Theme Music: Duke Ellington

Take the A Train

Cartoon: Bill Amend

FoxTrot
Some basic electrical ideas

- **Conductor** – a material that permits some of its charges to move freely within it.
  - Implication: If the charges in a conductor are not moving, the whole conductor is at the same $V$.
- **Insulator** – a material that permits some of its charges to move a little, but not freely.
- **Battery** – a device that creates and maintains a constant potential difference across its terminals.

\[ \Delta V = V_0 \text{ volts} \]

Why?
Charging a capacitor

- What is the potential difference between the plates?
- What is the field around the plates?
- How much charge is on each plate?
Ohm’s Law

- Current proportional to velocity
- Due to resistance, electric force proportional to velocity.
- Force proportional to “electric pressure drop” = “electric PE”
- Therefore, current proportional to “electric PE”

\[ \Delta V = IR \]

\[
\begin{align*}
I &= qnAv \\
\Rightarrow v &= \frac{I}{qnA} \\
qE &= bv \\
\Delta V &= EL \\
\Rightarrow E &= \frac{\Delta V}{L} \\
\Rightarrow \frac{q\Delta V}{L} &= \frac{bI}{qnA} \\
\Delta V &= I \left( \frac{bL}{q^2nA} \right) \equiv IR
\end{align*}
\]
Resistivity and Conductance

- The resistance factor in Ohm’s Law separates into a geometrical part \((L/A)\) times a part independent of the size and shape but dependent on the material.

- This coefficient is called the resistivity of the material \((\rho)\). Its reciprocal \((g)\) is called conductivity. The reciprocal of the resistance is called the conductance \((G)\).

\[
R = \left( \frac{bL}{q^2 nA} \right) = \rho \frac{L}{A} = \frac{1}{g} \frac{L}{A} = \frac{1}{G}
\]
Electric circuit elements

- **Batteries** — devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).

- **Resistances** — devices that have significant drag and oppose current. Pressure will drop across them.

- **Capacitors** — devices that can maintain a separation of charge if there is a potential difference maintained across them.

- **Wires** — have very little resistance. We can ignore the drag in them (mostly – as long as there are other resistances present).
Foothold ideas: Electric charges in fluids

- **Electroneutrality** – Opposite charges in materials attract each other strongly. Pulling them apart to create a charge unbalance costs energy. This tends to make small volumes of fluid electrically neutral.

- **Energy-Entropy balances** – When there are situations of non-uniformity, electrical forces (energy) can balance or be balanced by random thermal motion (entropy). Two important cases are:
  - **Debye shielding** – introduced unbalanced charge
  - **Nernst potential** – non-uniform concentrations of ions
Foothold ideas:

- **Debye length** – A charge imbedded in an ionic solution is shielded by the ions pulling up towards the charge. The amount of imbalance is determined by a balance of the thermal fluctuation energy against the repulsive electrostatic energy arising from the imbalance.

\[
\lambda_D = \sqrt{\frac{\kappa k_B T}{k_c q^2 c_0}}
\]

- **Nernst potential** – When a membrane permits only one kind of ion to pass, diffusion from the side with a greater concentration of that kind of ion will build up a potential difference due to ions moving to the side with the lower concentration.

\[
\Delta V = \frac{k_B T}{q} \ln \left( \frac{c_1}{c_2} \right)
\]
Foothold ideas:
Kirchhoff’s principles

1. **Flow rule**: The total amount of current flowing into any volume in an electrical network equals the amount flowing out.

