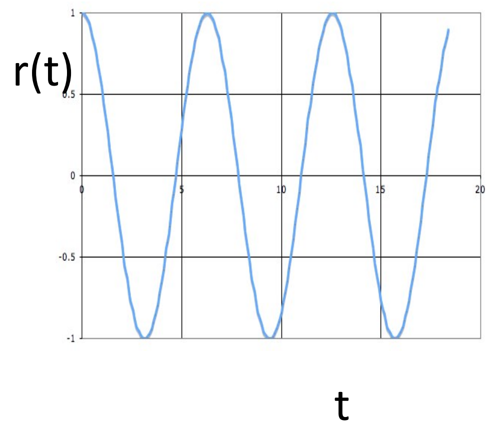
Waves

Oscillations and Waves

What's the difference?

Separation between two atoms in a molecule r(t)



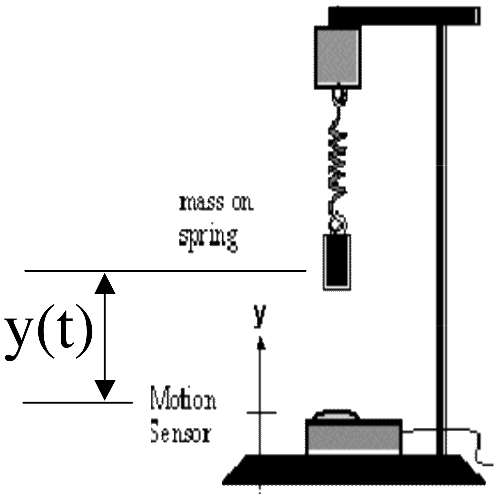
Oscillations involve a discrete set of quantities that vary in time (usually periodically).

Examples: pendula, vibrations of individual molecules, firefly lights, currents in circuits.

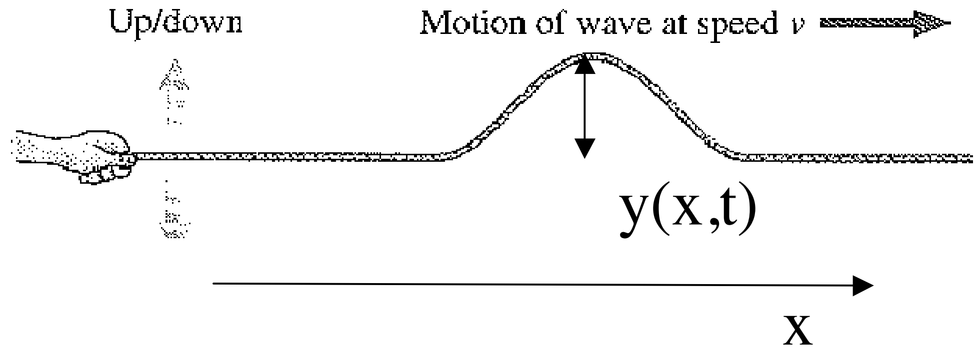
Waves involve continuous quantities the vary in both space and time. (variation may be periodic or not)

Examples: light waves, sound waves, elastic waves, surface waves, electro-chemical waves on neurons

Oscillations



Waves



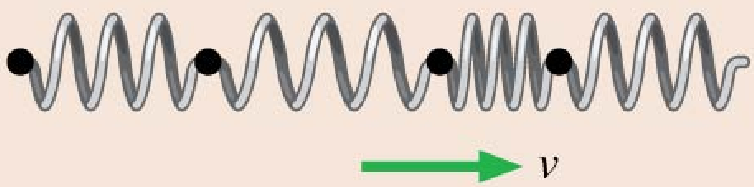
Waves involve <u>continuous</u> quantities the vary in <u>both space and time</u>. (variation may be periodic or not)

Mechanical Waves

Two Types of Waves

You'll find that waves come in two basic types:

Transverse waves: The displacement is perpendicular to the direction of travel.



Longitudinal waves: The displacement is parallel to the direction of travel.

Examples:

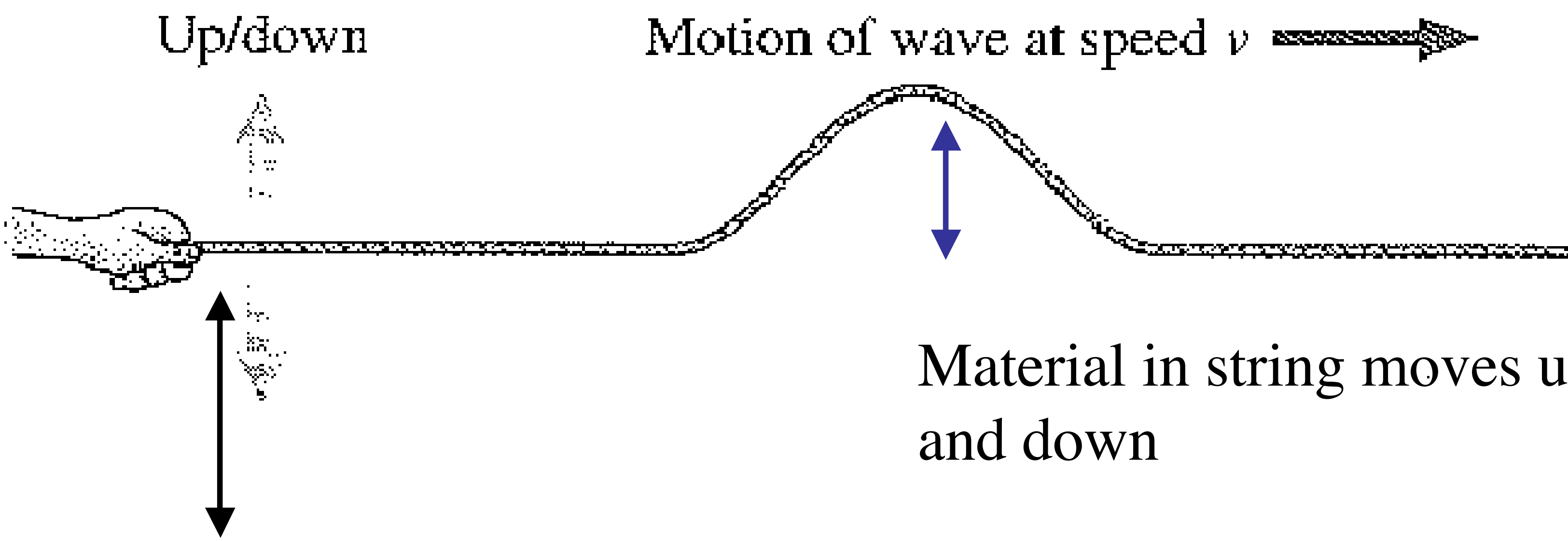
Transverse or Longitudinal?

