipe.



- In a practical optical fiber, a small-diameter glass core is surrounded by a layer of glass cladding.
- The glasses used for the core and the cladding have:

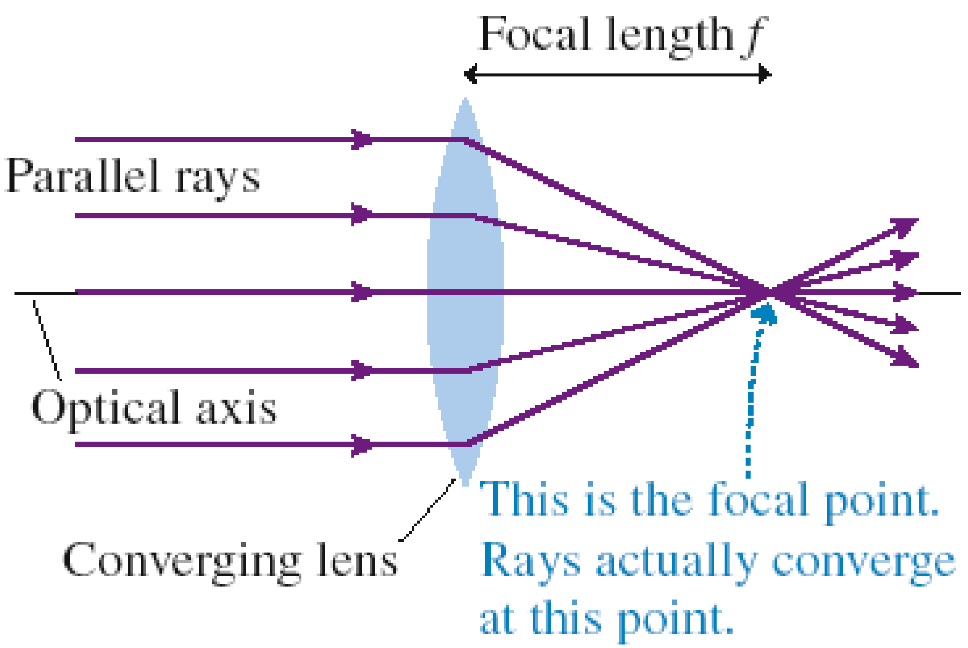
$$n_{\text{core}} > n_{\text{cladding}}$$



Lenses

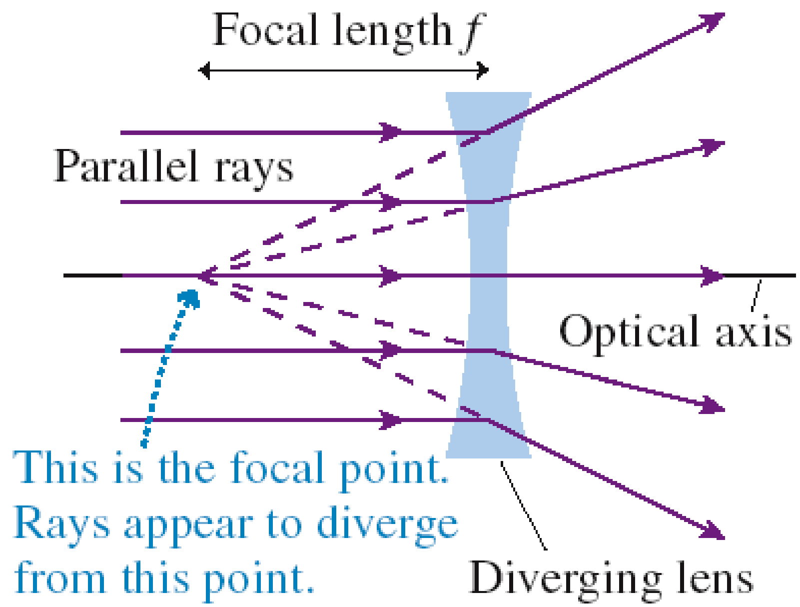
Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of converging and diverging lenses.

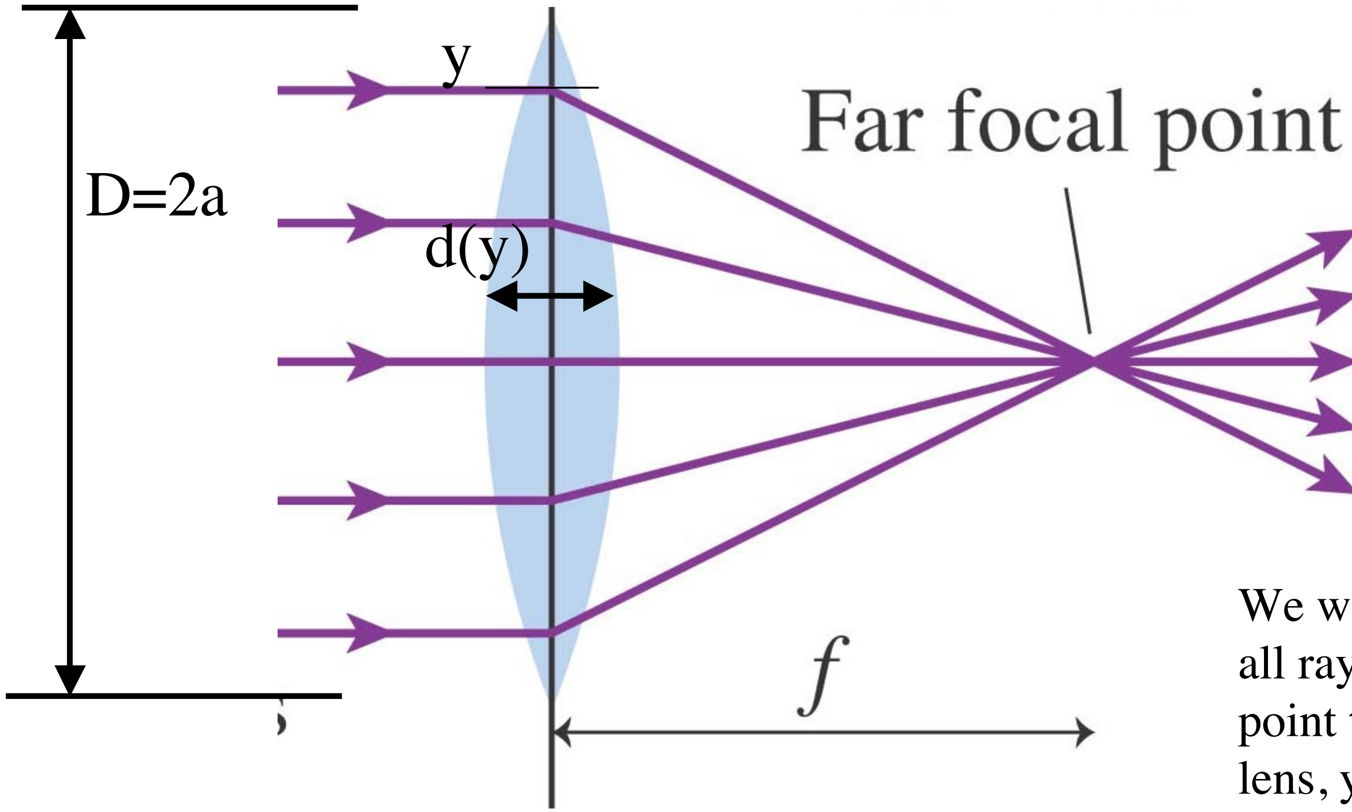


Thin Lenses: Ray Tracing

FIGURE 23.34 The focal point and focal length of converging and diverging lenses.



Thin lens approximation $d << D, f$



We would like to show that all rays, independent of the point they pass through the lens, y, focus to the same point f.

