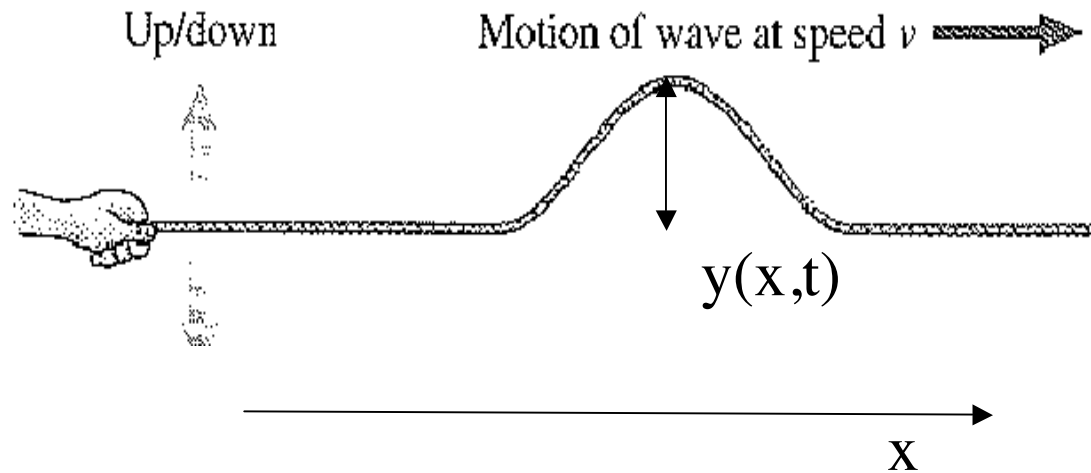


4/18/14

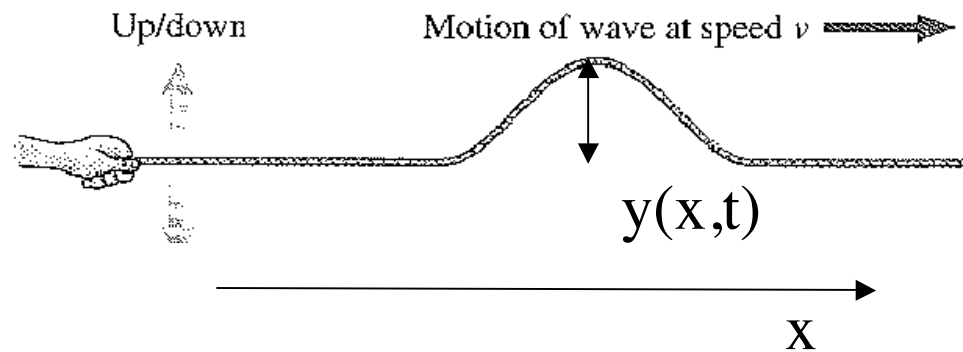


A solution to the wave equation

$$y(x,t) = f(x - v t)$$

Where  $f(\cdot)$  could be any function (It is determined by the hand at left).

What determines  $v$  ?



A solution to the wave equation

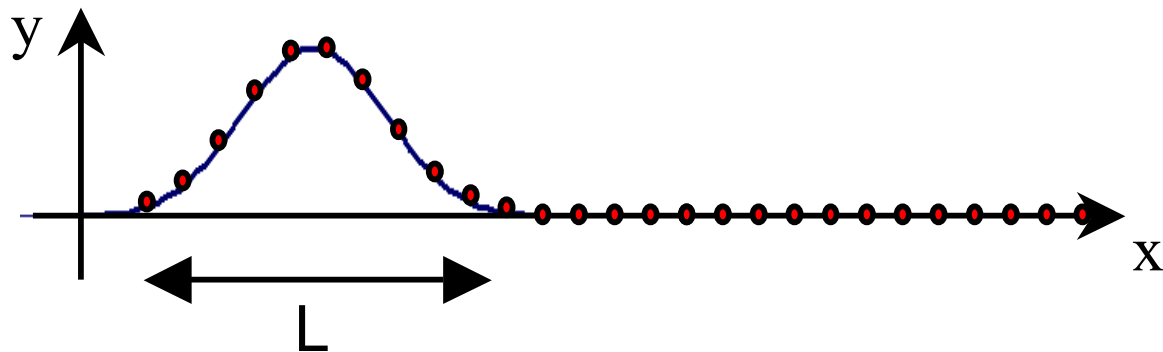
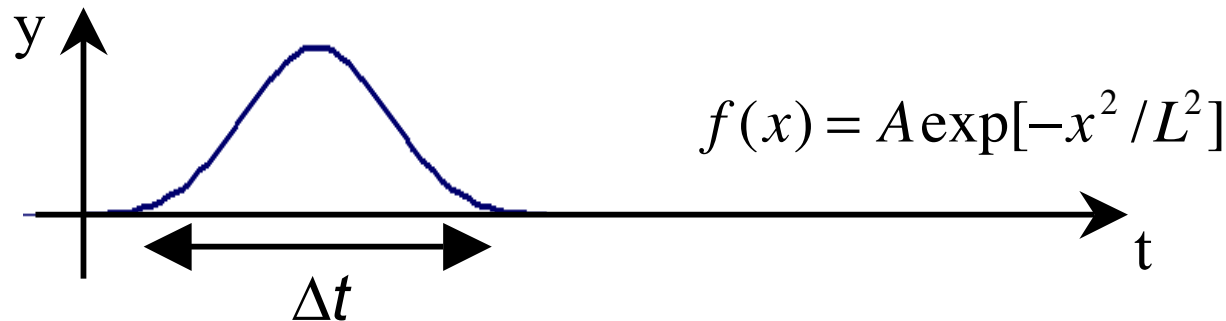
$$y(x,t) = f(x - v t)$$

Example: A Gaussian pulse

$$f(x) = A \exp[-x^2 / L^2]$$

# What controls the widths of the pulses in time and space?

$$y(x,t) = f(x - vt)$$

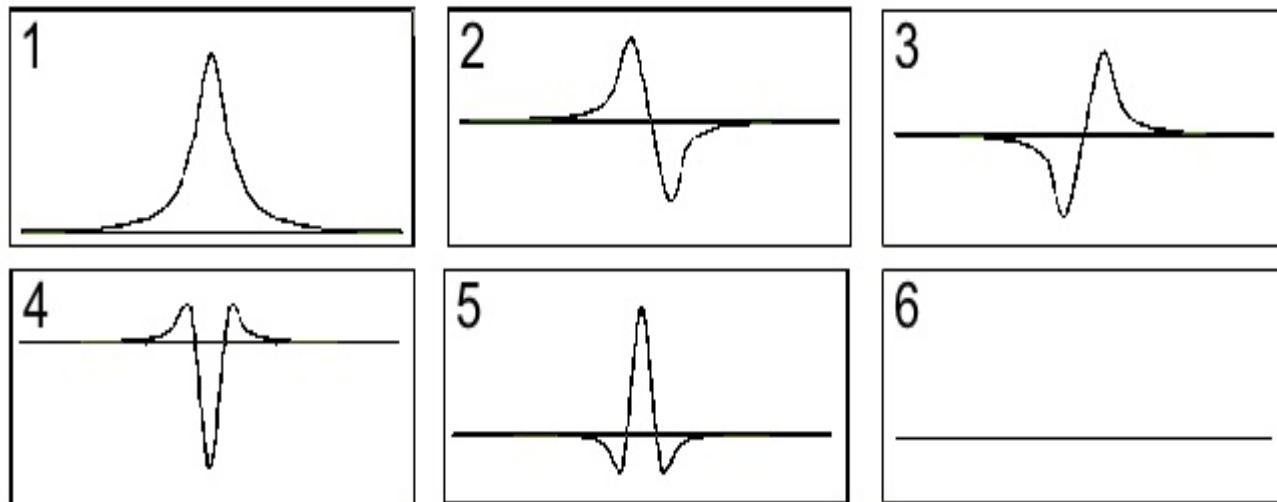


$$L = v\Delta t$$



A pulse is started on the string moving to the right. At a time  $t_0$  a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. ( $x$  is horizontal and right,  $y$  is vertical and up)