Wave	Wave Type, Trns.=A, Long.=B, Both=C

1. Waves on stretched strings:
2. Sound waves in Liquids/gases:
3. Elastic (Seismic) waves in solids:
4. Surface waves on water:
5. Electromagnetic waves:

Wave speed versus Material Speed

Disturbance (pulse) moves to right.
Speed of pulse determined by medium



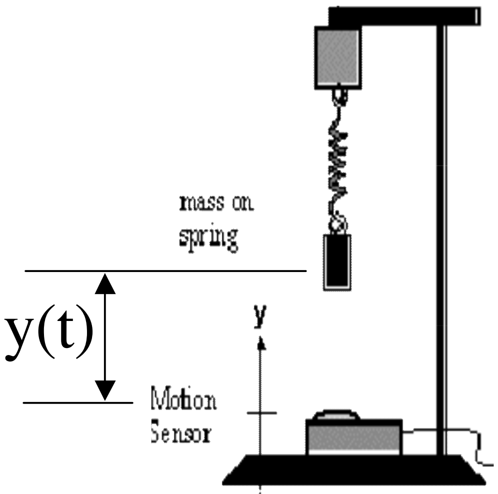
Material in string moves up and down

Shape of pulse determined by source of wave.

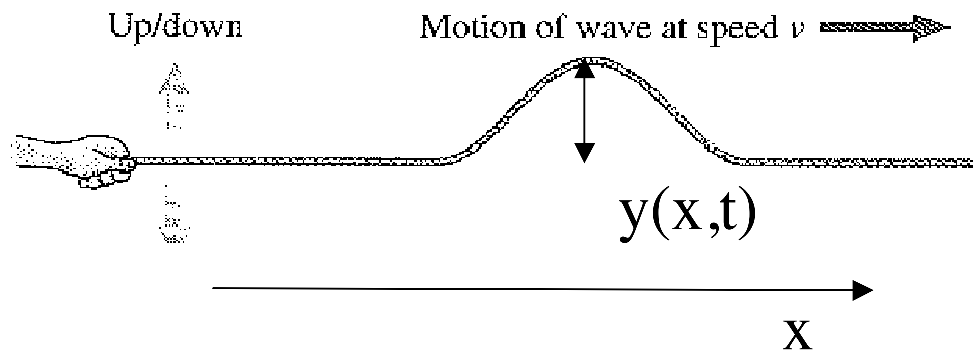
$$v = \sqrt{\frac{Tension}{LinearMassDensity}}$$

What is the restoring Force?

Oscillations



Waves



Newton says:

$$m\frac{d^2y(t)}{dt^2} = F_y = -ky(t)$$

Hooke's law

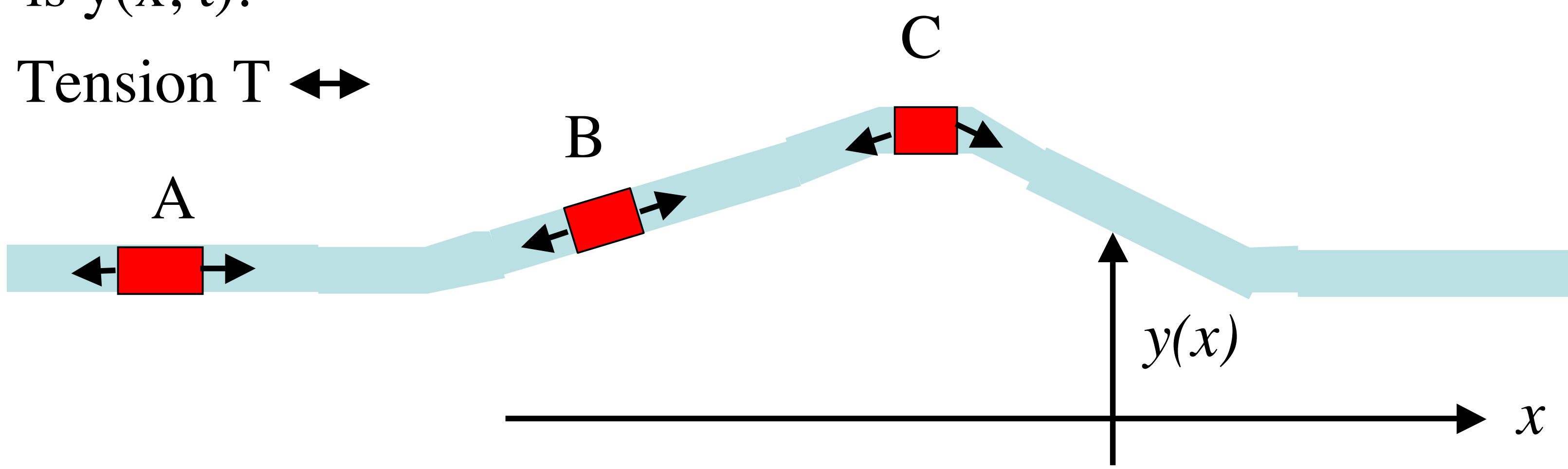
Newton says:

$$\mu\frac{d^2y(x,t)}{dt^2} = (F_y / l) = ?$$

mass per unit length

force per unit length

Consider three segments of a string under tension T.
The vertical displacement of the string at some instant of time is y(x, t).