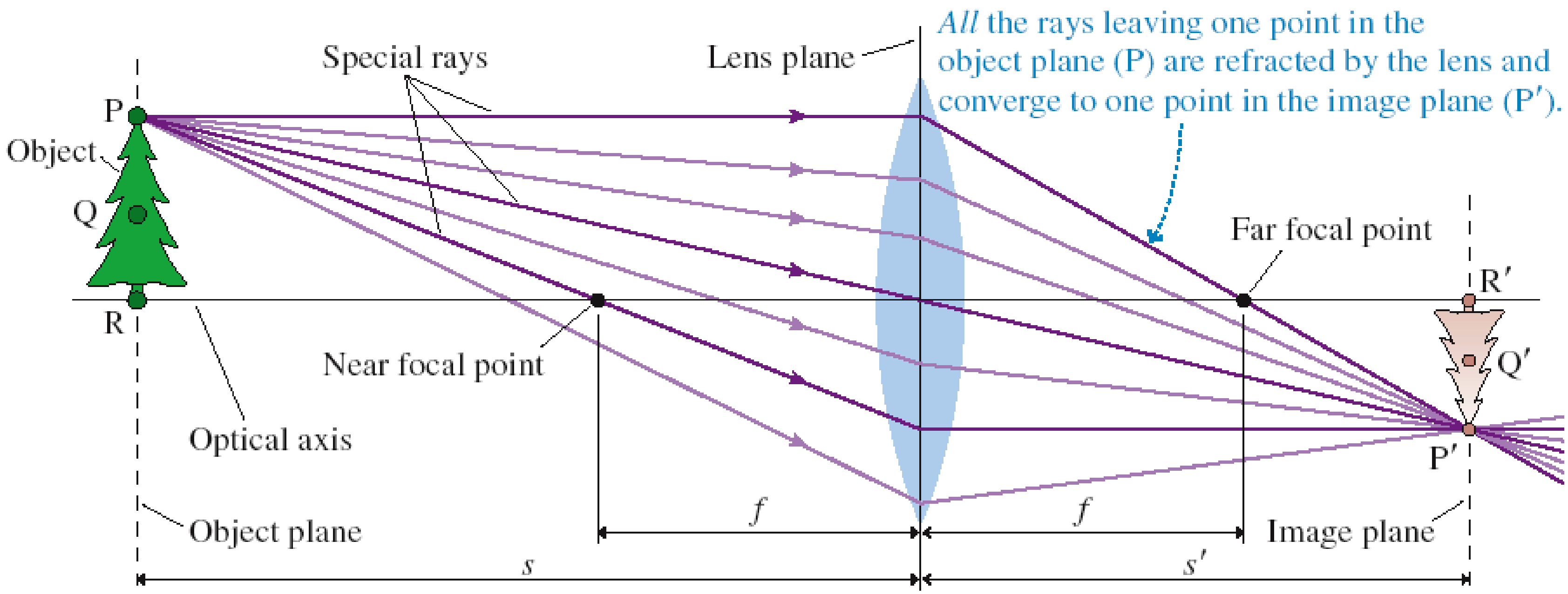
Lens has parabolic thickness

$$d(y) = \frac{a^2 - y^2}{2L}$$

Determines focal length

Thin Lenses: Ray Tracing

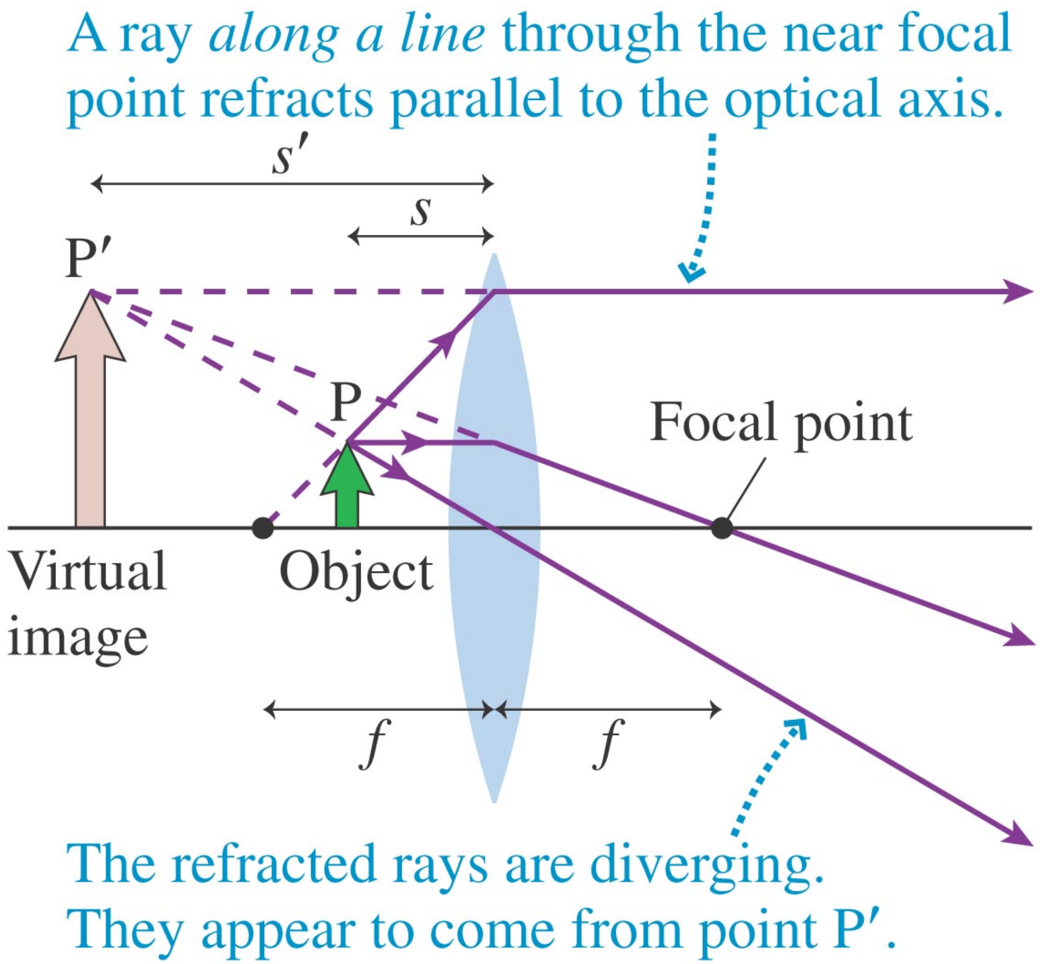
FIGURE 23.36 Rays from an object point P are refracted by the lens and converge to a real image at point P′.



Real Image

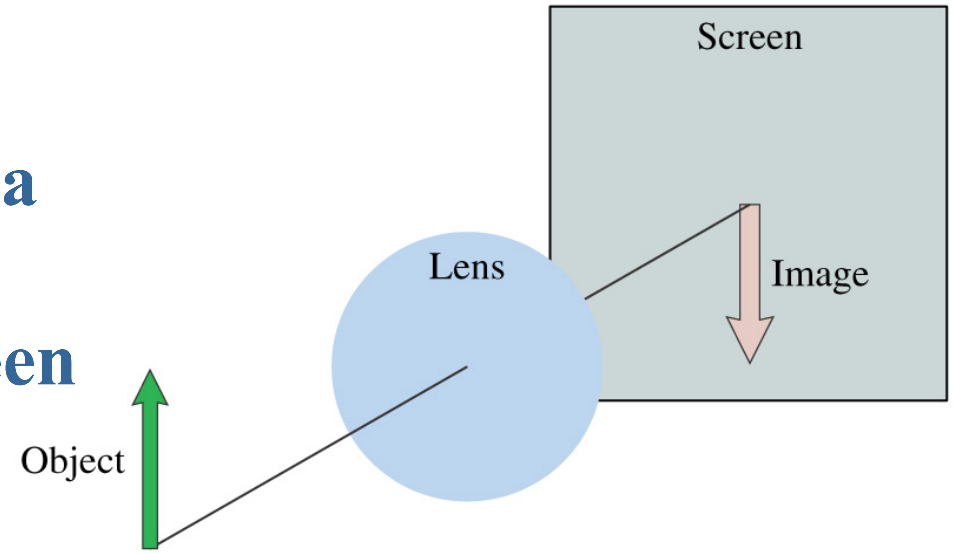
Object further than focal length, $s > f$

Suppose object is closer than focal point to lens $s < f$



Virtual mage located at $s' < 0$

A lens produces a sharply-focused, inverted image on a screen. What will you see on the screen if the lens is removed?



A. The image will be inverted and blurry.
B. The image will be as it was, but much dimmer.
C. There will be no image at all.
D. The image will be right-side-up and sharp.
E. The image will be right-side-up and blurry.

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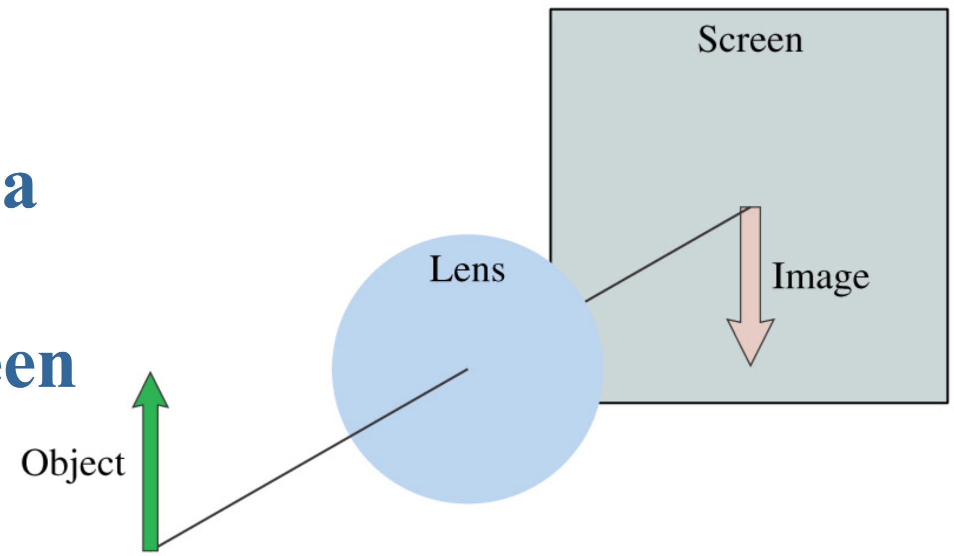

