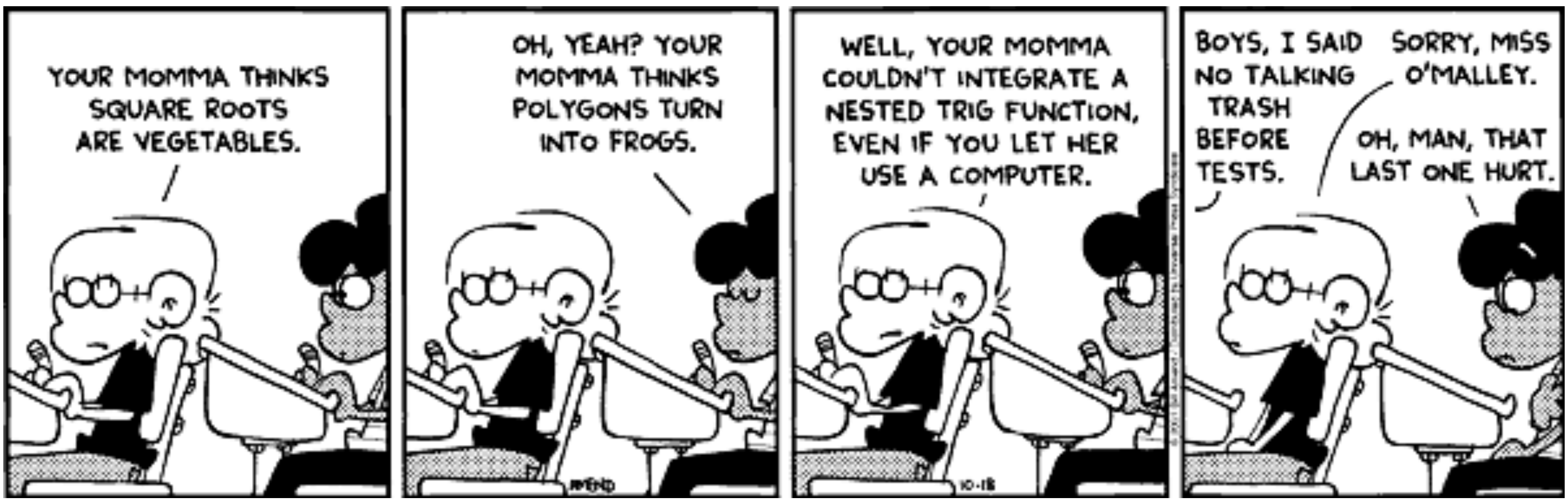
- **<u>Theme Music:</u> Duke Ellington**
  Take the A Train
- **<u>Cartoon:</u> Bill Amend**
  FoxTrot

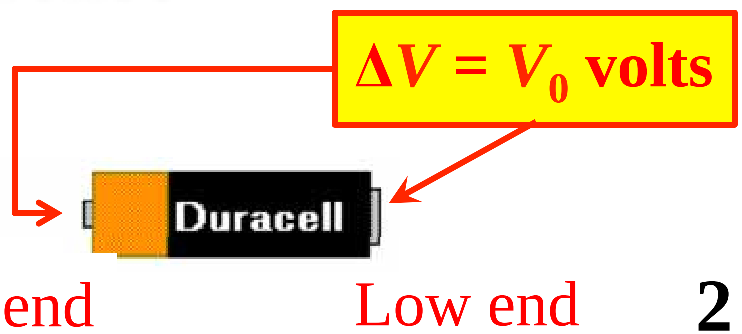


Some basic electrical ideas

- ***Conductor*** – a material that permits some of its charges to move freely within it.
  - ***Implication***: If the charges in a conductor are not moving, the whole conductor is as the same V.