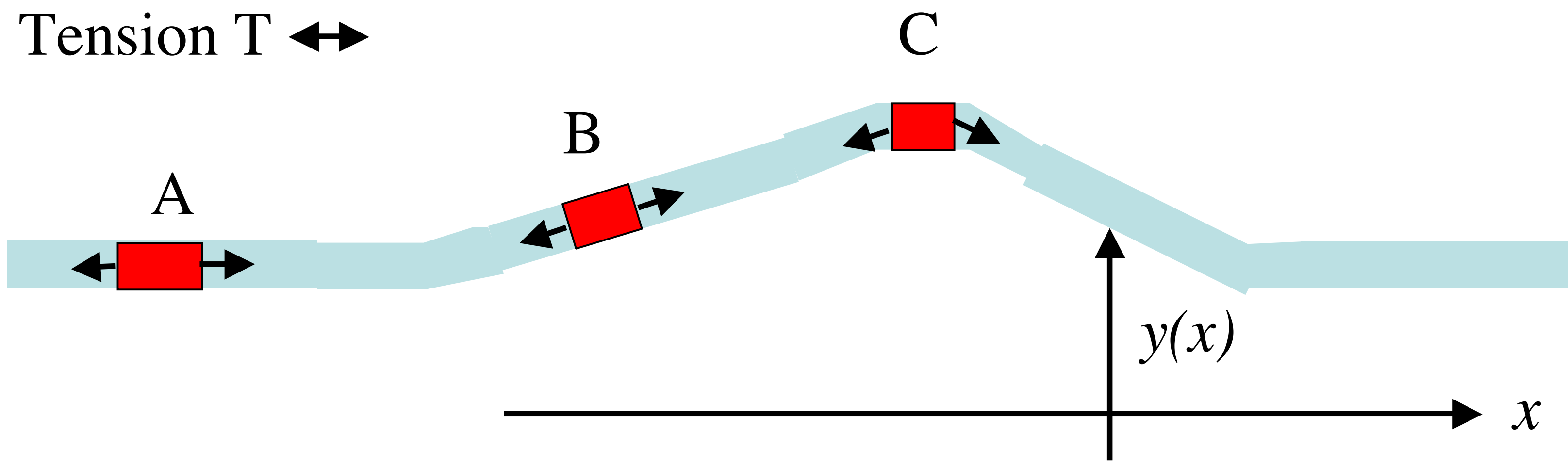
1. Which segment is feeling a vertical force from the string?

2. Is the force

A. up ?

B. down ?

Consider three segments of a string under tension T. The vertical displacement of the string is y(x).



3. Which best expresses the law giving the restoring force?

4. What else should the restoring force depend on?

A. $F_y \propto -y(x)$

B. $F_y \propto -\frac{dy(x)}{dx}$

C. $F_y \propto \frac{dy(x)}{dx}$

D. $F_y \propto -\frac{d^2y(x)}{dx^2}$

E. $F_y \propto -\frac{d^2y(x)}{dx^2}$

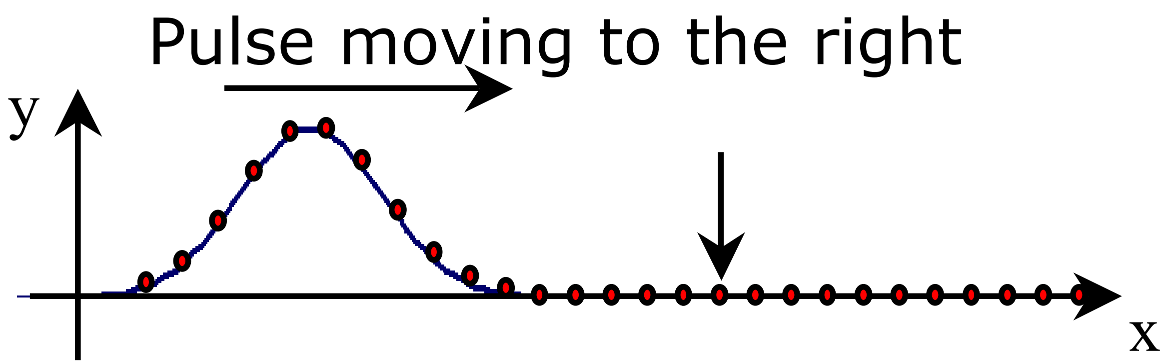
Whiteboard, TA & LA

The wave equation: $$\mu \frac{d^2 y(x,t)}{dt^2} = T \frac{d^2 y(x,t)}{dx^2}$$

$$\frac{\partial^2 y(x,t)}{\partial t^2} = v^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

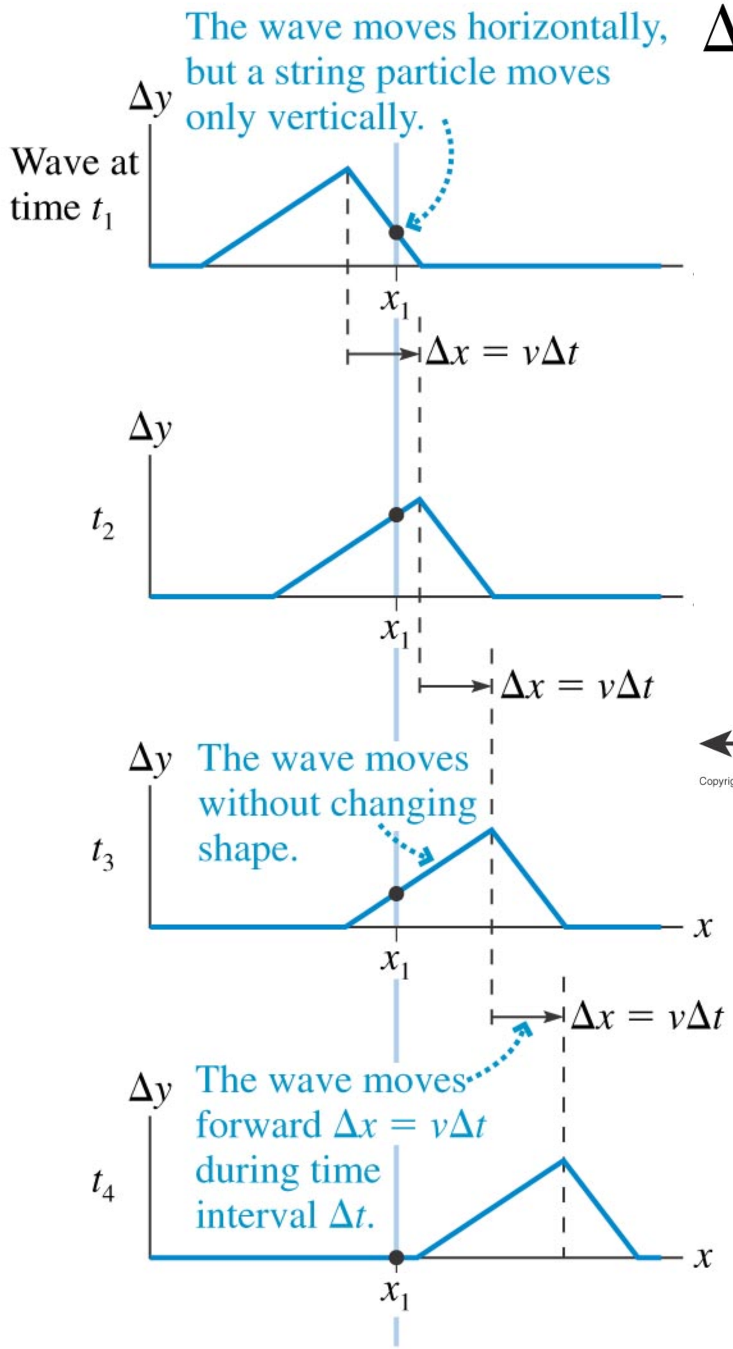
String	$v = \sqrt{T / \mu}$	Tension/Linear mass density
Sound	$v = \sqrt{\gamma P / \rho}$	Pressure/mass density
Elastic waves	$v = \sqrt{E / \rho}$	Young’s Modulus/mass density
EM waves	$v = c$	

How do the beads move?

