

April 12, 2013      Physics 132      Prof. E. F. Redish

- **Theme Music: Duke Ellington**  
*Take the A Train*
- **Cartoon: Bill Amend**  
*FoxTrot*

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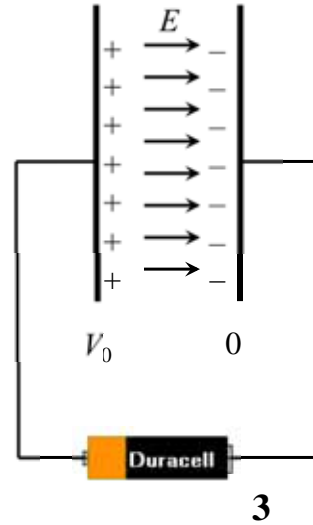
## Some basic electrical ideas

- **Conductor** – a material that permits some of its charges to move freely within it.
  - *Implication:* If the charges in a conductor are not moving, the whole conductor is at the same  $V$ . Why?
- **Insulator** – a material that permits some of its charges to move a little, but not freely.
- **Battery** – a device that creates and maintains a constant potential difference across its terminals.

4/12/13      High end      Low end      2

## Charging a capacitor

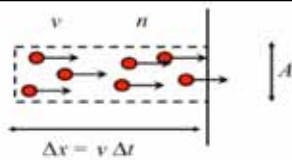
- What is the potential difference between the plates?
- What is the field around the plates?
- How much charge is on each plate?



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## Ohm's Law

- Current proportional to velocity
- Due to resistance, Electric force proportional to velocity.
- Force proportional to "electric pressure drop" = "electric PE"
- Therefore, current proportional to "electric PE"

$$I = qnAv \Rightarrow v = \frac{I}{qnA}$$

$$qE = bv$$

$$\Delta V = EL \Rightarrow E = \frac{\Delta V}{L}$$

$$\Rightarrow \frac{q\Delta V}{L} = \frac{bI}{qnA}$$

$$\Delta V = IR$$

$$\Delta V = I \left( \frac{bL}{q^2 nA} \right) \equiv IR$$

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## Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part ( $L/A$ ) times a part independent of the size and shape but dependent on the material.
- This coefficient is called the *resistivity* of the material ( $\rho$ ). Its reciprocal ( $g$ ) is called *conductivity*. The reciprocal of the resistance is called the *conductance* ( $G$ ).

$$R = \left( \frac{bL}{q^2 nA} \right) = \rho \frac{L}{A} = \frac{1}{g} \frac{L}{A} = \frac{1}{G}$$

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## Electric circuit elements

- Batteries — devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).
- Resistances — devices that have significant drag and oppose current. Pressure will drop across them.
- Capacitors — devices that can maintain a separation of charge if there is a potential difference maintained across the,
- Wires — have very little resistance. We can ignore the drag in them (mostly — as long as there are other resistances present).




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## Foothold ideas: Electric charges in fluids



- **Electroneutrality** – Opposite charges in materials attract each other strongly. Pulling them apart to create a charge unbalance costs energy. This tends to make small volumes of fluid electrically neutral.
- **Energy-Entropy balances** – When there are situations of non-uniformity, electrical forces (energy) can balance or be balanced by random thermal motion (entropy). Two important cases are:
  - **Debye shielding** – introduced unbalanced charge
  - **Nernst potential** – non-uniform concentrations of ions

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## Foothold ideas:



- **Debye length**– A charge imbedded in an ionic solution is shielded by the ions pulling up towards the charge. The amount of imbalance is determined by a balance of the thermal fluctuation energy against the repulsive electrostatic energy arising from the imbalance.
 
$$\lambda_D = \sqrt{\frac{\kappa k_B T}{k_c q^2 c_0}}$$

$$V(r) = \frac{k_c Q}{\kappa r} e^{-r/\lambda_D}$$
- **Nernst potential** – When a membrane permits only one kind of ion to pass, diffusion from the side with a greater concentration of that kind of ion will build up a potential difference due to ions moving to the side with the lower concentration.
 
$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_1}{c_2}\right)$$

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## Foothold ideas: Kirchhoff's principles



1. **Flow rule:** The total amount of current flowing into any volume in an electrical network equals the amount flowing out.
2. **Ohm's law:** in a resistor,  $\Delta V = IR$
3. **Loop rule:** Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).