2. **Ohm’s law**: in a resistor, \( \Delta V = IR \)

3. **Loop rule**: Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).
Very useful heuristic

- The Constant Potential Corollary (CPC)
  - Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant since in any circuit element

$$\Delta V = IR$$

$R = 0 \implies \Delta V = 0$

(even if $I \neq 0$)
Electric Power

The rate at which electric energy is depleted from a battery or dissipated (into heat or light) in a resistor is

\[ \text{Power} = I \Delta V \]
Units

- **Current** \((I)\)  \hspace{1cm} **Ampere** = Coulomb/sec
- **Voltage** \((V)\)  \hspace{1cm} **Volt** = Joule/Coulomb
- **E-Field** \((E)\)  \hspace{1cm} Newton/Coulomb = Volt/meter
- **Resistance** \((R)\)  \hspace{1cm} **Ohm** = Volt/Ampere
- **Capacitance** \((C)\)  \hspace{1cm} **Farad** = Volt/Coulomb
- **Power** \((P)\)  \hspace{1cm} **Watt** = Joule/sec
Foothold ideas: Harmonic oscillation

- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves, the force is in the direction of pushing it back to its equilibrium position.
- When it gets back to its equilibrium, it’s still moving so it overshoots.
Summary with Equations: Mass on a spring

\[ a = \frac{1}{m} F^{net} \]
\[ F^{net} = -kx \]
\[ a = -\omega_0^2 x \]
\[ \omega_0^2 = \frac{k}{m} \]

\[ x(t) = A \cos(\omega_0 t + \phi) \]
\[ \omega_0 = \frac{2\pi}{T} \]

Measured from where?
Interpret!
Summary with Equations: Mass on a spring (Energy)

\[ E = \frac{1}{2} mv^2 + mgh + \frac{1}{2} k(\Delta l)^2 \]

\[ E_i = E_f \]
Pendulum motion energy

\[ E_0 = \frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgL(1 - \cos \theta) \]

\[ \cos \theta \approx 1 - \frac{1}{2} \theta^2 \]

\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} [mgL] \theta^2 \]

\[ \theta \approx \sin \theta = \frac{x}{L} \]

\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \quad k = \frac{mg}{L} \]

Same as mass on a spring!
Just with a different \( \omega_0^2 = k/m = g/L \)

What’s the period? Why doesn’t it depend on \( m \)?
Foothold ideas:
Damped oscillator 1

Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

\[ ma = -kx - bv \]

\[ \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \quad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}} \]

Solution:

\[ x(t) = A_0 e^{-\frac{\gamma t}{2}} \cos(\omega_1 t + \phi) \]

\[ \omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} \]
Foothold ideas: Damped oscillator 2

■ Competing time constants:

\[ \frac{\gamma}{2} = \frac{1}{\tau}, \quad \frac{\omega_0}{2\pi} = \frac{1}{T} \]

\[ Q = \frac{\omega_0}{\gamma} = \pi \frac{\tau}{T} \]

Decay time \hspace{1cm} Period

■ If:

\( \omega_0 > \frac{\gamma}{2} \) \hspace{0.5cm} \text{underdamped: oscillates}

\( \omega_0 = \frac{\gamma}{2} \) \hspace{0.5cm} \text{critically damped: no oscillation, fastest decay}

\( \omega_0 < \frac{\gamma}{2} \) \hspace{0.5cm} \text{over damped: no oscillation, slower decay}

Tells which force dominates: restoring or damping.
Foothold ideas:
Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator, you get a big displacement (resonance). Otherwise, not much.
Foothold principles: Mechanical waves 1

- **Key concept**: We have to distinguish the motion of the bits of matter and the motion of the pattern.

- **Mechanism**: the pulse propagates by each bit of string pulling on the next.

- **Pattern speed**: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

\[ \nu_0 = \sqrt{\frac{T}{\mu}} \]

- **Matter speed**: the speed of the bits of matter depend on both the size and shape of the pulse and pattern speed.

- **Mechanism**: the pulse propagates by each bit of string pulling on the next.

\[ \nu_0 = \text{speed of pulse} \]
\[ T = \text{tension of spring} \]
\[ \mu = \text{mass density of spring} \ (M/L) \]
Foothold principles: Mechanical waves 2

- **Superposition**: when one or more disturbances overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)