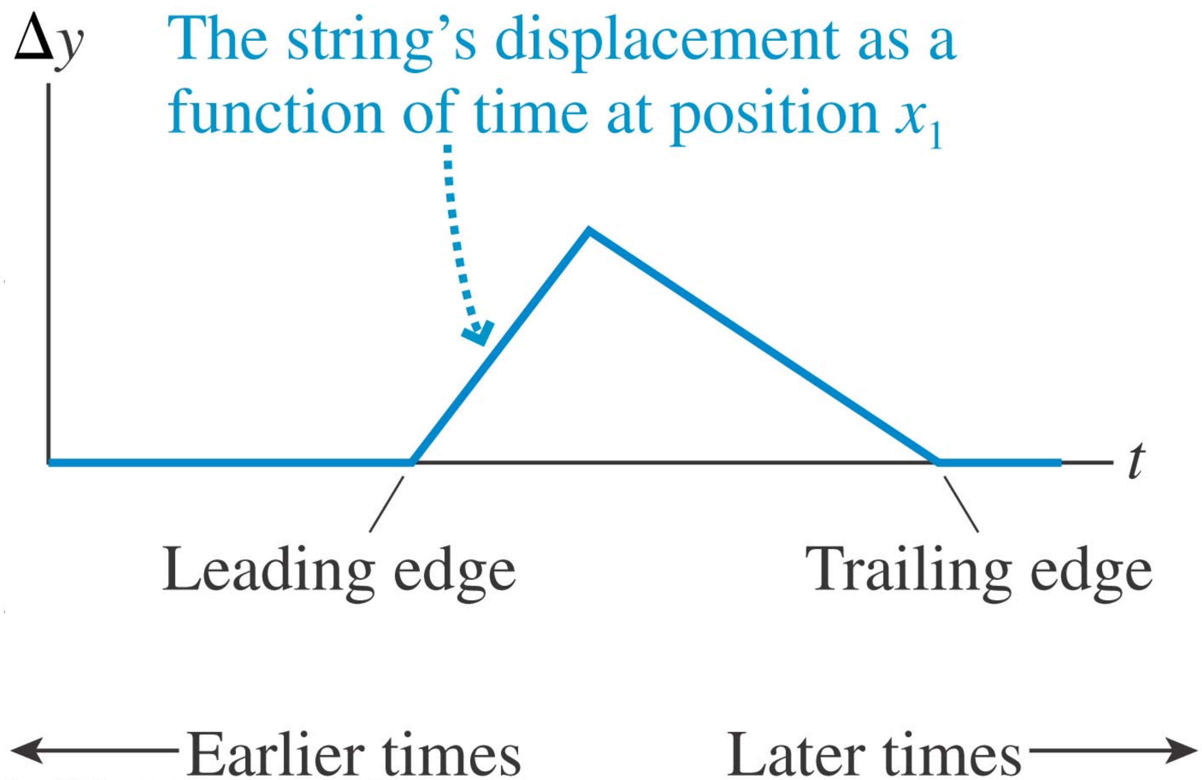


- Sketch the y position of the bead indicated by the arrow as a function of time

Whiteboard, TA & LA

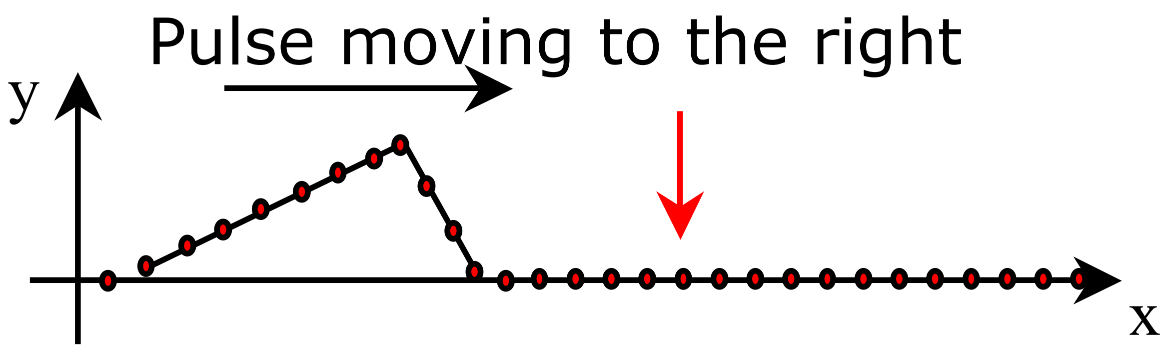


The wave moves horizontally, but a string particle moves only vertically.
Δy
Wave at time t_1
x_1
$\Delta x = v\Delta t$
Δy
t_2
x_1
$\Delta x = v\Delta t$
Δy
The wave moves without changing shape.
t_3
x
x_1
$\Delta x = v\Delta t$
Δy
The wave moves forward $\Delta x = v\Delta t$ during time interval Δt.
t_4
x
x_1