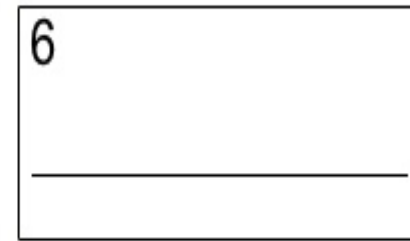
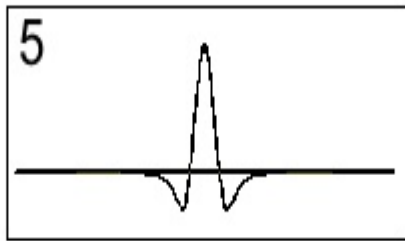
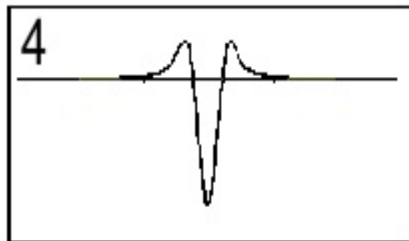
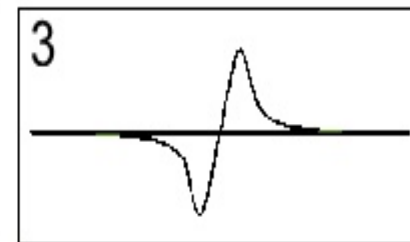
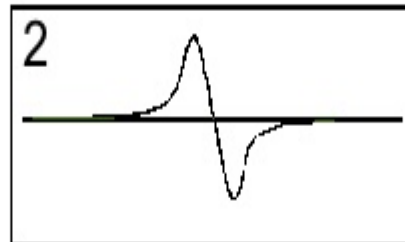
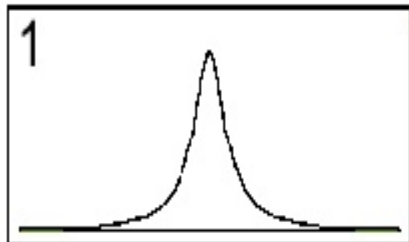
Which graph would look most like a graph of the **y displacement** of the spot as a **function of time**?



7 None of these

A pulse is started on the string moving to the right. At a time  $t_0$  a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. (*x is horizontal and right, y is vertical and up*)

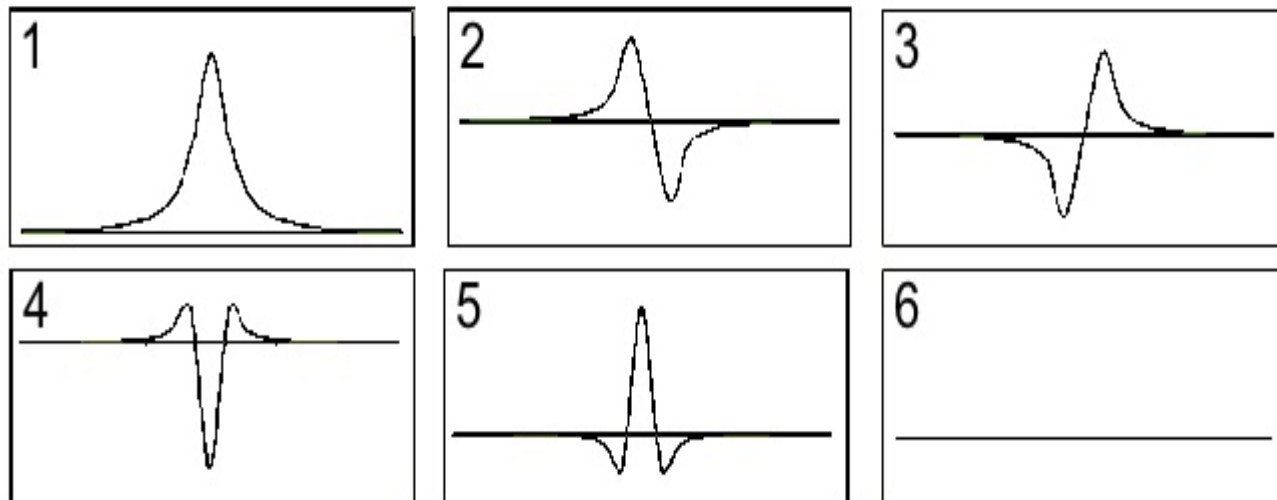
Which graph would look most like a graph of the **y velocity** of the spot as a function of time?



7 None of these

A pulse is started on the string moving to the right. At a time  $t_0$  a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. ( $x$  is *horizontal and right*,  $y$  is *vertical and up*)

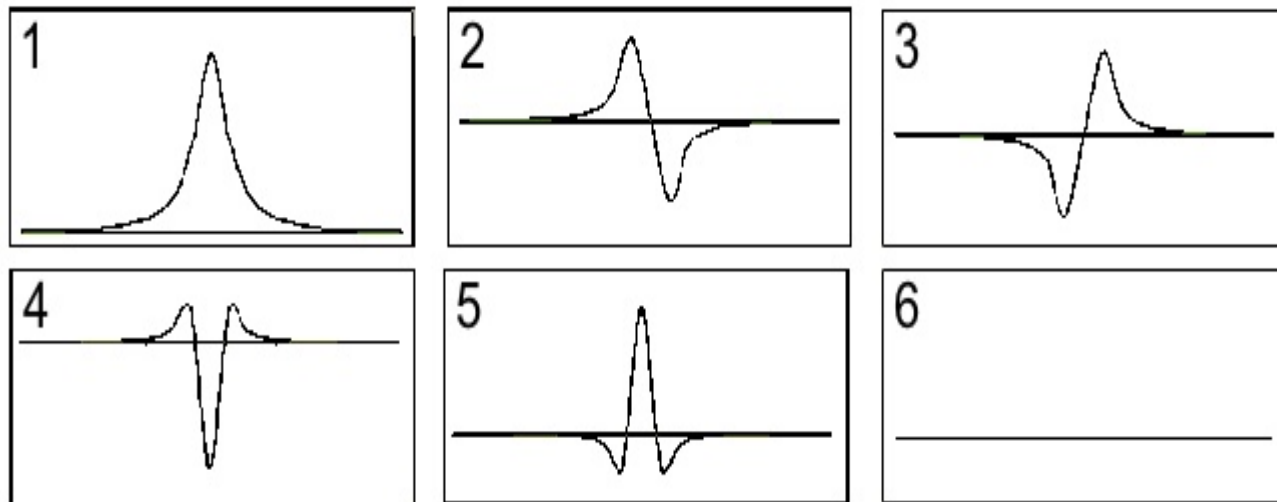
Which graph would look most like a graph of the **y force** on the spot as a **function of time**?



7 None of these

A pulse is started on the string moving to the right. At a time  $t_0$  a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. ( $x$  is horizontal and right,  $y$  is vertical and up)

Which graph would look most like a graph of the **x velocity** of the spot as a **function of time**?



7 None of these

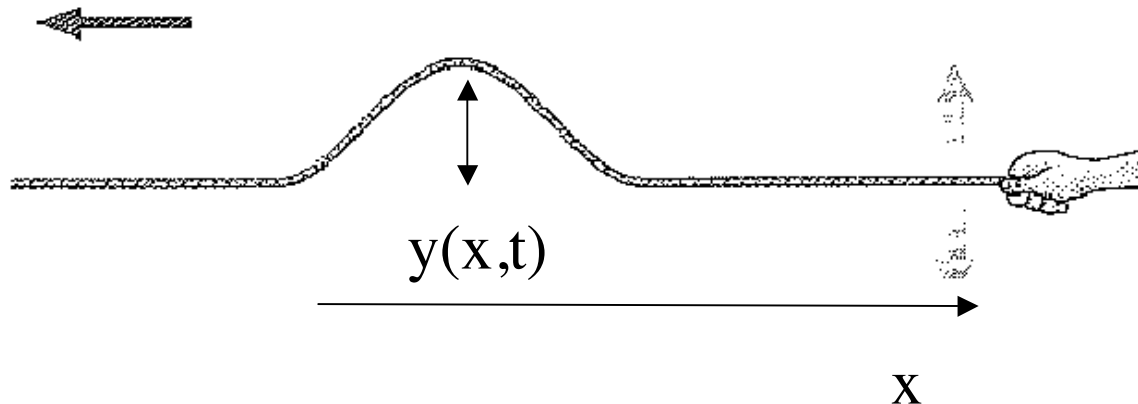
# What Controls the Speed of the Pulse on a Spring?