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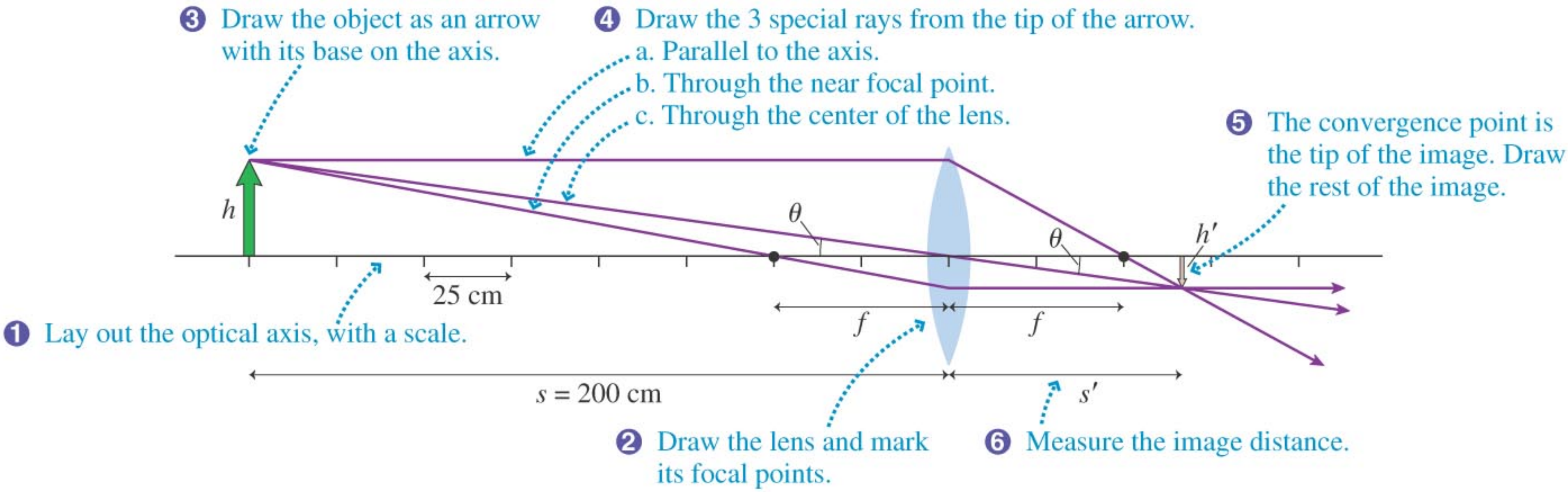
Graphically locating an image and determining its size
Three important rays (only two are needed)



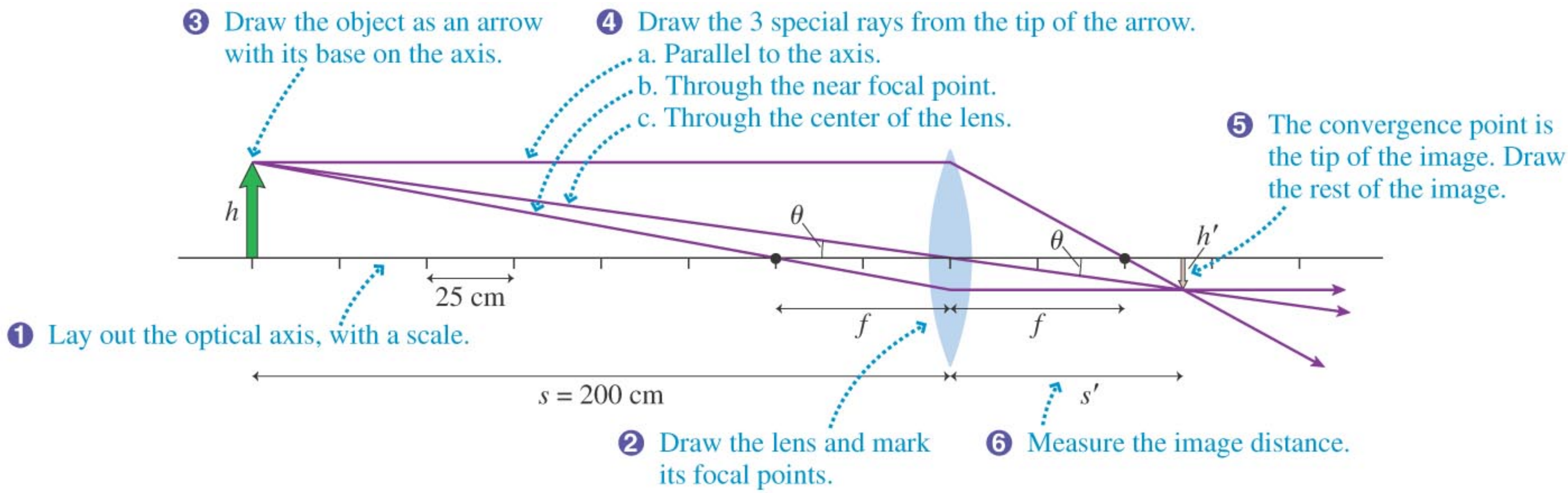
Important Quantities:

f - focal length,
s - distance of object to lens
h - height of object

These determine:

s’ - distance of image to lens
h’ - height of image

Graphically locating an image and determining its size
Three important rays (only two are needed)