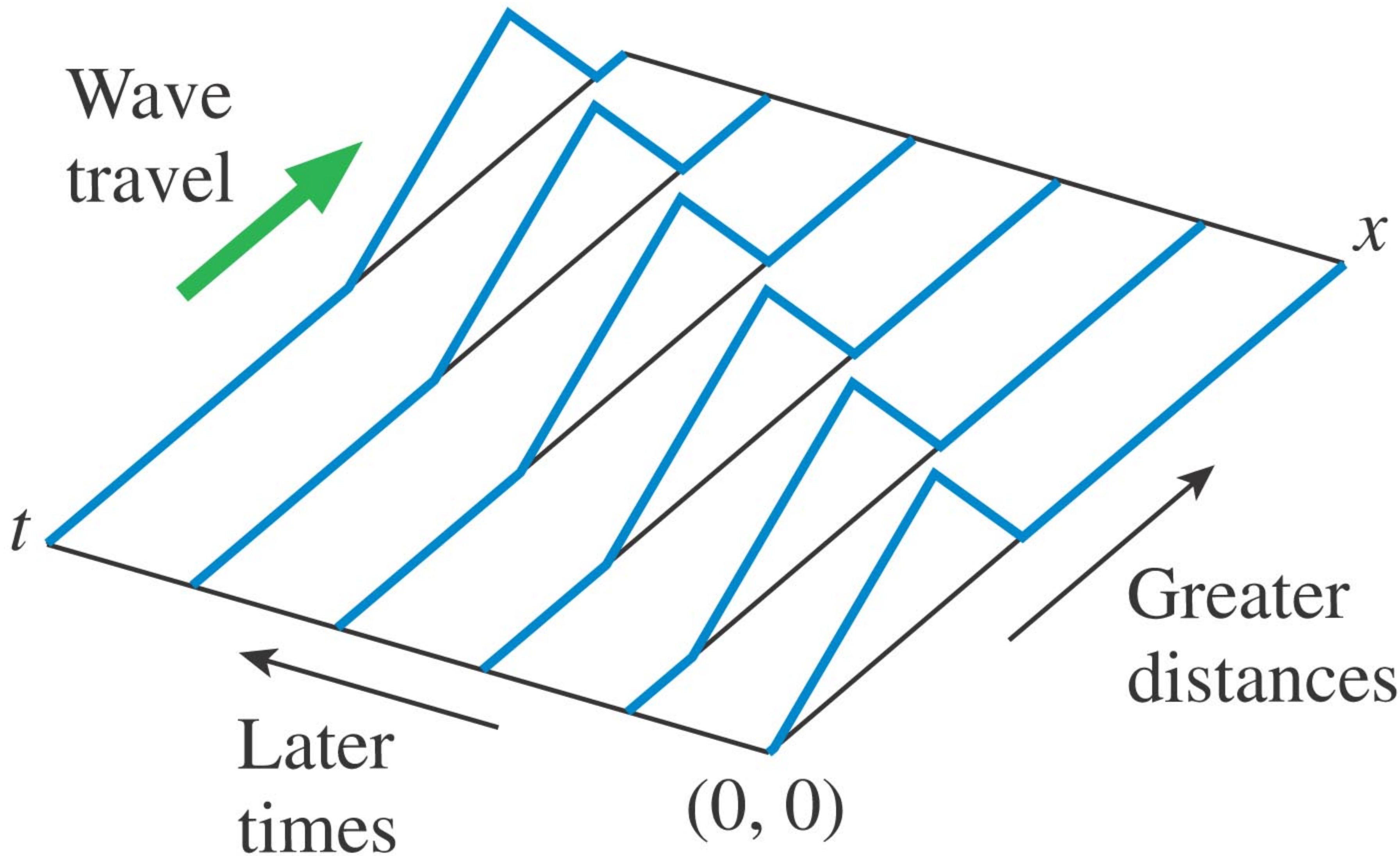


Δy
The string's displacement as a function of time at position x_1
t
Leading edge
Trailing edge
Earlier times
Later times

If this is the space-graph (photo at an instant of time) what does the time-graph look like for the bead marked
with a red arrow?



1. Choice One
2. Choice Two
3. Choice Three
4. Choice Four
5. Choice Five
6. Choice Six
7. Choice Seven
8. Choice Eight

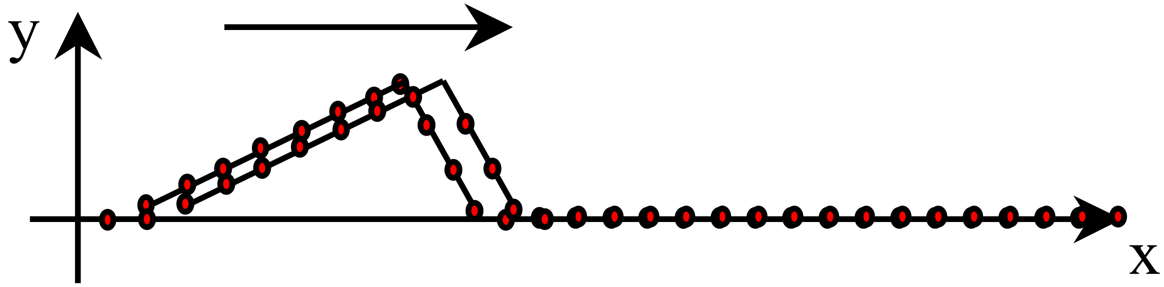


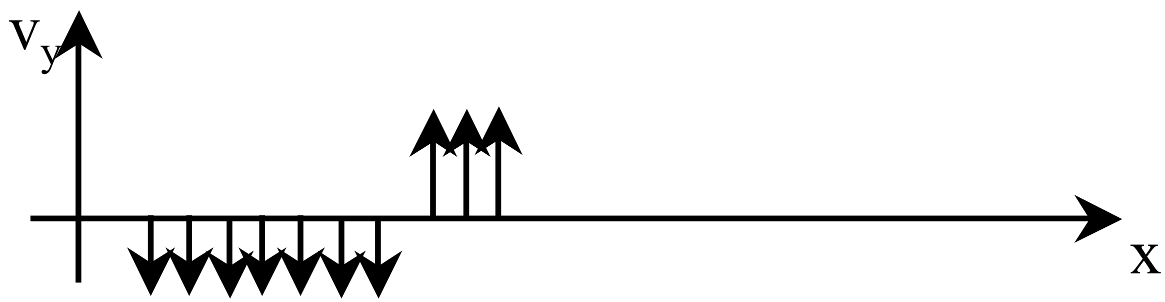
Wave
travel
x
t
Greater
distances
Later
times
(0, 0)