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## Very useful heuristic

- The Constant Potential Corollary (CPC)
  - Along any part of a circuit with 0 resistance, then  $\Delta V = 0$ , i.e., the voltage is constant since in any circuit element

$$\Delta V = IR$$

$$R = 0 \Rightarrow \Delta V = 0$$

(even if  $I \neq 0$ )

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## Electric Power

- The rate at which electric energy is depleted from a battery or dissipated (into heat or light) in a resistor is

$$Power = I\Delta V$$

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## Units


- Current ( $I$ )      **Ampere** = Coulomb/sec
- Voltage ( $V$ )      **Volt** = Joule/Coulomb
- E-Field ( $E$ )      Newton/Coulomb = Volt/meter
- Resistance ( $R$ )      **Ohm** = Volt/Ampere
- Capacitance ( $C$ )      **Farad** = Volt/Coulomb
- Power ( $P$ )      **Watt** = Joule/sec

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## Foothold ideas: Harmonic oscillation



- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves, the force is in the direction of pushing it back to its equilibrium position.
- When it gets back to its equilibrium, it's still moving so it overshoots.

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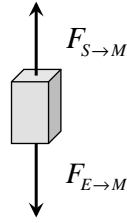
## Summary with Equations: Mass on a spring

$$a = \frac{1}{m} F^{net} \quad F^{net} = -kx$$

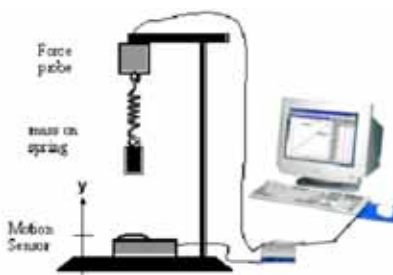
$$a = -\omega_0^2 x \quad \omega_0^2 = \frac{k}{m}$$

$$x(t) = A \cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$



Interpret!



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## Summary with Equations: Mass on a spring (Energy)

Measured  
from where?

$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}k(\Delta l)^2$$

$$E_i = E_f$$

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## Pendulum motion energy

$$E_0 = \frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2 + mgL(1 - \cos\theta)$$

$$\cos\theta \approx 1 - \frac{1}{2}\theta^2$$

$$E_0 \approx \frac{1}{2}mv^2 + \frac{1}{2}[mgL]\theta^2$$

$$\theta \approx \sin\theta = \frac{x}{L}$$

$$E_0 \approx \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad k = \frac{mg}{L}$$


Same as mass on a spring!  
Just with a different  $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

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## Foothold ideas: Damped oscillator 1



- Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

$$ma = -kx - bv$$


$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \quad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

- Solution:  $x(t) = A_0 e^{-\gamma t/2} \cos(\omega_1 t + \phi)$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

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## Foothold ideas: Damped oscillator 2



- Competing time constants:

$$\frac{\gamma}{2} = \frac{1}{\tau} \quad \frac{\omega_0}{2\pi} = \frac{1}{T}$$

Decay time
Period


$$Q = \frac{\omega_0}{\gamma} = \pi \frac{\tau}{T}$$

Tells which force dominates: restoring or damping.

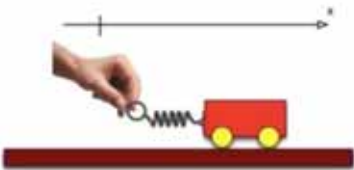
- If:
  - $\omega_0 > \gamma/2$  underdamped: oscillates
  - $\omega_0 = \gamma/2$  critically damped: no oscillation, fastest decay
  - $\omega_0 < \gamma/2$  over damped: no oscillation, slower decay

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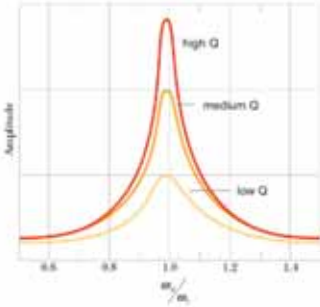
## Foothold ideas: Driven oscillator



- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not much.




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## Foothold principles: Mechanical waves 1



- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

$$v_0 = \sqrt{T/\mu}$$

$v_0$  = speed of pulse

$T$  = tension of spring

$\mu$  = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and pattern speed.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.

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## Foothold principles: Mechanical waves 2

- *Superposition*: when one or more disturbances overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)