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(thin-lens equation)}$$

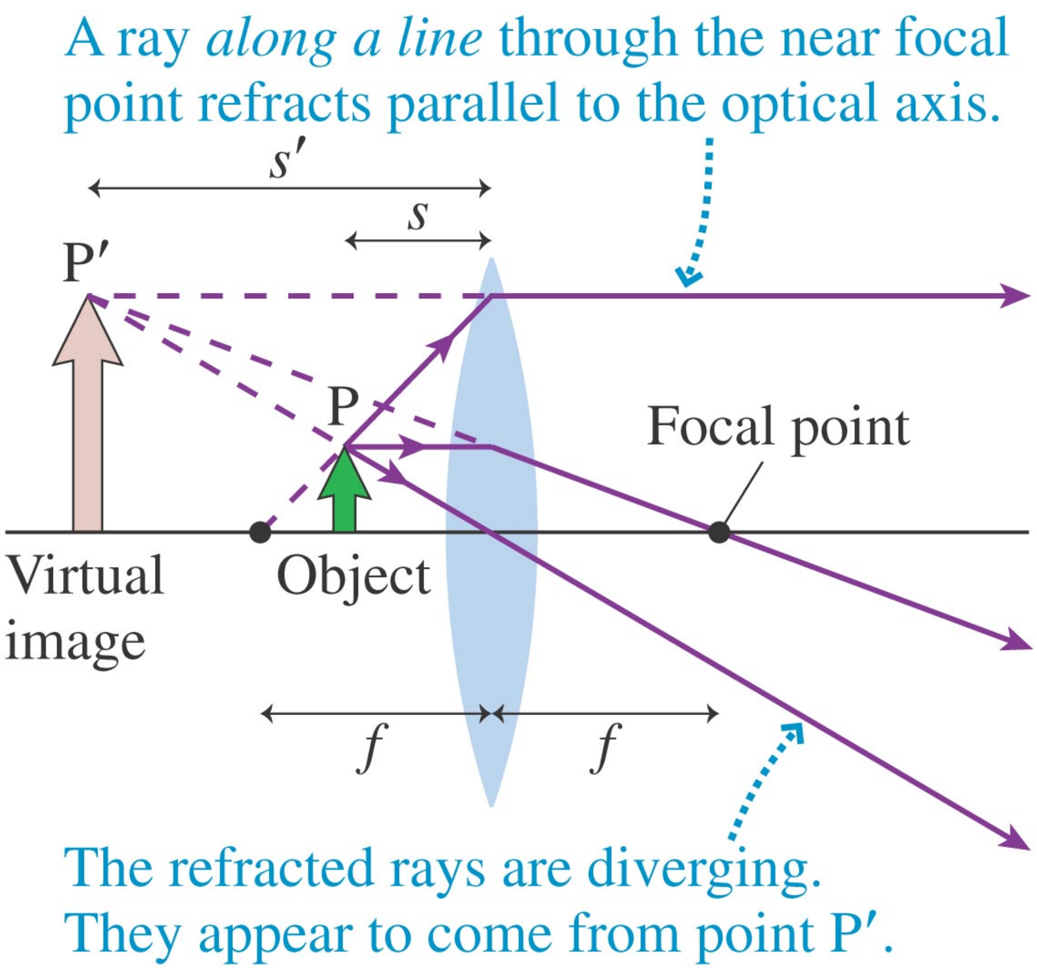
$$\frac{h'}{h} = -\frac{s'}{s} = m$$

magnification

f - focal length,
s - distance of object to lens
h - height of object

s’ - distance of image to lens
h’ - height of image

Suppose object is closer than focal point to lens

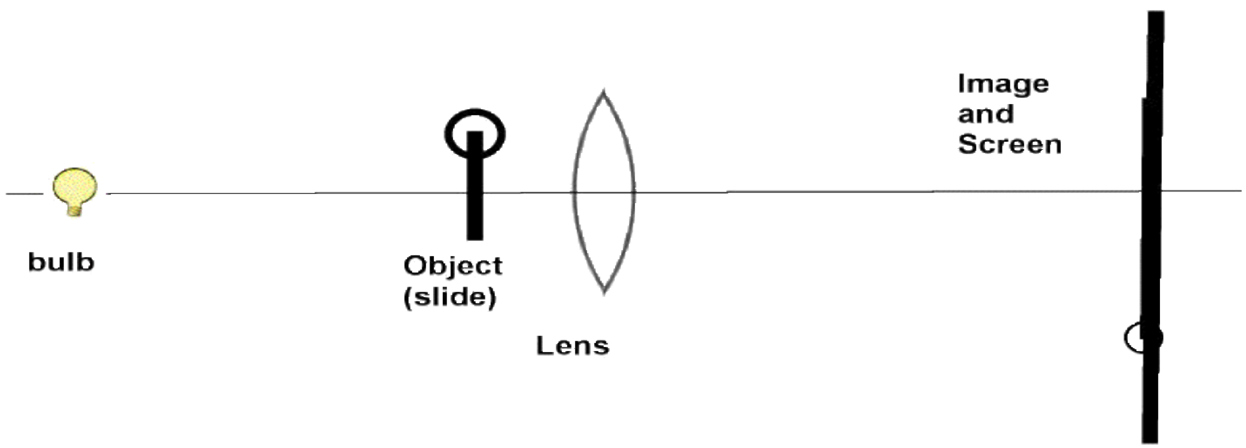


Virtual mage located at $s' < 0$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(thin-lens equation)}$$

$$\frac{h'}{h} = -\frac{s'}{s} = m$$

A projector has an arrangement of lenses as shown in the figure below. A bulb illuminates an object (a slide) and the light then passes through a lens that creates an image on a distant screen as shown. The lens has a focal length of 3 cm. How far from the lens should the slide be placed in order for the image to focus on a screen 3 m away from the lens? (The figure is not to scale!)



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Whiteboard, TA & LA

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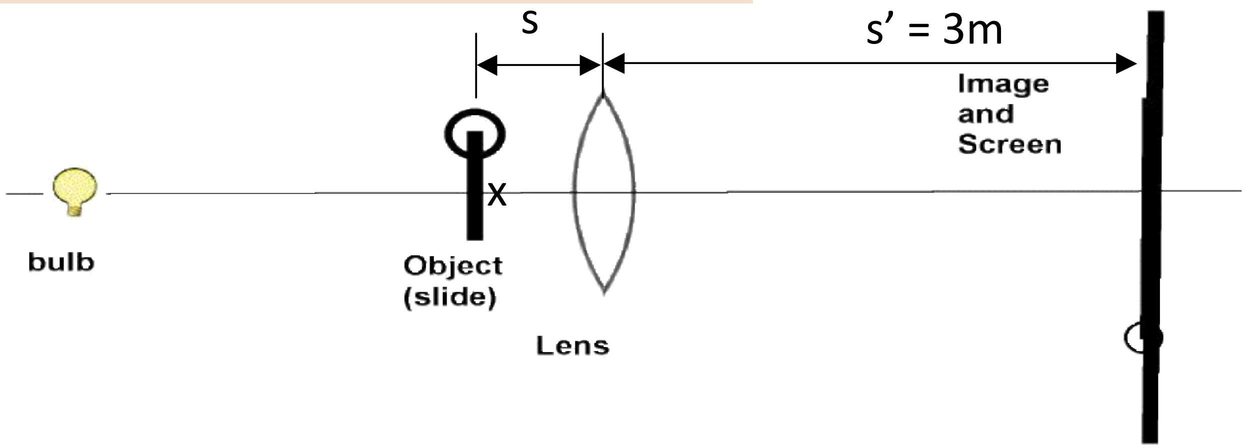


f = .03m

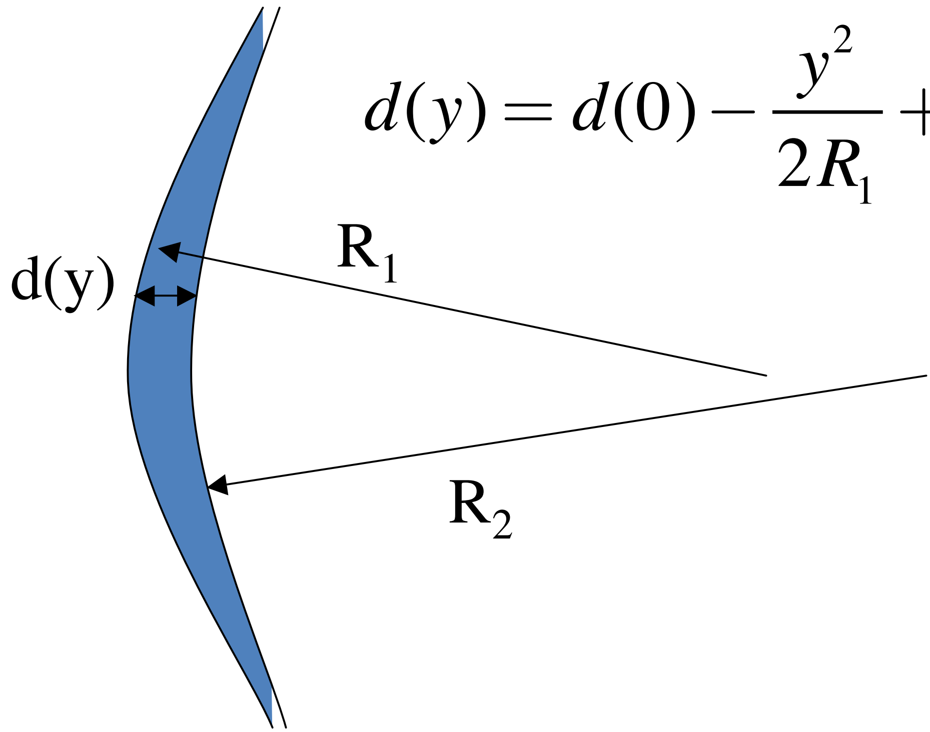
s' = 3m

1/s = 1/f-1/s'=1/.03-1/3 = 33.000 m^{-1}

s=3.030 cm

Whiteboard, TA & LA

Lens Maker Formula: two surfaces defined by two radii of curvature



$$d(y) = d(0) - \frac{y^2}{2R_1} + \frac{y^2}{2R_2}$$

Compare with

$$d(y) = \frac{a^2 - y^2}{2L}$$

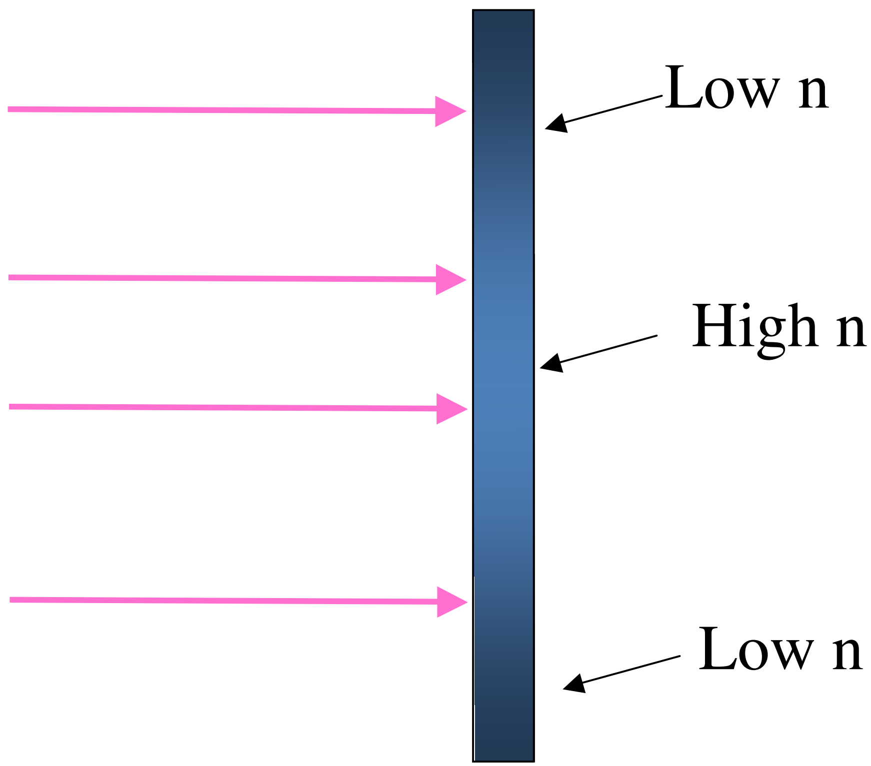
The same coefficient of y^2 if

$$\frac{1}{2L} = \frac{1}{2R_1} - \frac{1}{2R_2}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Works for both converging and diverging lens