Describing the motion of the beads

- Sketch the velocity of each bead in the top figure at the time shown

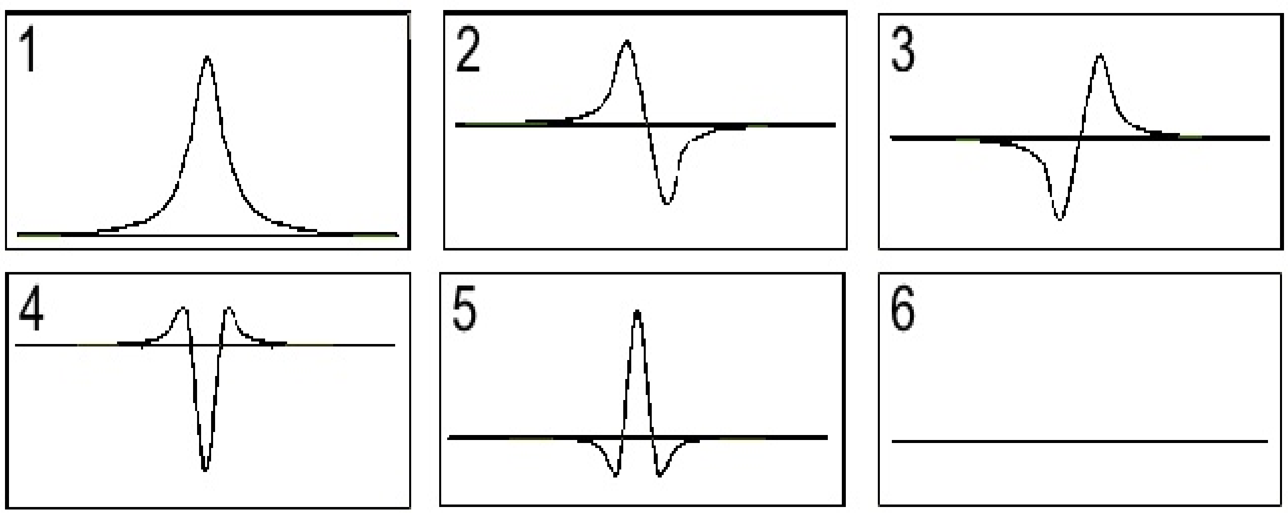
Pulse moving to the right





A pulse is started on the string moving to the right. At a time t_0 a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. *(x is horizontal and right, y is vertical and up)*

Which graph would look most like a graph of the **y displacement** of the spot as a function of time?



7 None of these

A pulse is started on the string moving to the right. At a time t_0 a photograph of the string would look like figure 1 below. A point on the string to the right of the pulse is marked by a spot of paint. *(x is horizontal and right, y is vertical and up)*

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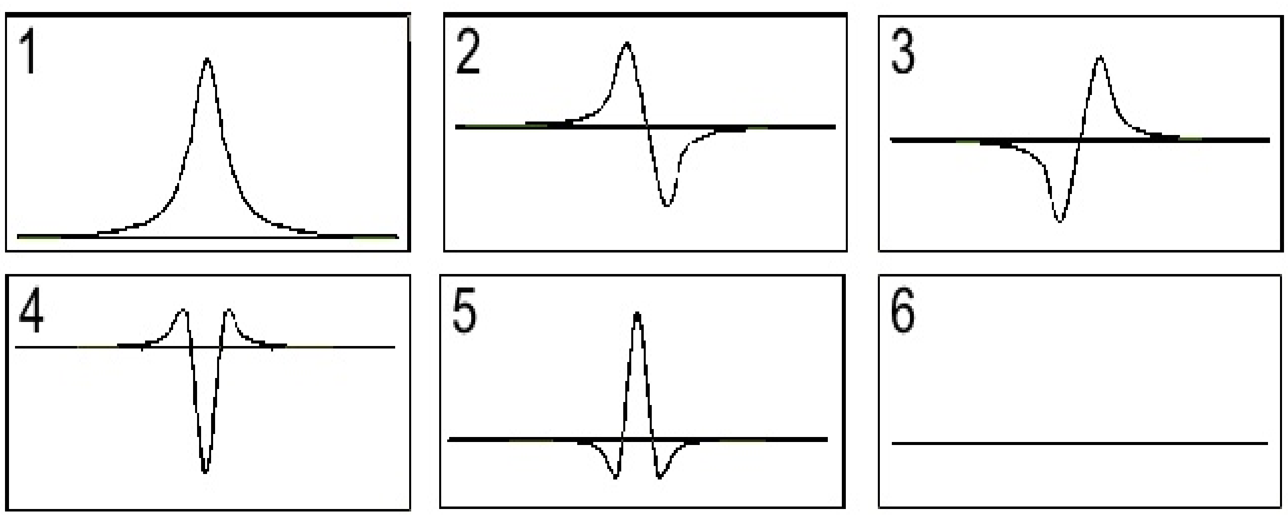

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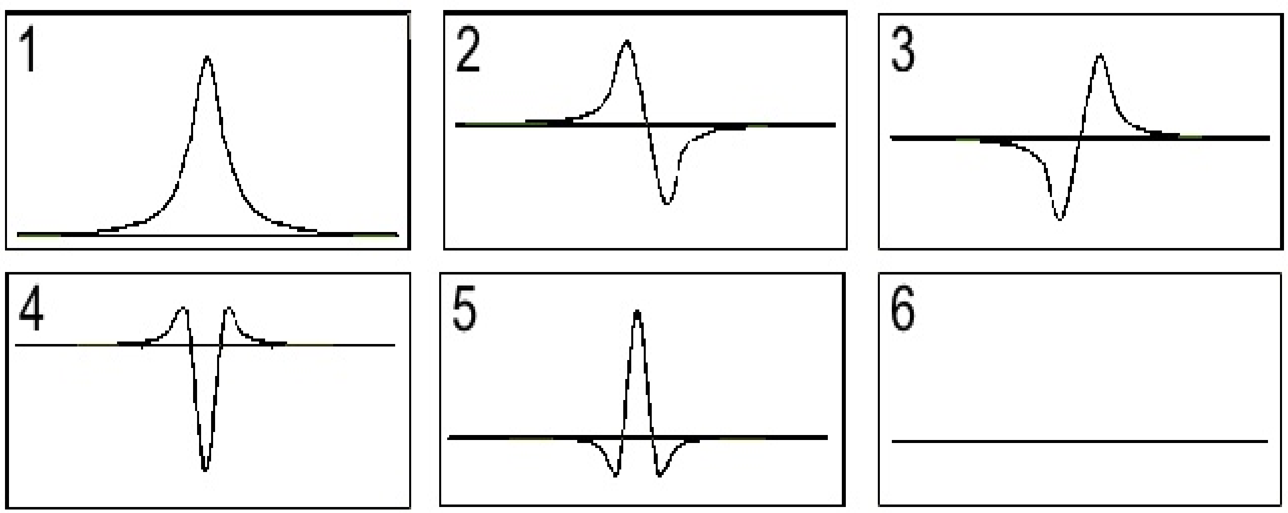
Which graph would look most like a graph of the **y force** on the spot as a function of time?