To make the pulse go to the wall faster

1. Move your hand up and down more quickly (but by the same amount).
2. Move your hand up and down more slowly (but by the same amount).
3. Move your hand up and down a larger distance in the same time.
4. Move your hand up and down a smaller distance in the same time.
5. Use a heavier string of the same length under the same tension.
6. Use a string of the same density but decrease the tension.
7. Use a string of the same density but increase the tension.
8. Put more force into the wave,
9. Put less force into the wave.



Do waves only propagate in one direction?



A solution to the wave equation

$$y(x,t) = f(x + vt)$$

Where  $f(\cdot)$  could be any function (It is determined by the hand at right).

[http://phet.colorado.edu/simulations/sims.php?sim=Wave\\_on\\_a\\_String](http://phet.colorado.edu/simulations/sims.php?sim=Wave_on_a_String)

# Superposition

If  $f_1(x-vt)$  and  $f_2(x-vt)$  are two solutions then so is

$$f_3(x-vt) = f_1(x-vt) + f_2(x-vt)$$

Most general solution:

$$y(x,t) = f_1(x-vt) + f_2(x+vt)$$

# Sinusoidal waves


- Suppose we make a continuous wiggle. When we start our clock ( $t = 0$ ) we might have created shape something like

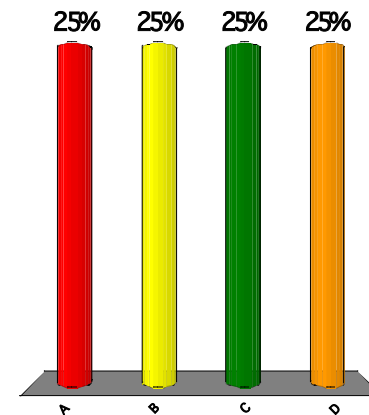
$$y(x, t = 0) = A \sin[kx]$$

- If this moves in the  $+x$  direction, at later times it would look like



If this moves in the  
+ $x$  direction, at  
later times it will  
look like

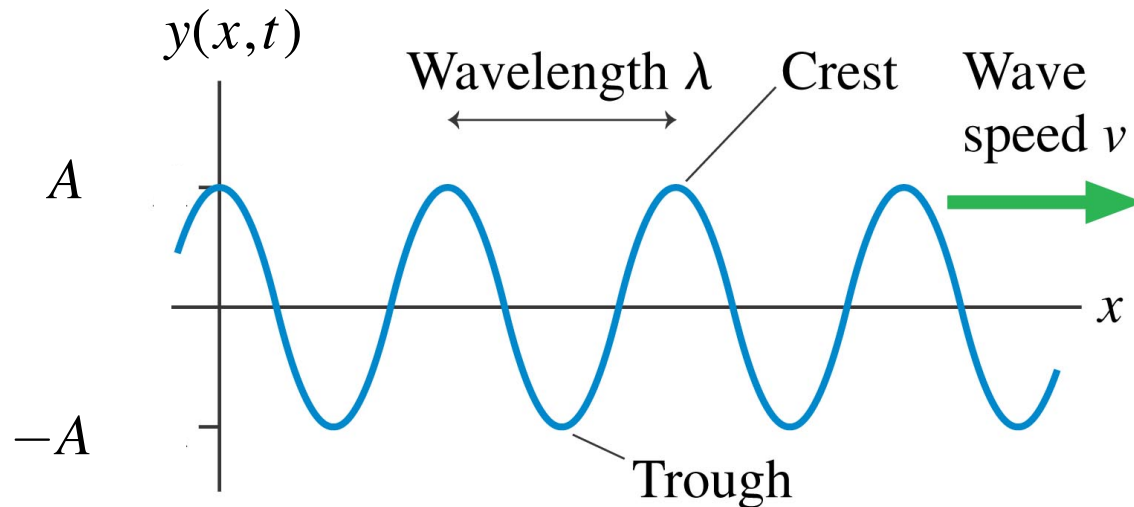
- A.  $y(x,t) = A \sin[k(x + vt)]$
-  B.  $y(x,t) = A \sin[k(x - vt)]$
- C.  $y(x,t) = A \cos[k(x + vt)]$
- D.  $y(x,t) = A \cos[k(x - vt)]$