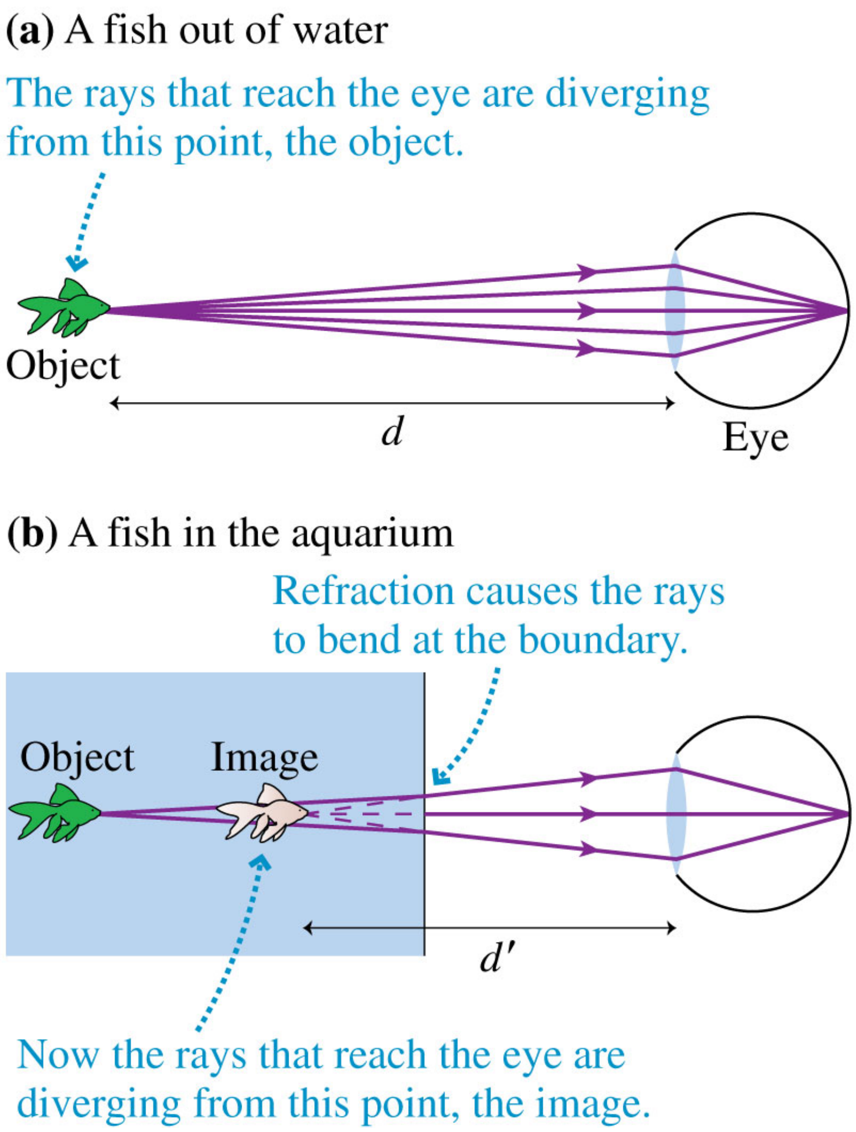
A lens is made of a material with two flat parallel surfaces. The material has a non-uniform index of refraction



Will the rays

a) Converge
b) Diverge
c) Go straight
d) Spiral
e) Become so frustrated that the fall down to the ground

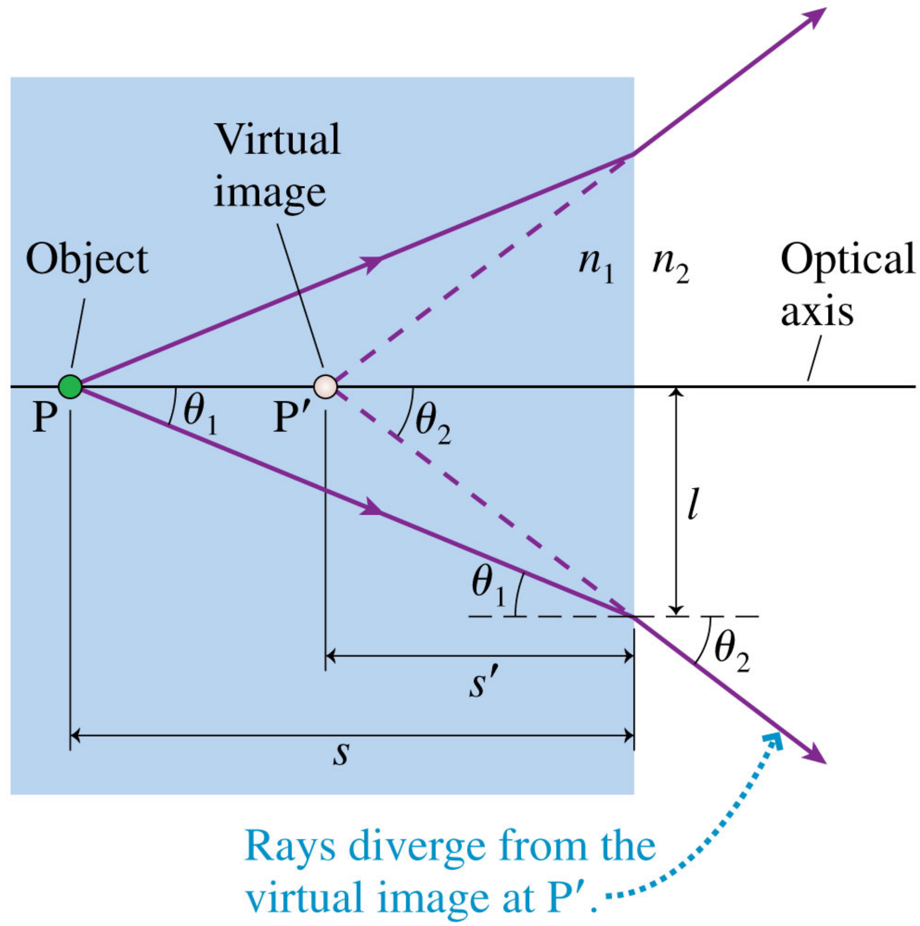
If you see a fish that appears to be swimming close to the front window of the aquarium, but then look through the side of the aquarium, you'll find that the fish is actually farther from the window than you thought.



- Rays emerge from a material with $n_1 > n_2$.
- Consider only **paraxial rays**, for which θ_1 and θ_2 are quite small.
- In this case:

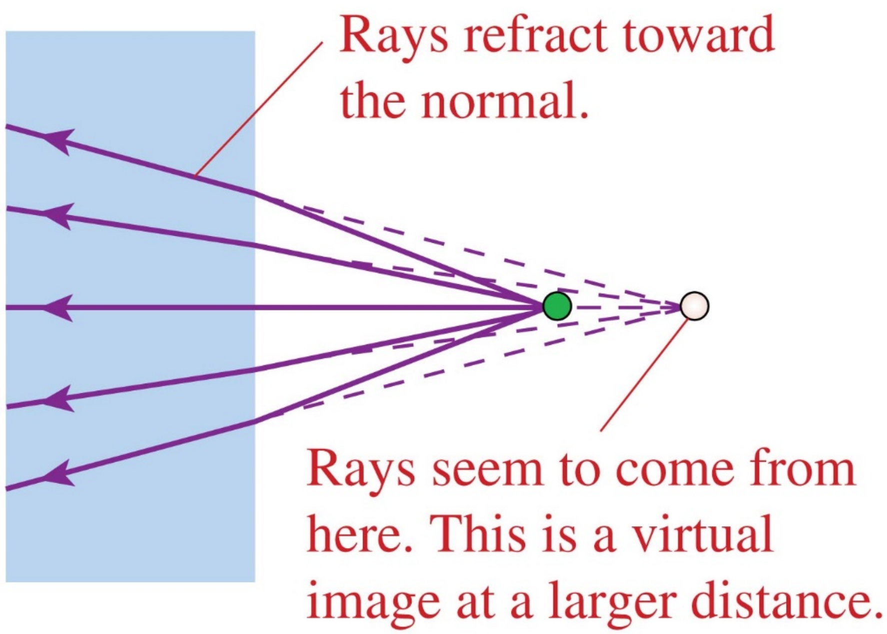
$$s' = \frac{n_2}{n_1} s$$

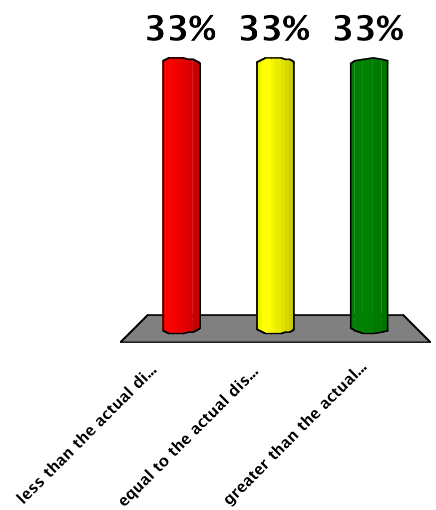
where s is the **object distance** and s' is the **image distance.**