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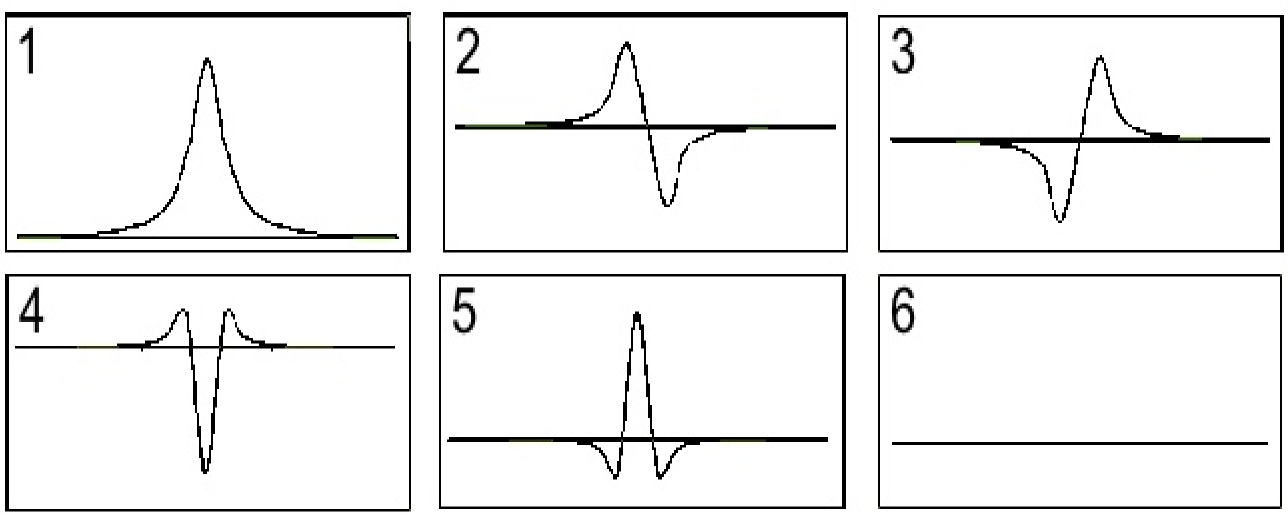
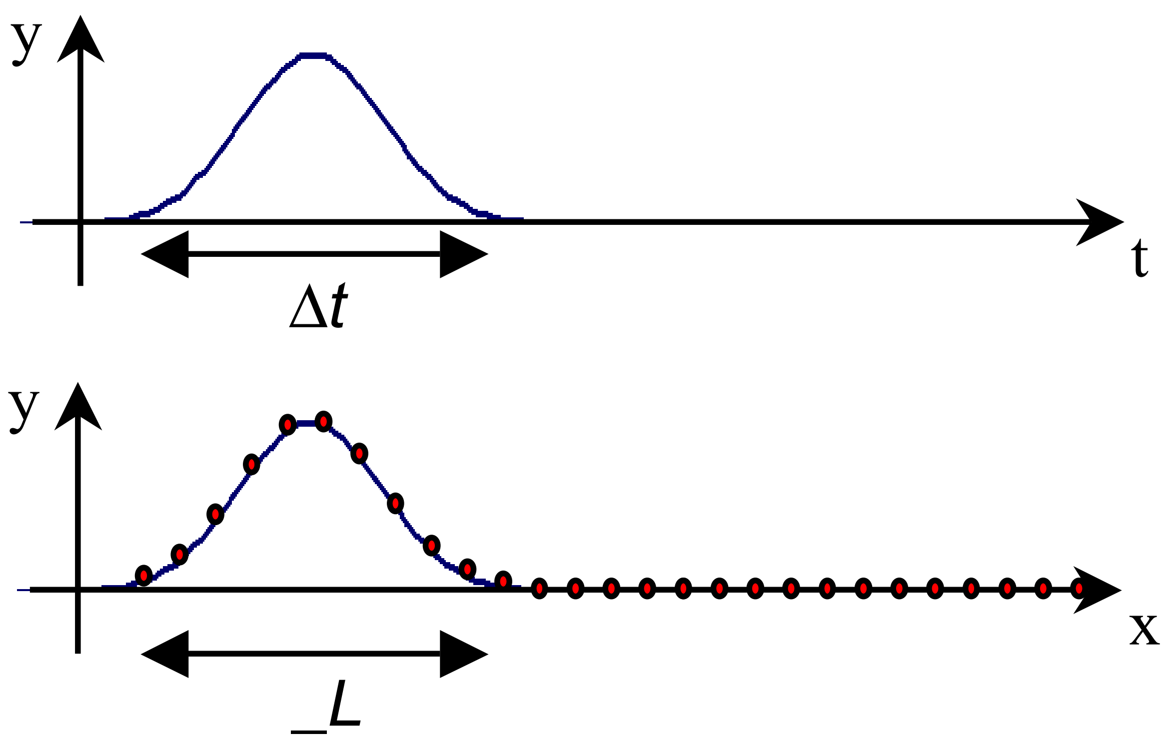
Which graph would look most like a graph of the **x velocity** of the spot as a function of time?



7 None of these

What controls the widths of the pulses in time and space?

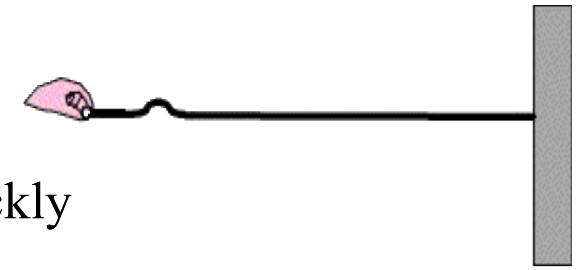


Width of a pulse

- The amount of time the demonstrator's hand was displaced up and down determines the time width of the t-pulse, Δt.
- The speed of the signal propagation on the string controls the width of the x-pulse, ΔL.
  - The leading edge takes off with some speed, v_0.
  - The pulse is over when the trailing edge is done.
  - The width is determined by "how far the leading edge got to" before the displacement was over.

$$\Delta L = v_0 \Delta t$$

What Controls the Speed of the Pulse on a Spring?



To make the pulse go to the wall faster

1. Move your hand up and down more quickly (but by the same amount).
2. Move your hand up and down more slowly (but by the same amount).
3. Move your hand up and down a larger distance in the same time.
4. Move your hand up and down a smaller distance in the same time.
5. Use a heavier string of the same length under the same tension.
6. Use a string of the same density but decrease the tension.
7. Use a string of the same density but increase the tension.
8. Put more force into the wave,
9. Put less force into the wave.

Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish between the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)
- *Matter speed*: the speed of the bits of matter depend on both the **Amplitude** and **shape of the pulse** and **pattern speed**.