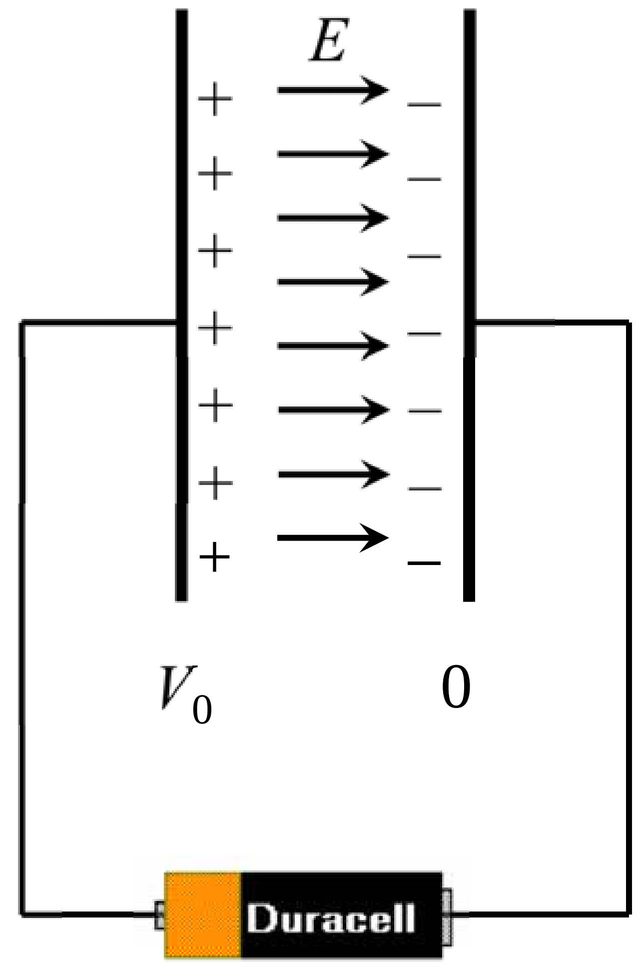
Why?

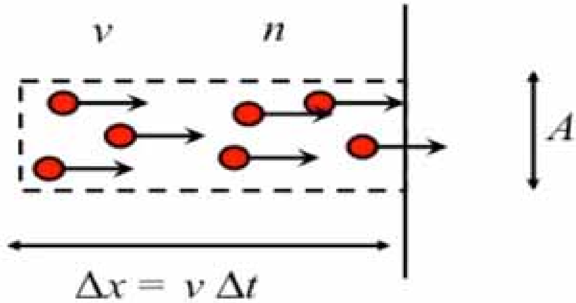
- ***Insulator*** – a material that permits some of its charges to move a little, but not freely.
- ***Battery*** – a device that creates and maintains a constant potential difference across its terminals.



Charging a capacitor

- What is
  the potential difference
  between the plates?
- What is the field
  around the plates?
- How much charge
  is on each plate?





Ohm's Law

- Current proportional to velocity
- Due to resistance,
  Electric force proportional to velocity.
- Force proportional to "electric pressure drop" = "electric PE"
- Therefore, current proportional to "electric PE"

$$I = qnAv \quad \Rightarrow \quad v = \frac{I}{qnA}$$

$$qE = bv$$

$$\Delta V = EL \quad \Rightarrow \quad E = \frac{\Delta V}{L}$$

$$\Rightarrow \quad \frac{q\Delta V}{L} = \frac{bI}{qnA}$$

$$\Delta V = IR$$

$$\Delta V = I\left(\frac{bL}{q^2 nA}\right) \equiv IR$$

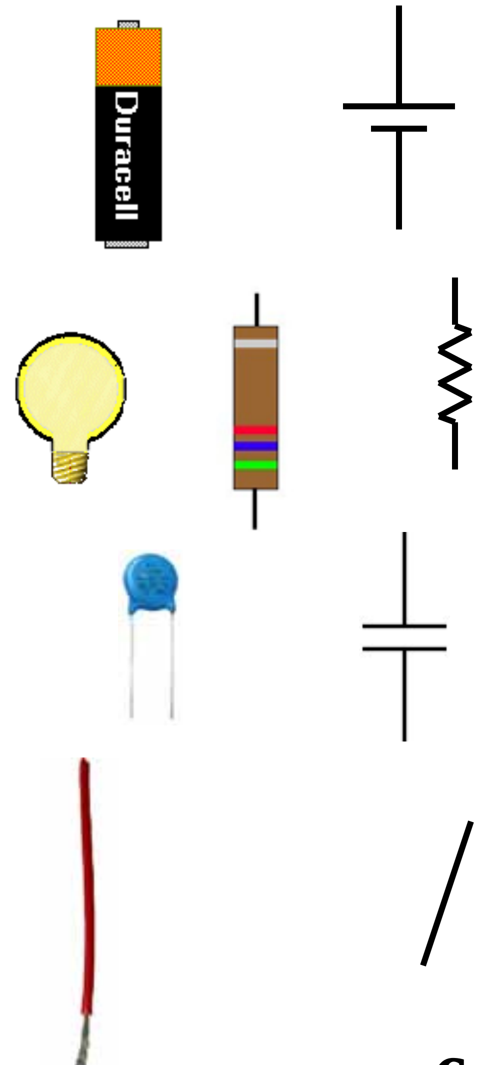
Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part (*L/A*) times a part independent of the size and shape but dependent on the material.
- This coefficient is called the *resistivity* of the material (ρ). Its reciprocal (g) is called *conductivity*. The reciprocal of the resistance is called the *conductance* (G).