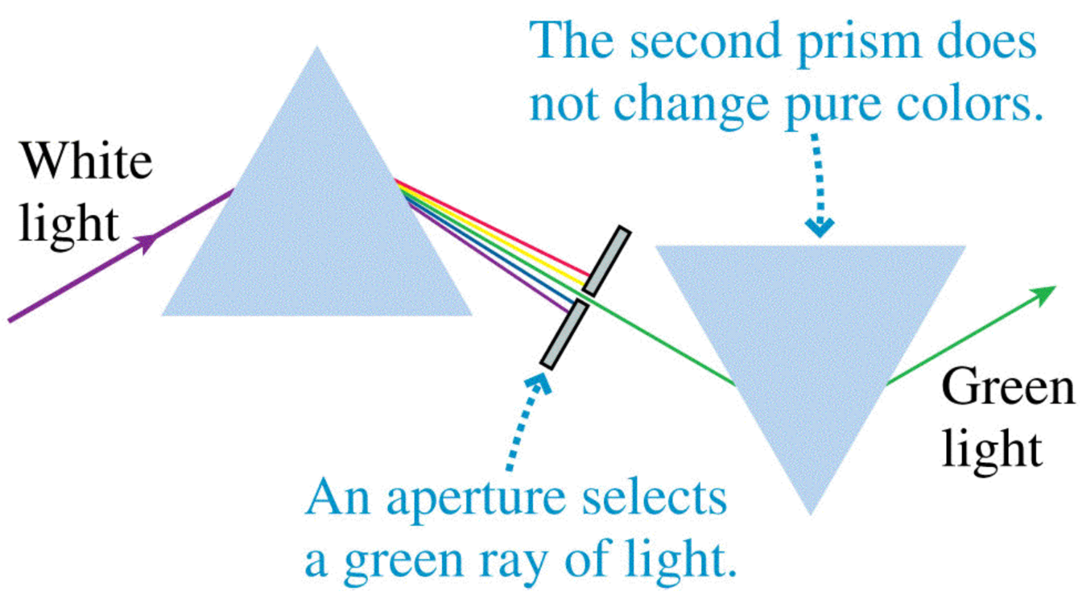
A fish in an aquarium with flat sides looks out at a hungry cat. To the fish, the distance to the cat appears to be

A. less than the actual distance.
B. equal to the actual distance.
C. greater than the actual distance.





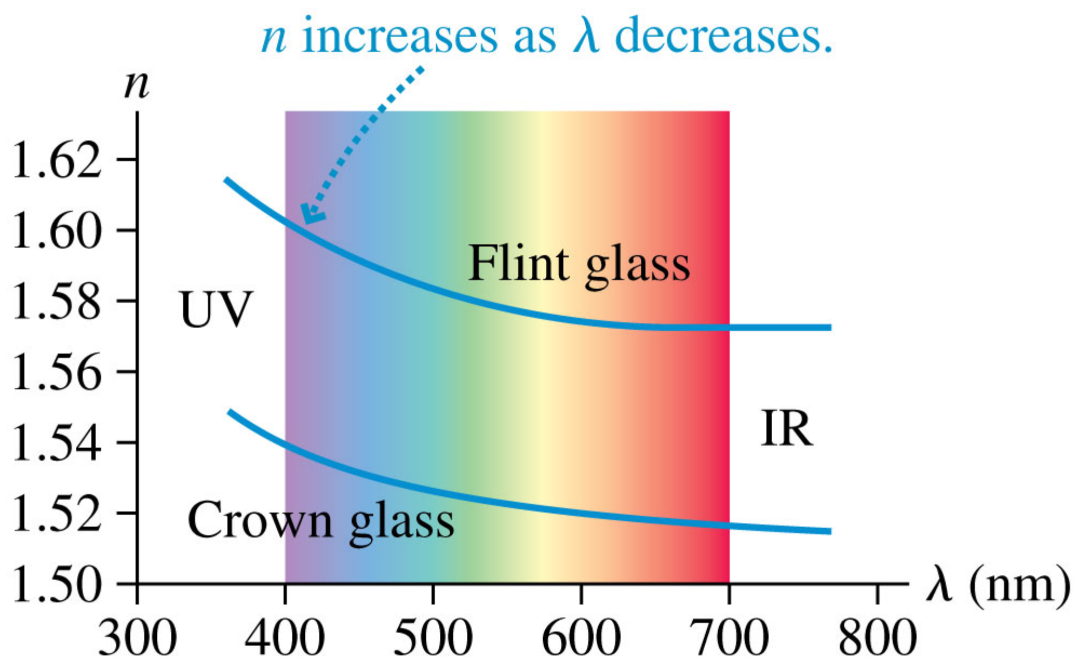
- A prism *disperses* white light into various colors.
- When a particular color of light enters a prism, its color does not change.



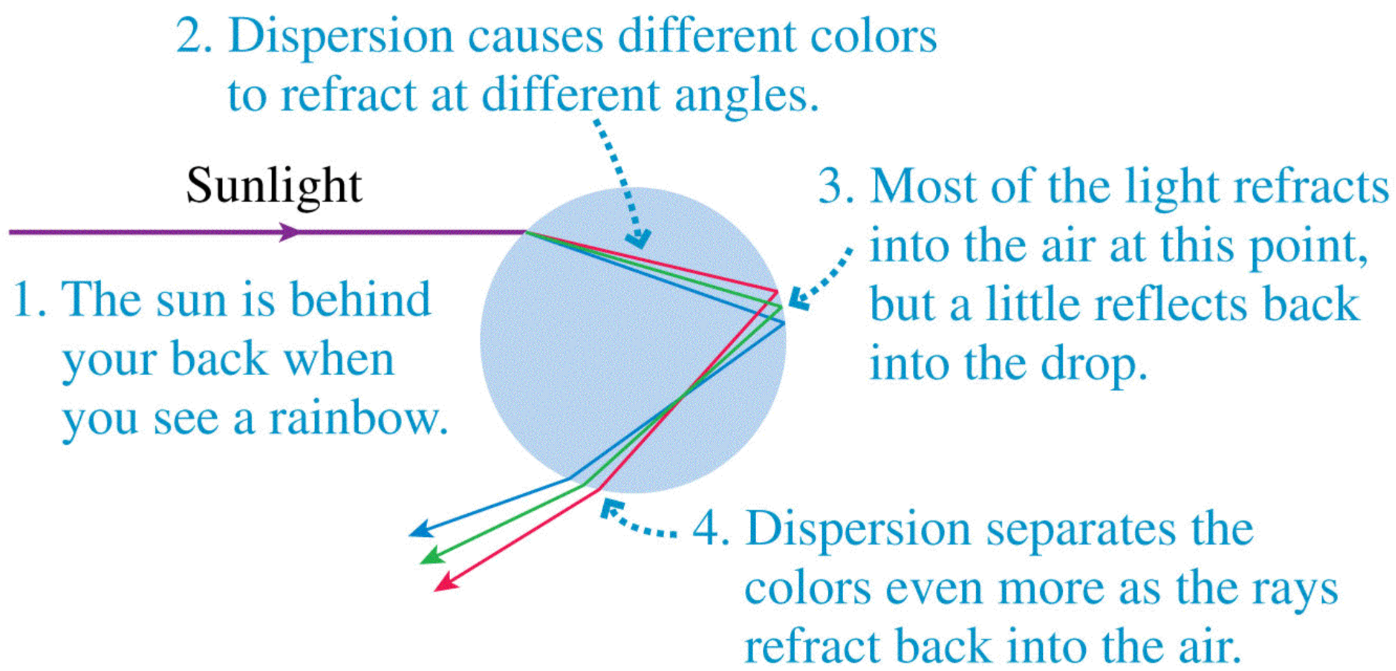
- Different colors are associated with light of different wavelengths.
- The longest wavelengths are perceived as red light and the shortest as violet light.
- What we perceive as white light is a mixture of all colors.