## Special Case Sinusoidal Waves

$$y(x,t) = f(x - vt) = A \cos[k(x - vt)]$$

**(b)** A snapshot graph at one instant of time



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Wavenumber and wavelength

$$k = 2\pi / \lambda$$

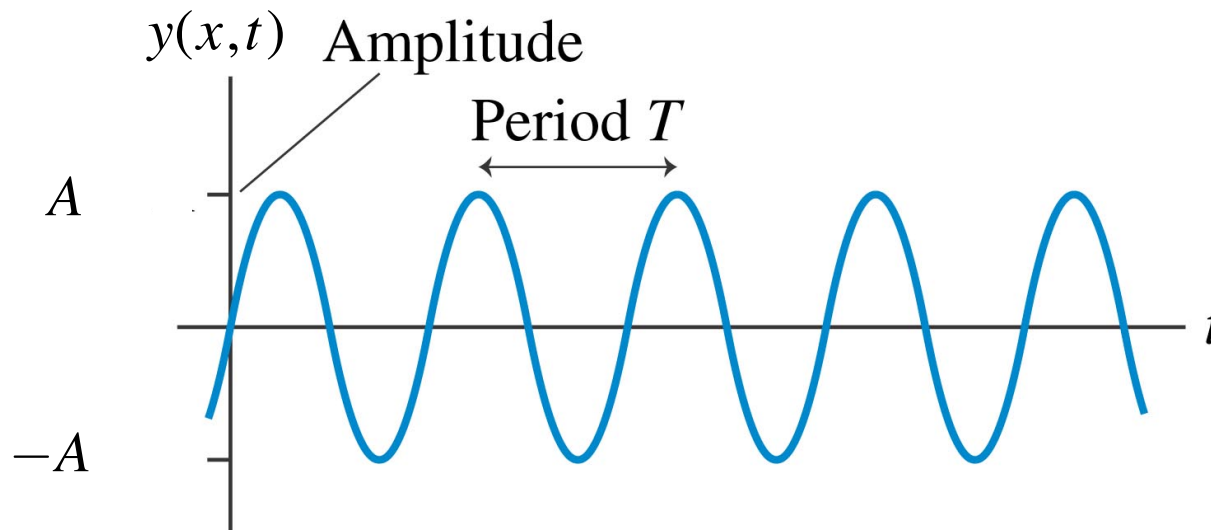
$$\lambda = 2\pi / k$$

These two contain the same information

## Special Case Sinusoidal Waves

$$y(x,t) = f(x - vt) = A \cos[k(x - vt)]$$

(a) A history graph at one point in space



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$$2\pi = kvT$$

Introduce

$$\omega = 2\pi / T$$

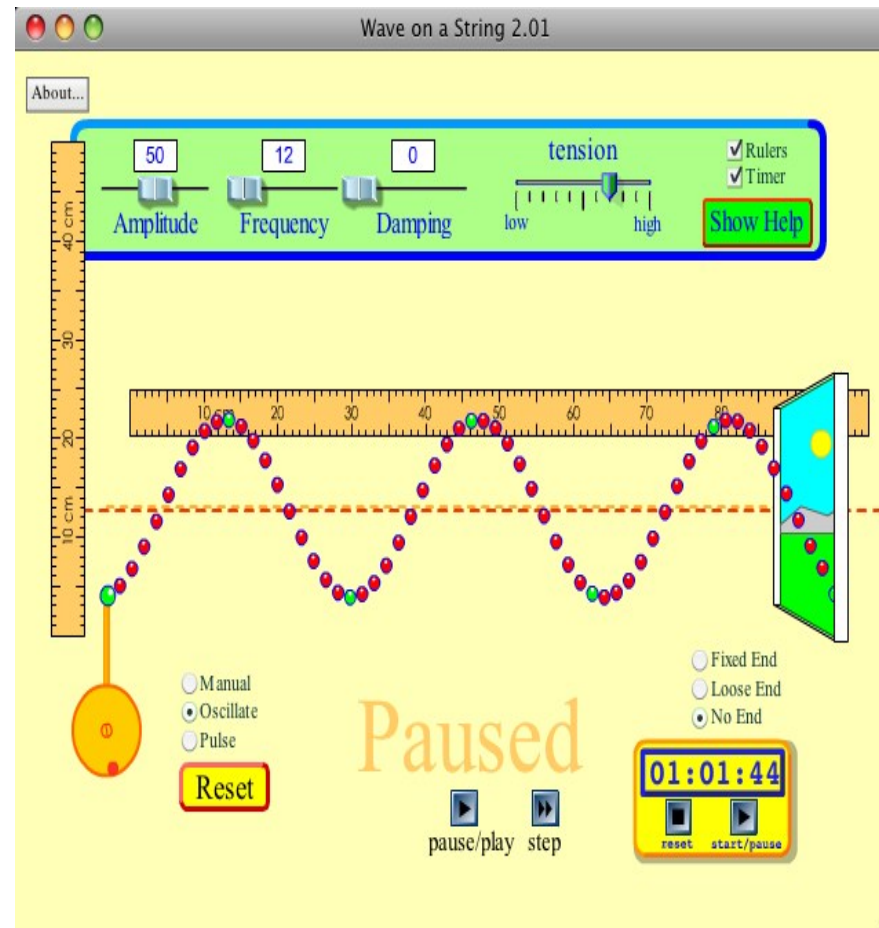
$$f = 1 / T$$

Different ways of saying the same thing:

$$\omega / k = v$$

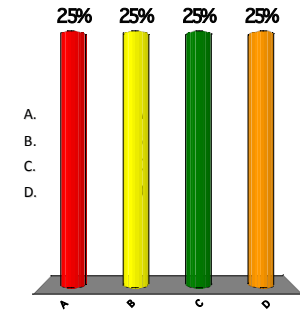
$$f\lambda = v$$

# Sinusoidal Waves

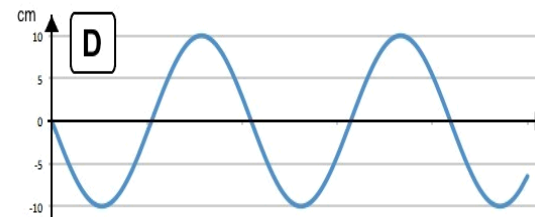
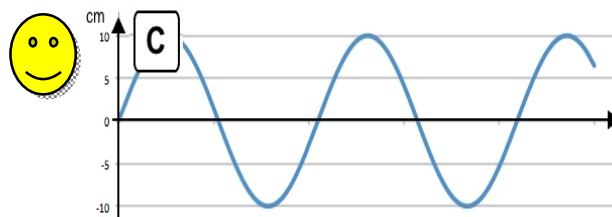
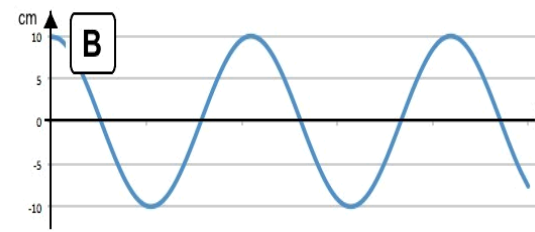
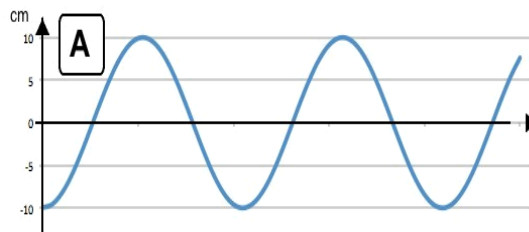
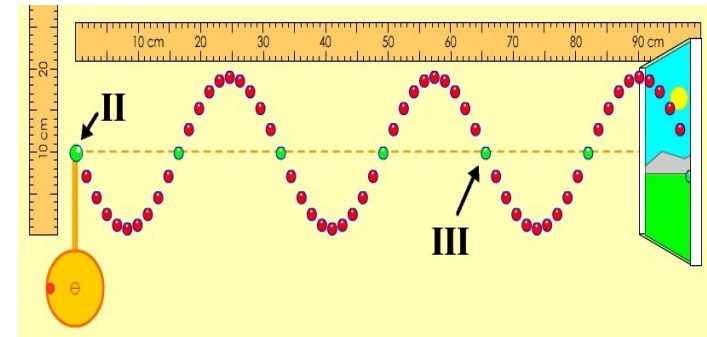
$$y(x,t) = A \sin k(x - v_0 t)$$


[http://phet.colorado.edu/simulations/sims.php?sim=Wave\\_on\\_a\\_String](http://phet.colorado.edu/simulations/sims.php?sim=Wave_on_a_String)

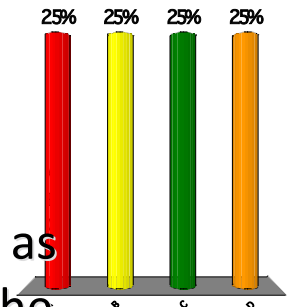
The driving wheel has generated a traveling wave of amplitude 10 cm moving to the right. (The string continues on for a long way to the right as indicated by its going “out the window.”) The figure shows  $t = 0$ , when the green bead marked “II” is passing through its equilibrium point.



Which of the graphs could serve as the graph of **the vertical displacement of bead II** as a function of **time**?

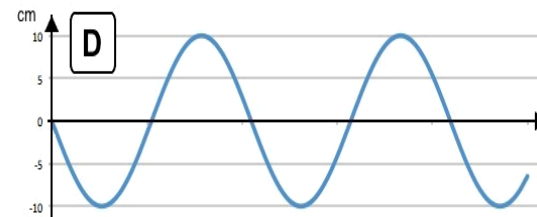
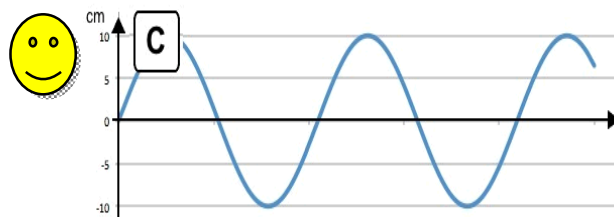
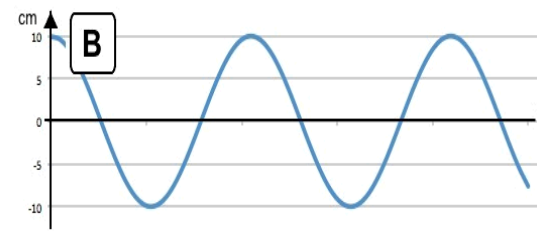
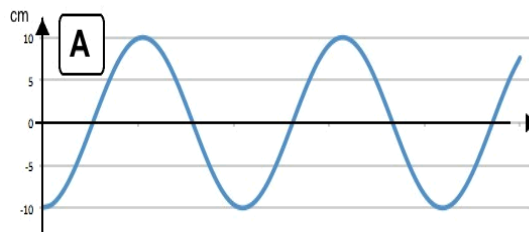
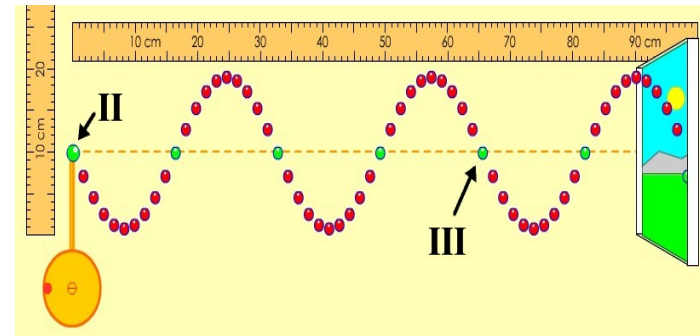


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The driving wheel has generated a traveling wave of amplitude 10 cm moving to the right. (The string continues on for a long way to the right as indicated by its going “out the window.”) The figure shows  $t = 0$ , when the green bead marked “II” is passing through its equilibrium point.