$$R = \left(\frac{bL}{q^2 nA} \right) = \rho \frac{L}{A} = \frac{1}{g} \frac{L}{A} = \frac{1}{G}$$

Electric circuit elements

- *Batteries* —devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).
- *Resistances* —devices that have significant drag and oppose current. Pressure will drop across them.
- *Capacitors* – devices that can maintain a separation of charge if there is a potential difference maintained across the,
- *Wires* — have very little resistance. We can ignore the drag in them (mostly – as long as there are other resistances present).



Foothold ideas: Electric charges in fluids

- ***Electroneutrality*** – Opposite charges in materials attract each other strongly. Pulling them apart to create a charge unbalance costs energy. This tends to make small volumes of fluid electrically neutral.
- ***Energy-Entropy balances*** – When there are situations of non-uniformity, electrical forces (energy) can balance or be balanced by random thermal motion (entropy). Two important cases are:
  - **Debye shielding** – introduced unbalanced charge
  - **Nernst potential** – non-uniform concentrations of ions

Foothold ideas:

- ***Debye length***– A charge imbedded in an ionic solution is shielded by the ions pulling up towards the charge. The amount of imbalance is determined by a balance of the thermal fluctuation energy against the repulsive electrostatic energy arising from the imbalance.

$$\lambda_D = \sqrt{\frac{\kappa k_B T}{k_C q^2 c_0}}$$

$$V(r) = \frac{k_C Q}{\kappa r} e^{-r/\lambda_D}$$

- ***Nernst potential*** – When a membrane permits only one kind of ion to pass, diffusion from the side with a greater concentration of that kind of ion will build up a potential difference due to ions moving to the side with the lower concentration.

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_1}{c_2}\right)$$

Foothold ideas: Kirchhoff's principles

1. ***Flow rule***: The total amount of current flowing into any volume in an electrical network equals the amount flowing out.
2. ***Ohm's law***: in a resistor, $\Delta V = IR$
3. ***Loop rule***: Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).

Very useful heuristic

- The Constant Potential Corollary (CPC)
  – Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant since in any circuit element

$$\Delta V = IR$$

$$R = 0 \Rightarrow \Delta V = 0$$

$$(\text{even if } I \neq 0)$$

Electric Power

- The rate at which electric energy is depleted from a battery or dissipated (into heat or light) in a resistor is

$$Power = I\Delta V$$

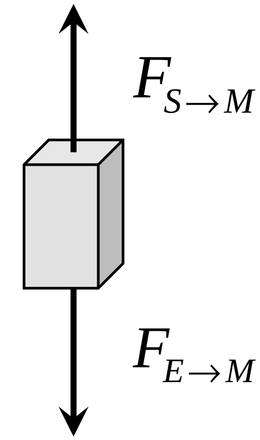
Units

- Current (I) — **Ampere** = Coulomb/sec
- Voltage (V) — **Volt** = Joule/Coulomb
- E-Field (E) — Newton/Coulomb = Volt/meter
- Resistance (R) — **Ohm** = Volt/Ampere
- Capacitance (C) — **Farad** = Volt/Coulomb
- Power (P) — **Watt** = Joule/sec

Foothold ideas: Harmonic oscillation

- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves, the force is in the direction of pushing it back to its equilibrium position.
- When it gets back to its equilibrium, it's still moving so it overshoots.

Summary with Equations: Mass on a spring



Measured from where?