Color	Approximate wavelength
Deepest red	700 nm
Red	650 nm
Green	550 nm
Blue	450 nm
Deepest violet	400 nm

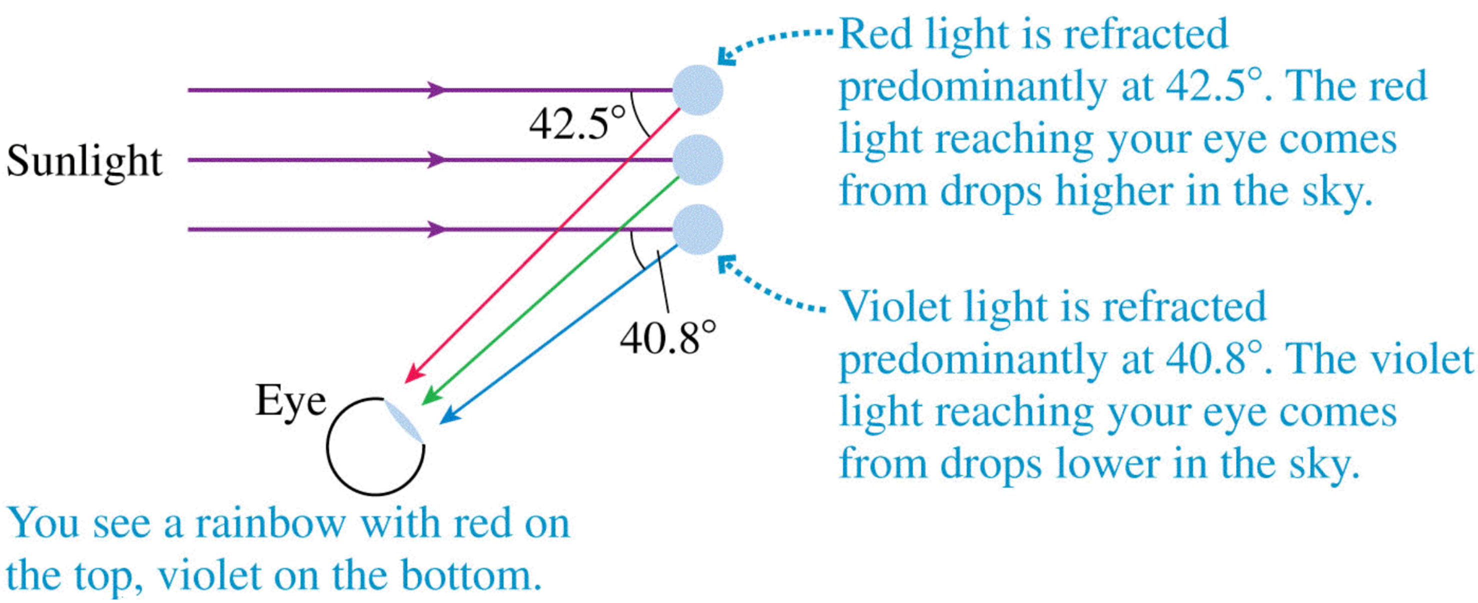
- The slight variation of index of refraction with wavelength is known as **dispersion.**
- Shown is the dispersion curves of two common glasses.
- Notice that ***n*** **is *larger* when the wavelength is *shorter***, thus violet light refracts more than red light.



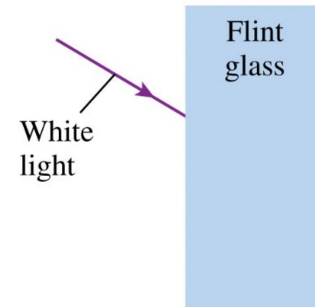
- One of the most interesting sources of color in nature is the rainbow.
- The basic cause of the rainbow is a combination of refraction, reflection, and dispersion.



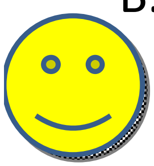
- A ray of red light reaching your eye comes from a drop *higher* in the sky than a ray of violet light.
- You have to look higher in the sky to see the red light than to see the violet light.



A narrow beam of white light is incident at an angle on a piece of flint glass. As the light refracts into the glass,

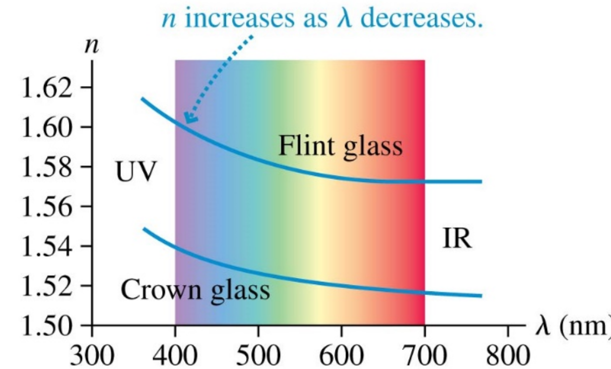


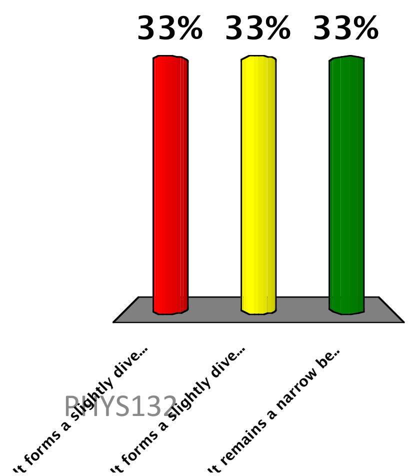
A. It forms a slightly diverging cone with red rays on top, violet rays on the bottom.



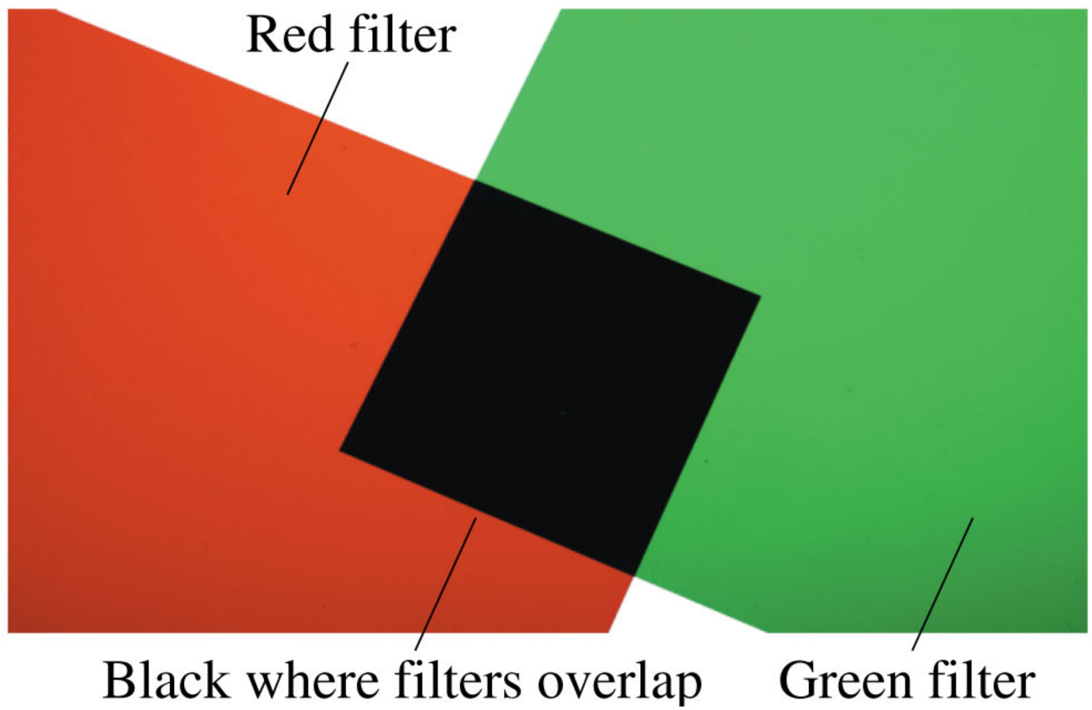
B. It forms a slightly diverging cone with violet rays on top, red rays on the bottom.

C. It remains a narrow beam of white light because all the colors of white were already traveling in the same direction.

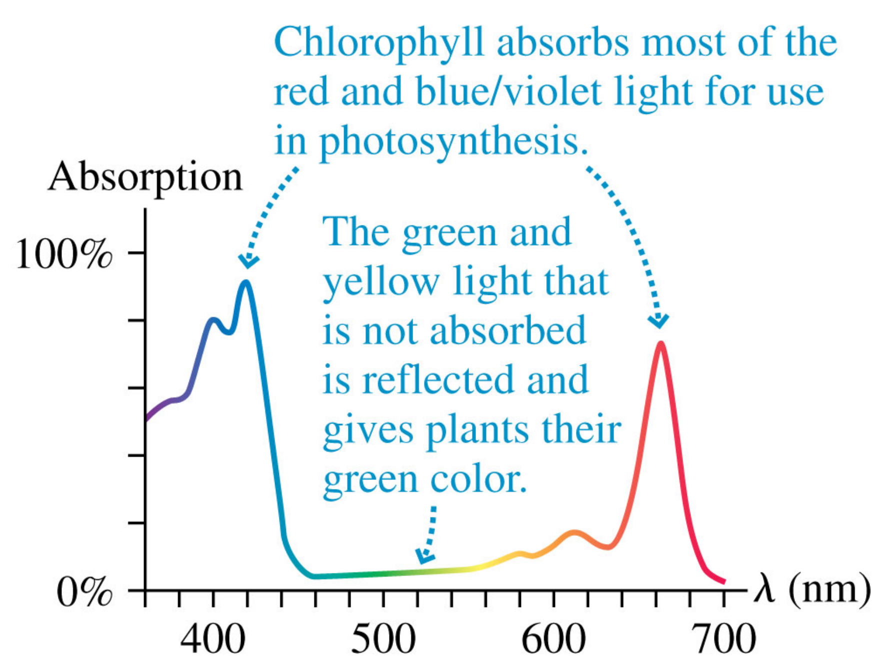




- Green glass is green because it absorbs any light that is “not green.”
- If a green filter and a red filter are overlapped, no light gets through.
- The green filter transmits only green light, which is then absorbed by the red filter because it is “not red.”