Dimensional analysis

- Square brackets are used to indicate a quantities dimensions – mass ($\mathcal{M}$), length ($\mathcal{L}$), or time ($\mathcal{T}$)
  - $[m] = \mathcal{M}$
  - $[L] = \mathcal{L}$
  - $[t] = \mathcal{T}$
  - $[F] = \mathcal{M}\mathcal{L}$

- Build a velocity using mass (m), length (L), and tension (T) of the string:
  - $[v] = \mathcal{L}/\mathcal{T}$
  - $[T] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$
  - $[T/m] = \mathcal{L}/\mathcal{T}^2$
  - $[TL/m] =$

$$v_0^2 = \frac{TL}{m}$$

or, using $\mu = m/L$ $\qquad v_0 = \sqrt{\frac{T}{\mu}}$

Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
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$$v_0 = \sqrt{T/\mu}$$

v_0 = speed of pulse
T = tension of spring
_ = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the

We now want to expand the picture in the following way:

EM waves propagate in 3D not just 1D as we have considered.
- Diffraction - waves coming from a finite source spread out.

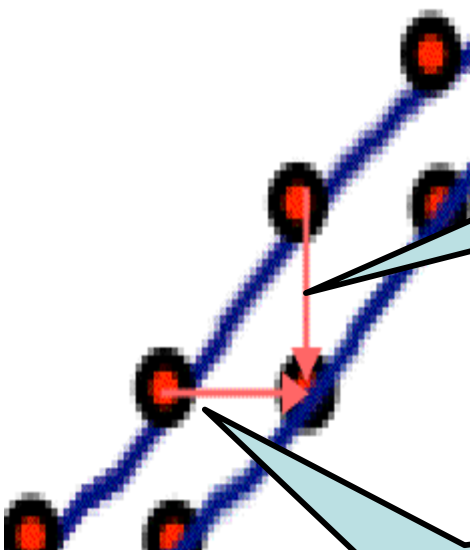
EM waves propagate through material and are modified.
- Dispersion - waves are slowed down by media, different frequency waves travel with different speeds

- Reflection - waves encounter boundaries between media. Some energy is reflected.
- Refraction - wave trajectories are bent when crossing from one medium to another.

EM waves can take multiple paths and arrive at the same point.
- Interference - contributions from different paths add or cancel.

Speed of a bead

- The speed the bead moves depends on how fast the pulse is moving and how far it needs to stay



dy = how far bead moves in time dt

dx = how far pulse moves in time dt

$$v(x,t) = \frac{dy(x,t)}{dt} = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{dy}{dx}\right) v_0$$

slope of pulse

speed of bead

speed of pulse

Properties of electromagnetic waves in vacuum:

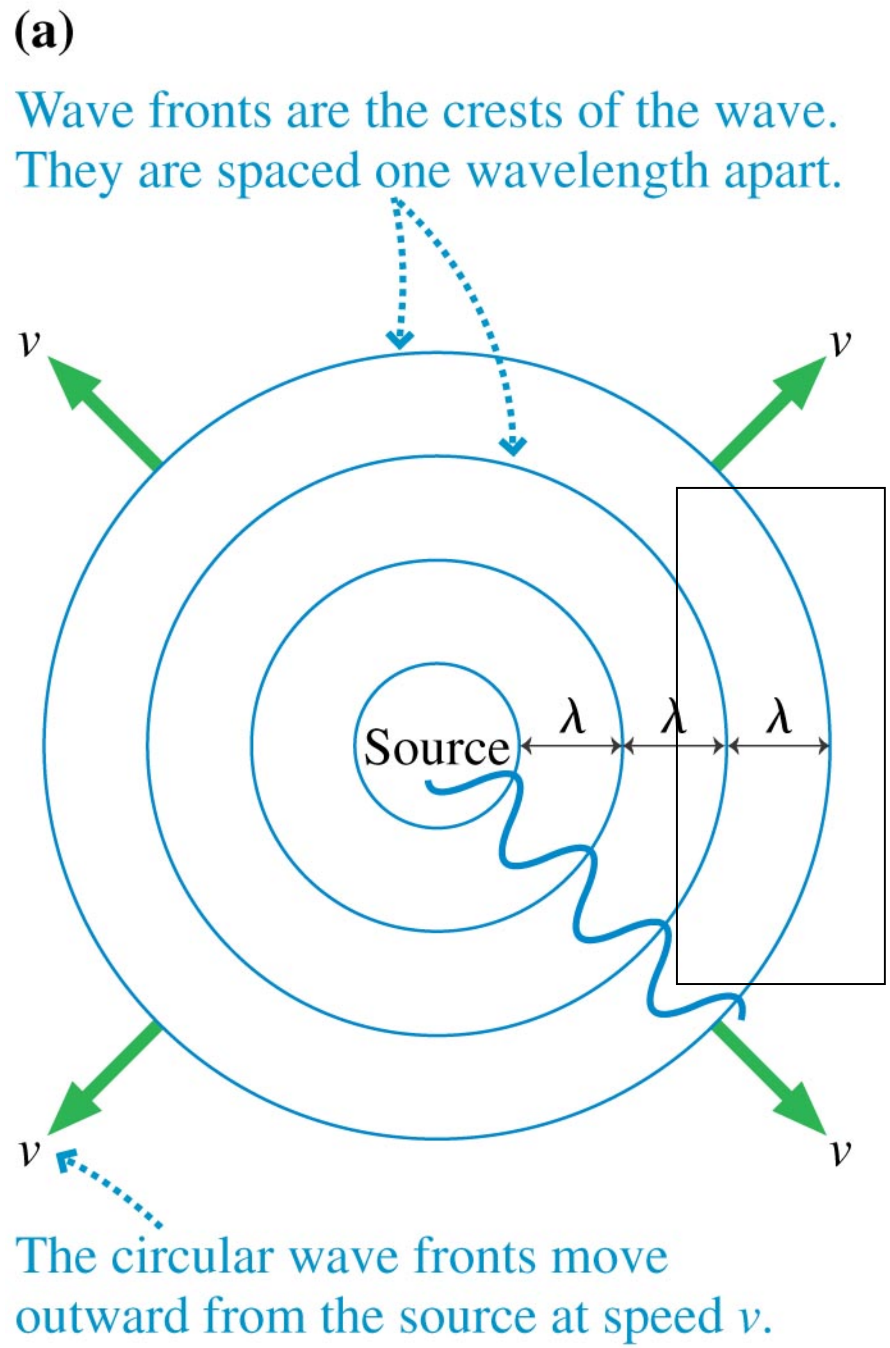
Waves propagate through vacuum (no medium is required like sound waves)

All frequencies have the same propagation speed, c in vacuum.

Electric and magnetic fields are oriented transverse to the direction of propagation. (transverse waves)

Waves carry both energy and momentum.

Waves emanating from a point source



Displacements on an elastic string / spring

- Each bit of the string can move up or down (perpendicular to its length) – transverse waves
- Each bit of string can also move toward/away along the string length if the string is elastic (most notable on very deformable strings such as slinky, rubber band). – longitundinal waves