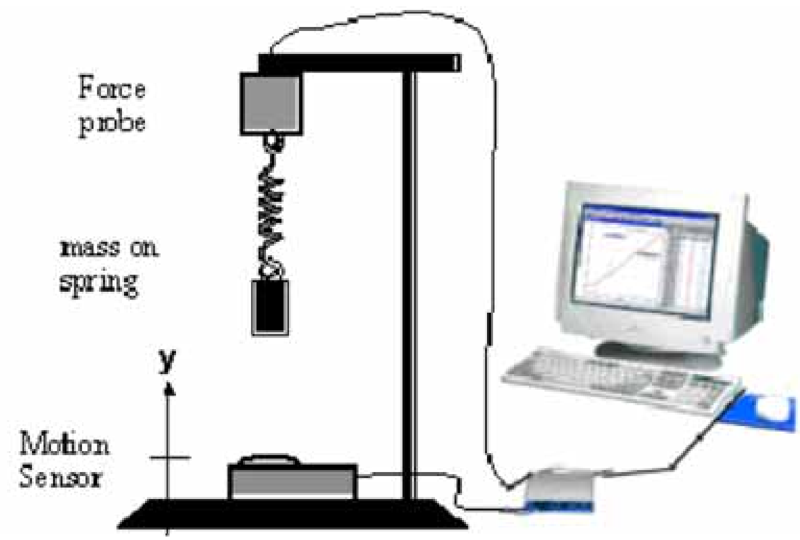
$$a = \frac{1}{m}F^{net} \qquad F^{net} = -kx$$

$$a = -\omega_0^2 x \qquad \omega_0^2 = \frac{k}{m}$$

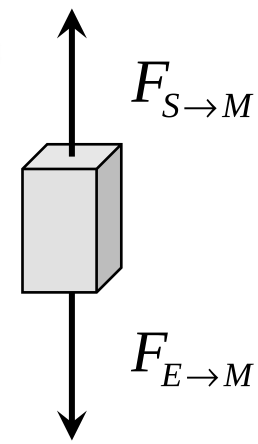
$$x(t) = A\cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



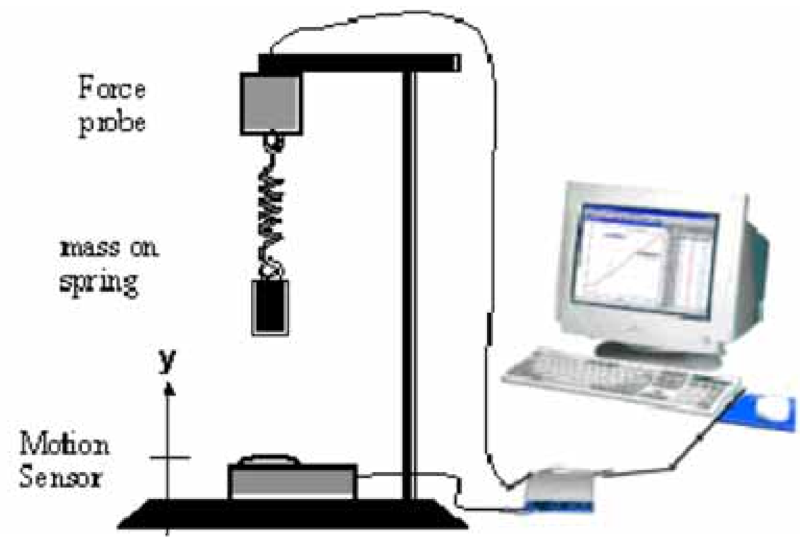
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k\left(\Delta l\right)^2$$