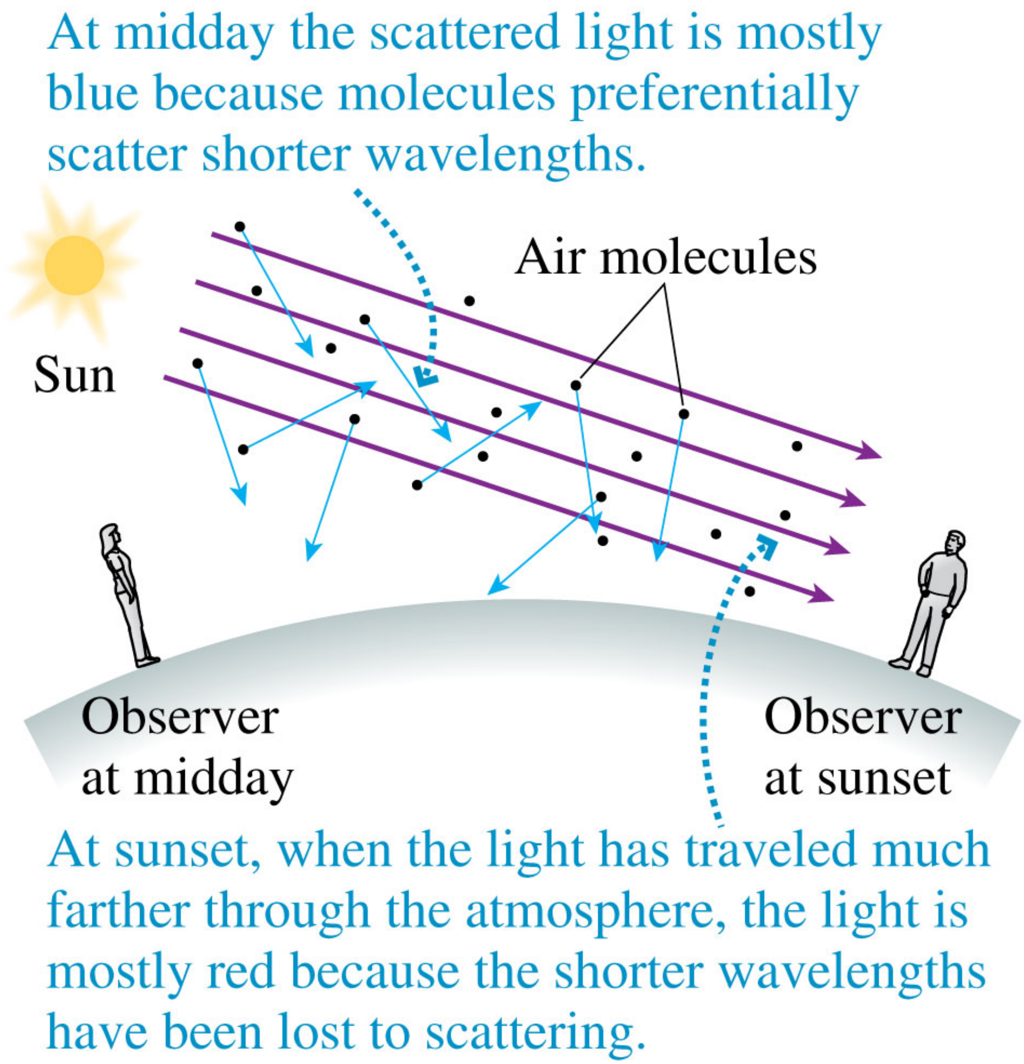


- The figure below shows the absorption curve of *chlorophyll*, which is essential for photosynthesis in green plants.
- The chemical reactions of photosynthesis absorb red light and blue/violet light from sunlight and puts it to use.
- When you look at the green leaves on a tree, you're seeing the light that was reflected because it *wasn't* needed for photosynthesis.

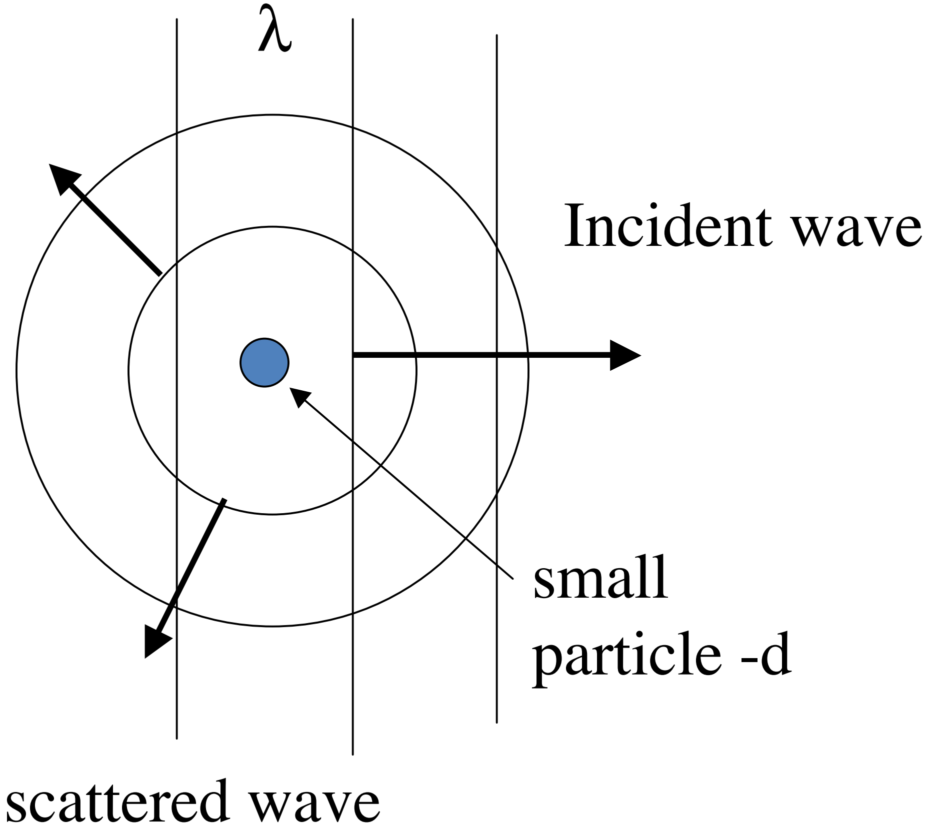


- Light can scatter from small particles that are suspended in a medium.
- **Rayleigh scattering** from atoms and molecules depends inversely on the fourth power of the wavelength:

$$I_{\text{scattered}} \propto \lambda^4$$

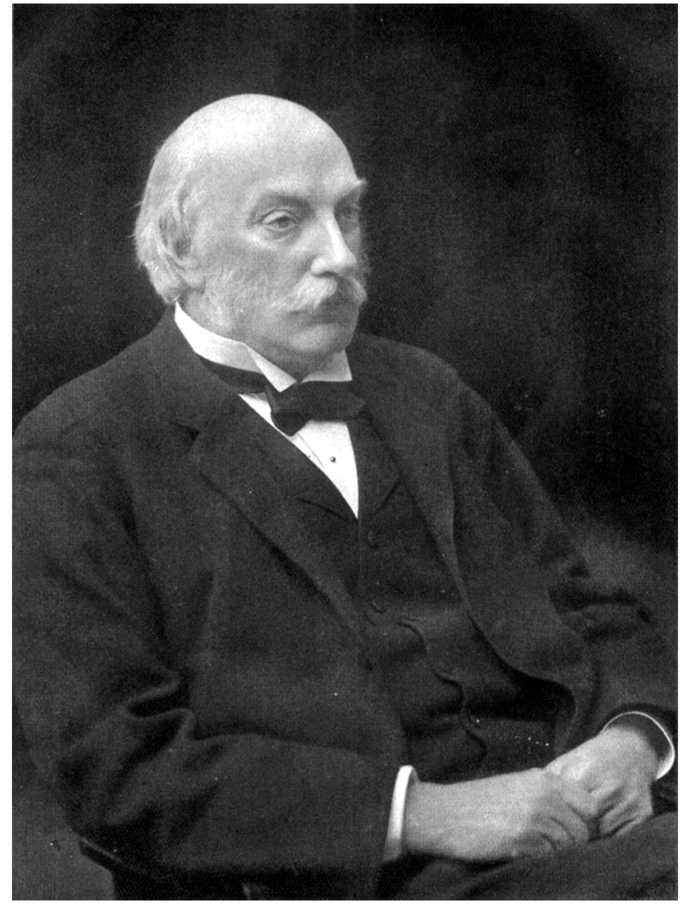


Rayleigh Scattering



$$I \propto \frac{d^6}{\lambda^4 R^2}$$

scattered intensity is higher for shorter wavelengths



John William Strutt
3rd Baron Rayleigh

Wikimedia commons

Sunsets are red because all the blue light has scattered as the sunlight passes through the atmosphere.