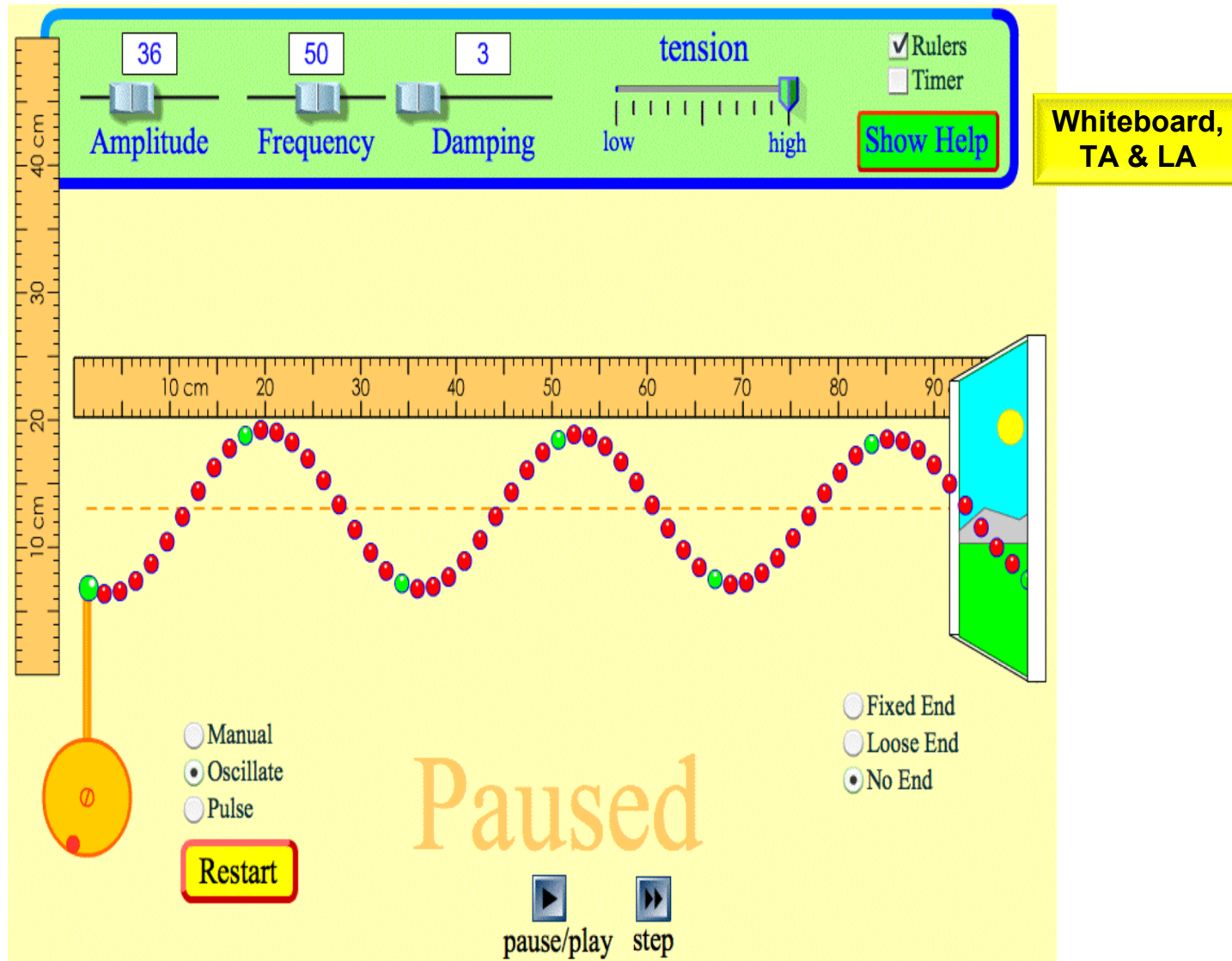
Which of the graphs could serve as the graph of **the vertical displacement of bead III** as a function of **time**?



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This is the state of the PhET wave-on-a-string simulation when the string is very long so reflection can be ignored. What is the speed of the wave (assuming that the frequency is given in cycles/min?)

# What happens at a fixed end?

1. Wave passes through as before
2. Wave stops and dies
3. Wave bounce back right side up
4. Bounce back upside down
5. 3 but delayed
6. 4 but delayed



# What happens at a loose end?

1. Pass through like before
2. Stop and die
3. Bounce back right side up
4. Bounce back upside down
5. 3 but delayed
6. 4 but delayed

## Standing waves: 2 Sinusoidal Waves, same frequency, going in opposite directions

$$y(x,t) = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

Using trig identities (sc+cs...) we can show

$$y(x,t) = 2A \sin(kx) \cos(\omega t)$$

For each point on the string labeled “x” it oscillates with an amplitude that depends on where it is

but all parts of the string go up and down together.

## Adding Sinusoidal Waves – an example

$$y = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$y = 2A \sin(kx) \cos(\omega t)$$

Is there a position for which this function is zero at all times?

The function is also zero wherever  $kx$  is a multiple of  $\pi$

$$\sin(kx = n\pi) = 0$$

# Standing Waves

- Some points in this pattern (values of  $x$  for which  $kx = n\pi$ ) are always 0. (NODES)
- To wiggle like this (all parts oscillating together in a “standing wave”) we need to have the end fixed

$$L = n \frac{\lambda}{2}$$

- We still have

$$v = \frac{\omega}{k} \quad \text{that is} \quad v = \lambda f$$

If we start our beaded string off in a sinusoidal shape

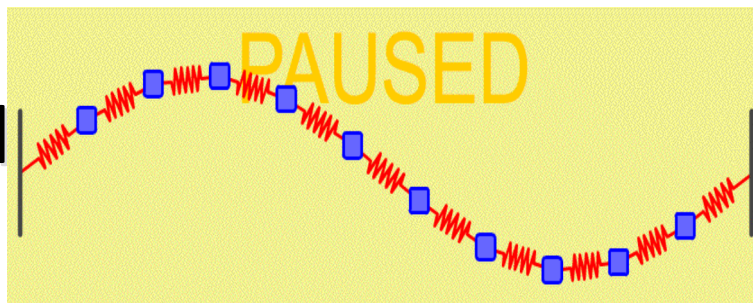
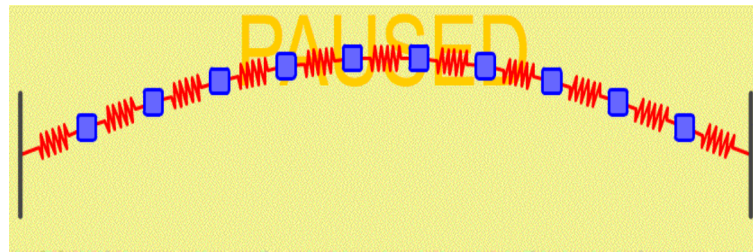
$$y(x) = A \sin(\pi x/L)$$

it will oscillate with a frequency  $f_0 = v/2L$ . If we start it out with a shape

$$y(x) = A \sin(2\pi x/L)$$

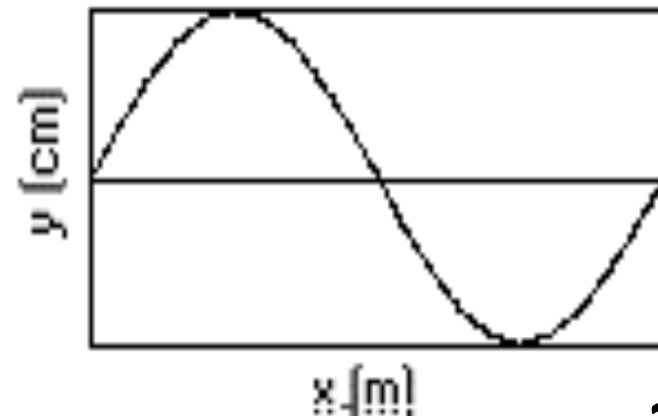
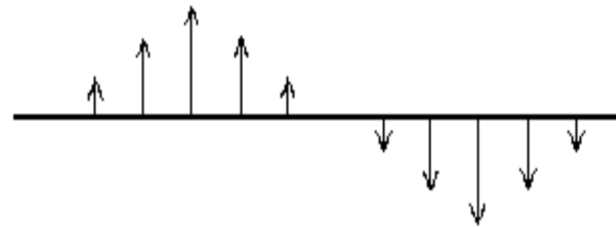
with what frequency will it oscillate?