$$E_i = E_f$$



Pendulum motion energy

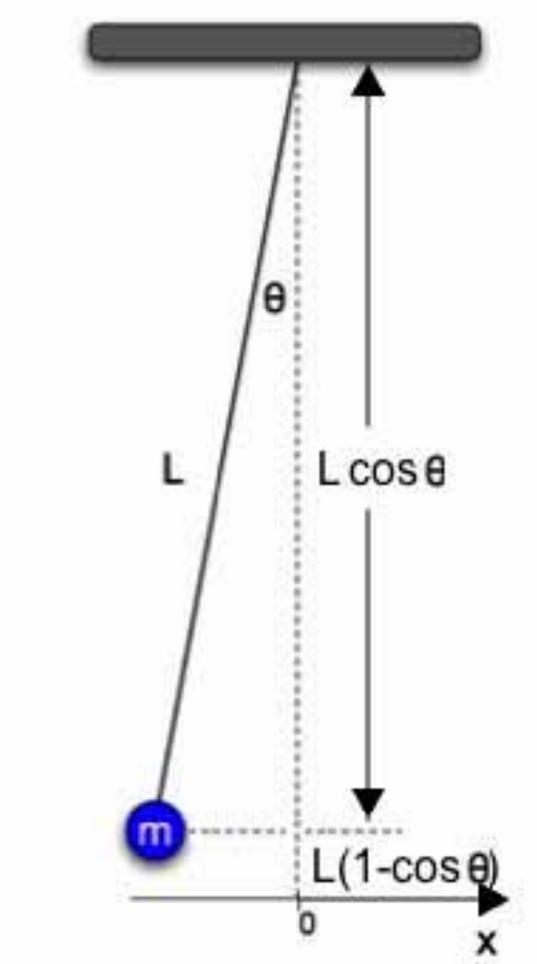
$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1-\cos\theta)$$

$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left[mgL\right]\theta^2$$

$$\theta \approx \sin\theta = \frac{x}{L}$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



Same as mass on a spring!
Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

$$ma = -kx - bv$$

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

- Solution:

$$x(t) = A_0 e^{-\gamma t/2}\cos(\omega_1 t + \phi)$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

Foothold ideas: Damped oscillator 2

- Competing time constants:

$$\frac{\gamma}{2} = \frac{1}{\tau} \qquad \frac{\omega_0}{2\pi} = \frac{1}{T} \qquad Q = \frac{\omega_0}{\gamma} = \pi \frac{\tau}{T}$$

Decay time

Period

Tells which force dominates: restoring or damping.

- If:

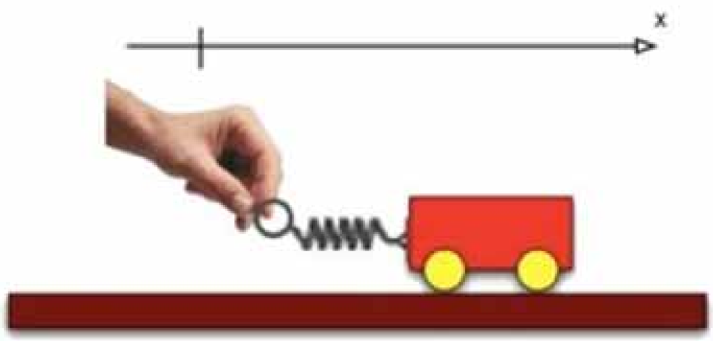
$\omega_0 > \gamma/2$ underdamped: oscillates

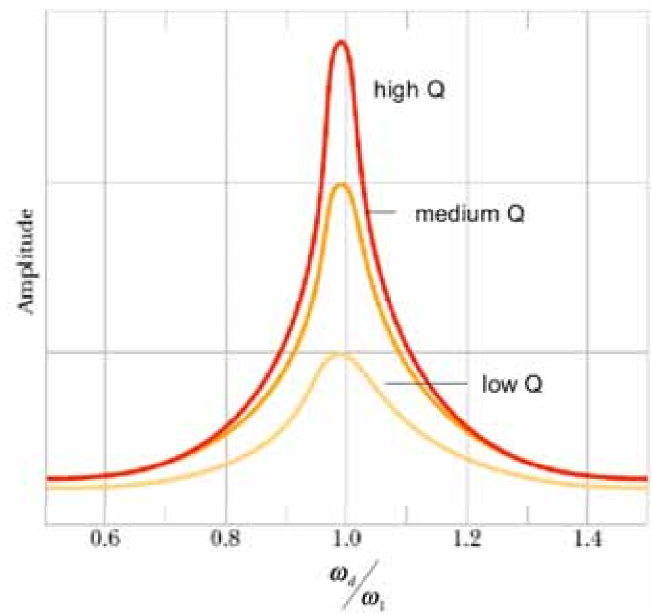
$\omega_0 = \gamma/2$ critically damped: no oscillation, fastest decay

$\omega_0 < \gamma/2$ over damped: no oscillation, slower decay

Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not much.





Foothold principles: Mechanical waves 1

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

$$v_0 = \sqrt{T/\mu}$$

v_0 = speed of pulse
T = tension of spring
μ = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and pattern speed.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.

Foothold principles: Mechanical waves 2

- *Superposition*: when one or more disturbances overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)