1.  $f_0$
2.  $2f_0$
3.  $f_0/2$
4. Something else



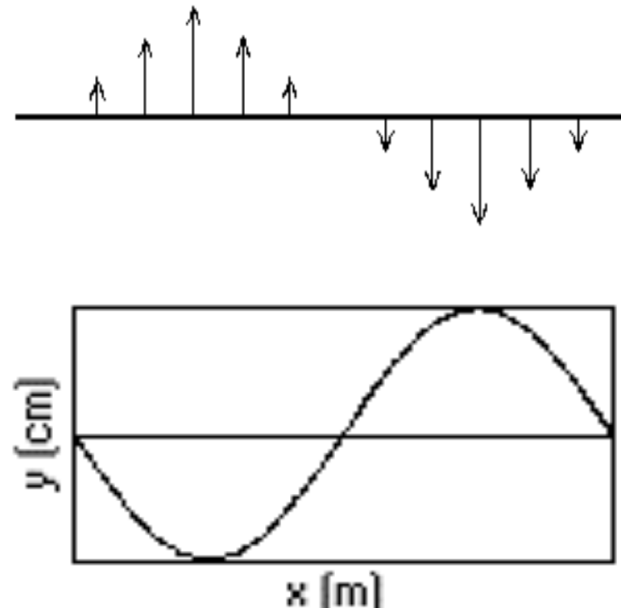
In the figure below is shown a picture of a string at a time  $t_1$ . The pieces of the string are each moving with velocities indicated by arrows. (Vertical displacements are small and don't show up in the picture.) if the shape of the string at time  $t_1$  is that shown below (displacement magnified by X100) then the motion of the string is

1. Left traveling wave
2. Right traveling wave
3. A standing wave increasing in amplitude
4. A standing wave decreasing in amplitude
5. None of these.



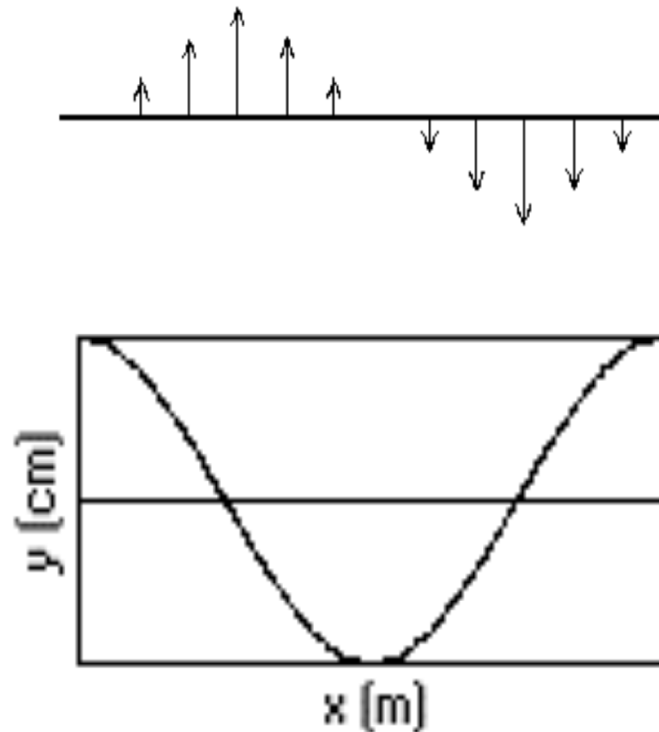
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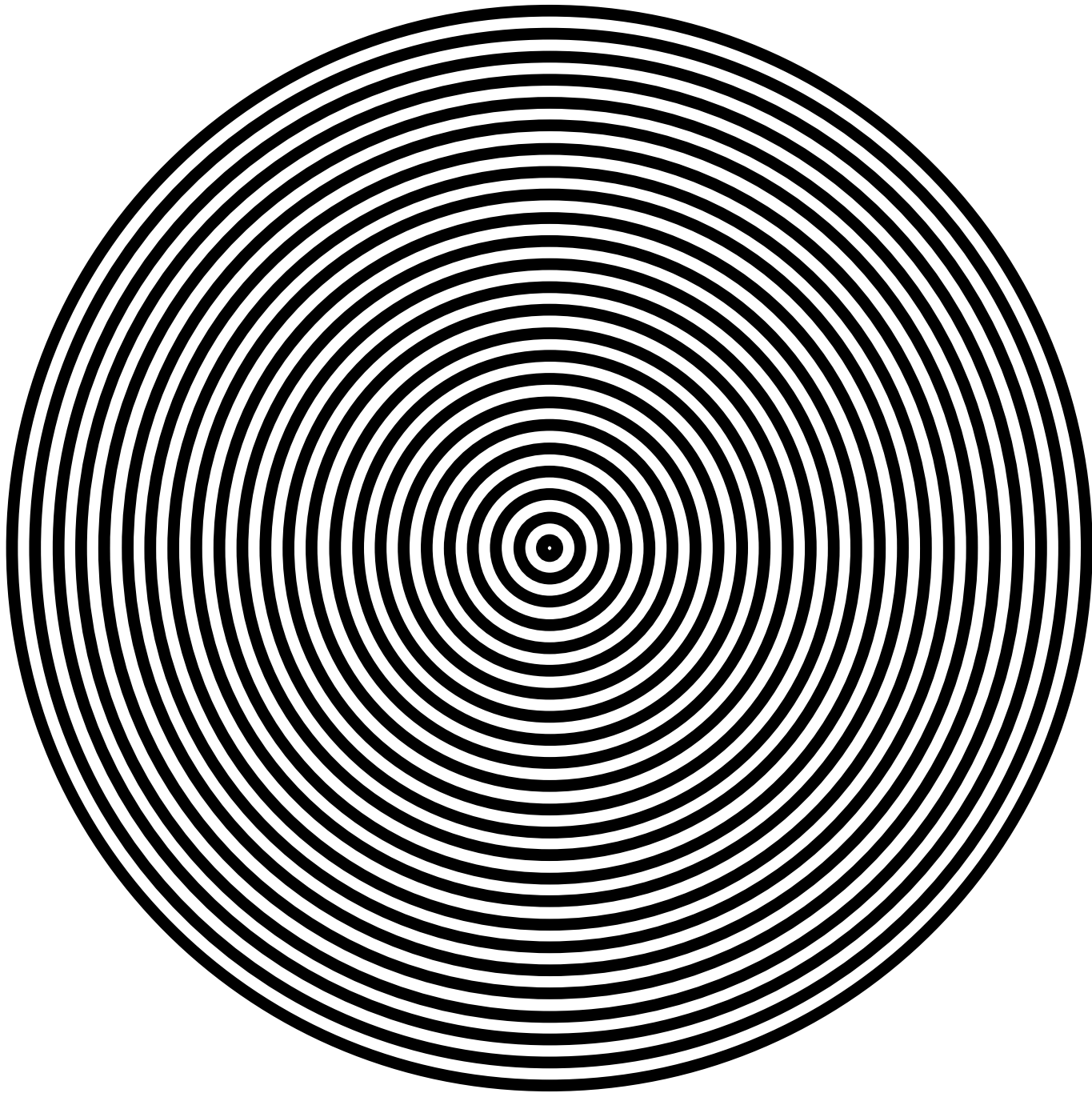
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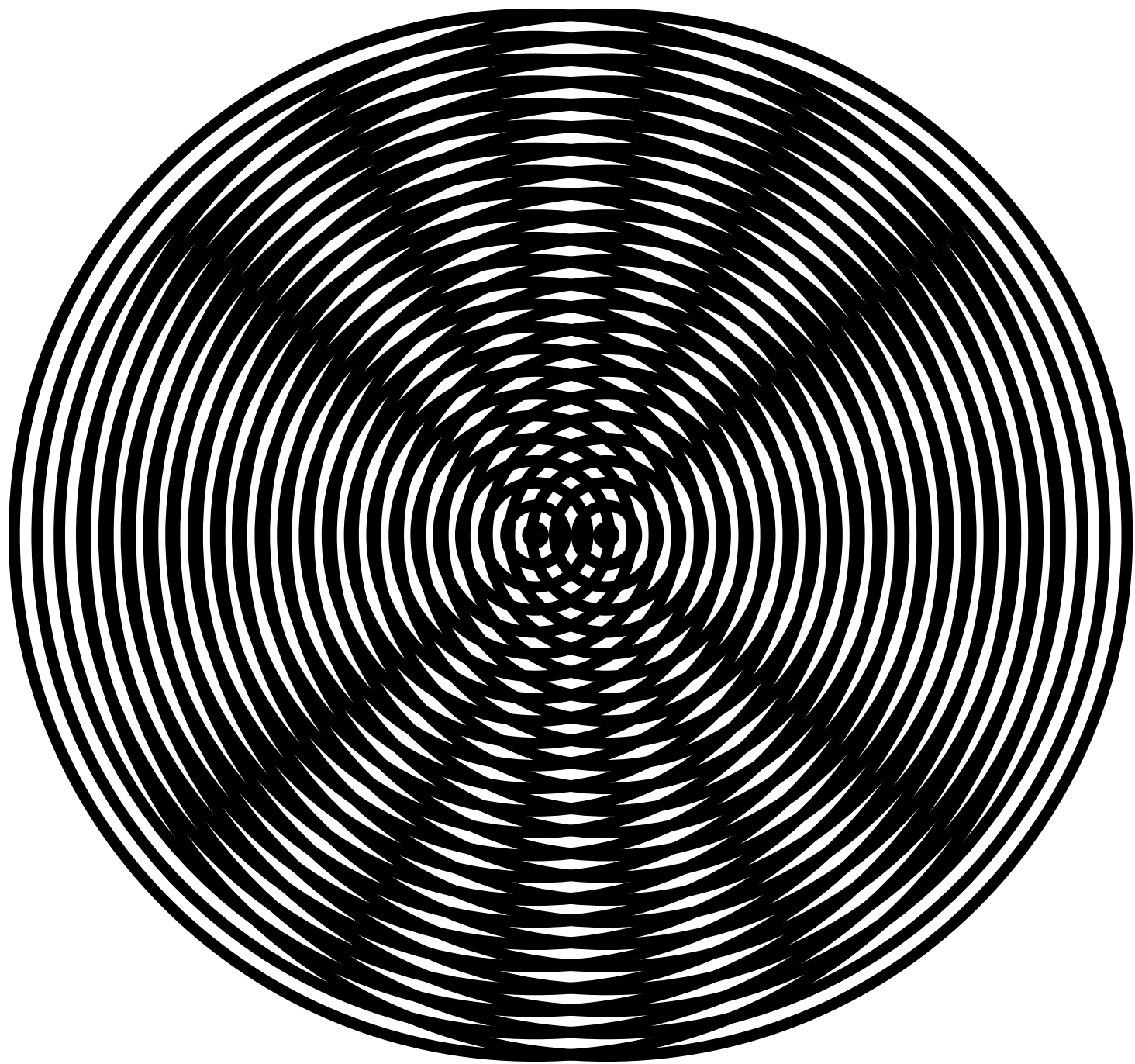
1. Left traveling wave
2. Right traveling wave
3. A standing wave increasing in amplitude
4. A standing wave decreasing in amplitude
5. None of these.





# Extra slides





Speed of sound

$$v = \sqrt{\gamma P / \rho}$$

Pressure / mass density

Q: An ideal gas with  $n$  degrees of freedom per molecule is compressed/expanded from volume  $V_1$  to volume  $V_2$ . The pressure changes from  $P_1$  to  $P_2$ . How are these related?

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma$$

$$\gamma = \frac{n+2}{n}$$

also called the ratio of specific heats

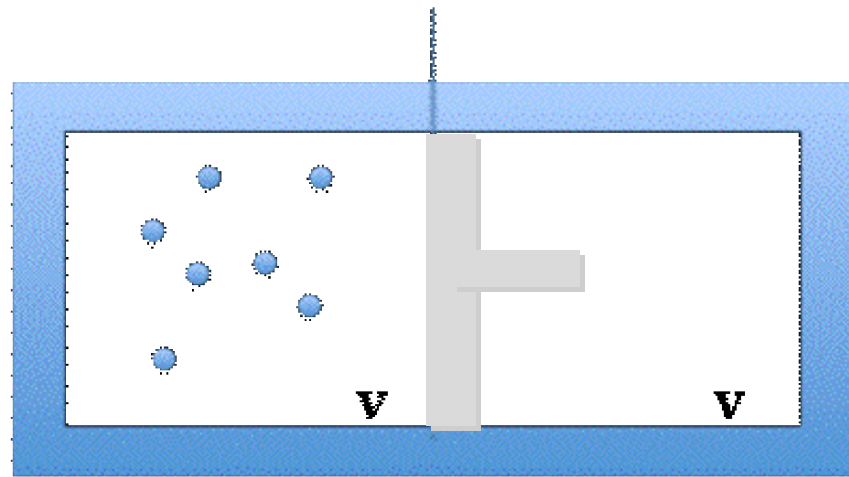
Need to use the following:

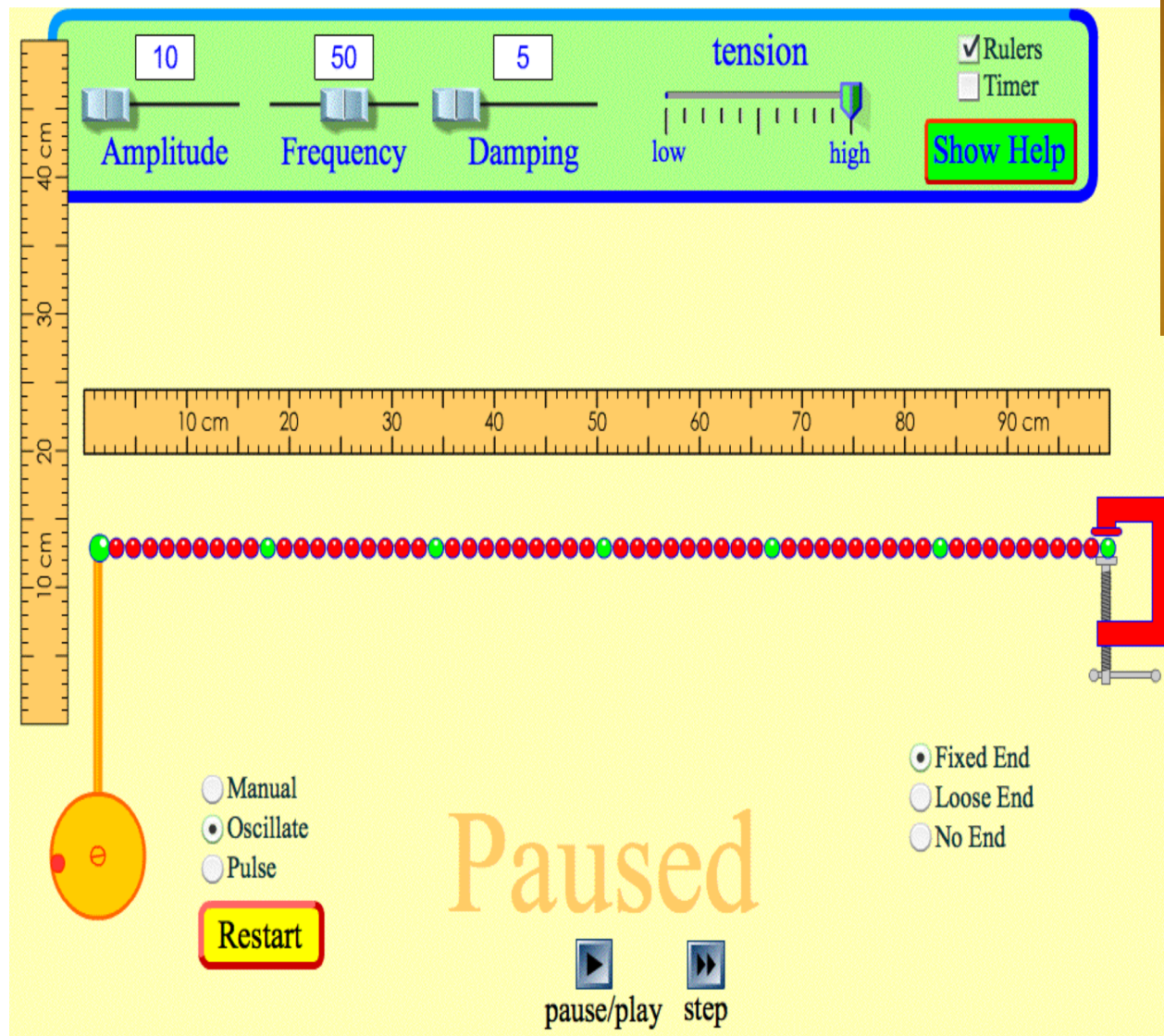
$$PV = N kT$$

$$U = \frac{n}{2} N kT$$

$$\Delta U = Q - P\Delta V$$

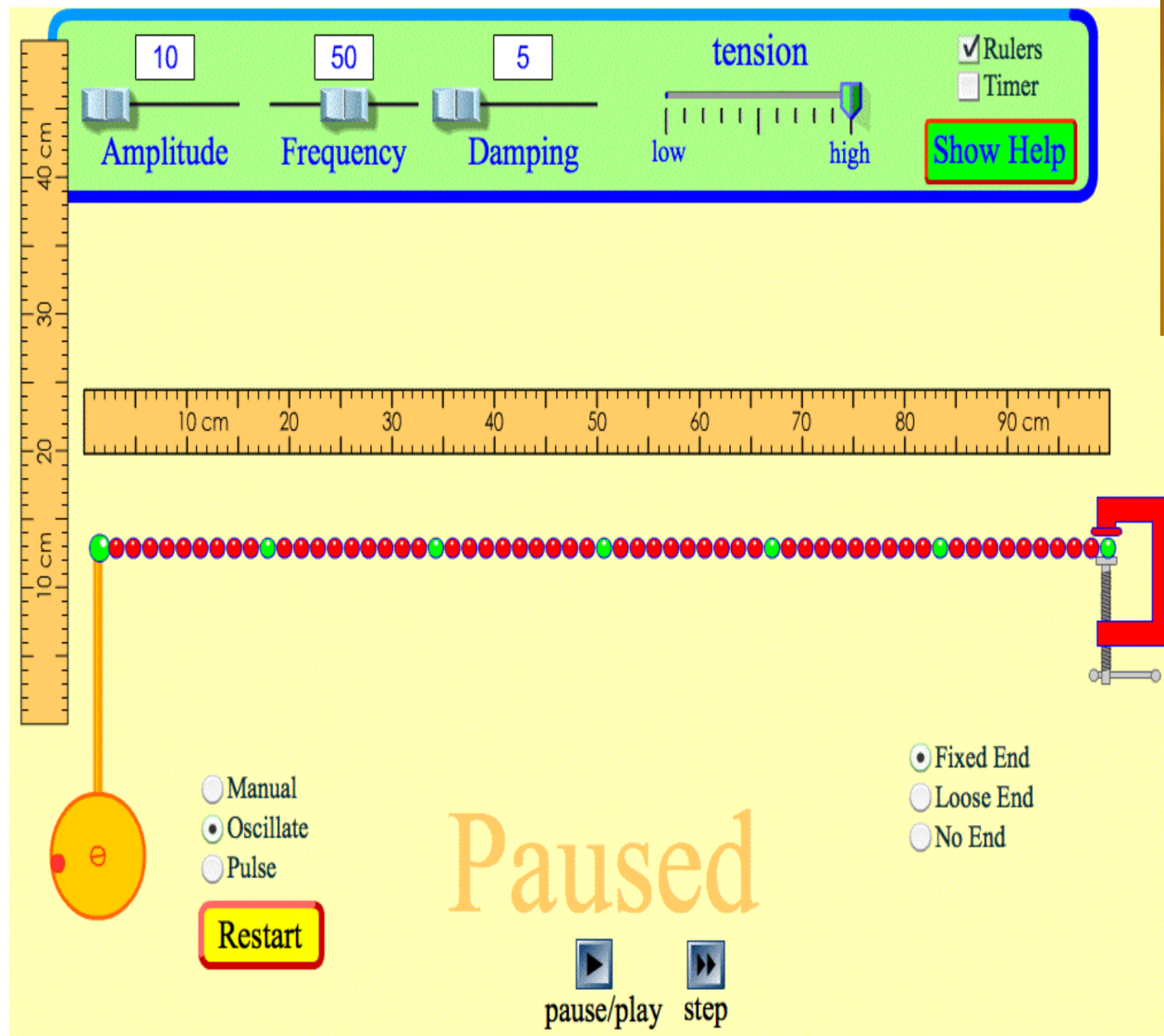
$$Q = 0$$





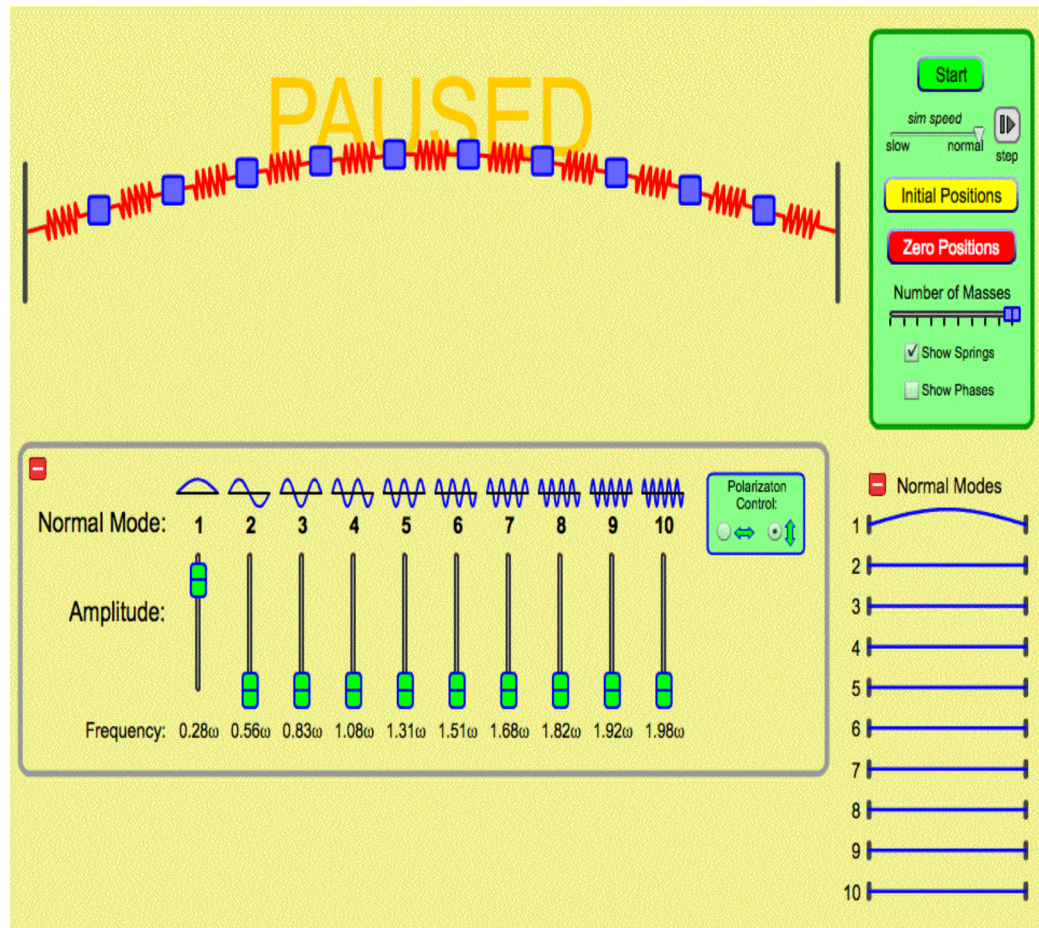
For what frequencies will I generate a large (resonant) standing wave if I drive it with a small amplitude?





For what frequencies will I generate a large (resonant) standing wave if I drive it with a small amplitude?

# Explore with a simulation



<http://phet.colorado.edu/en/simulation/normal-modes>