- **Theme Music:** **Superchunk**
  The Question is How Fast
- **Cartoon:** **Bill Amend**
  Foxtrot

Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

$$v_0 = \sqrt{T/\mu}$$

v_0 = speed of pulse
T = tension of spring
μ = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and pattern speed.

Dimensional analysis

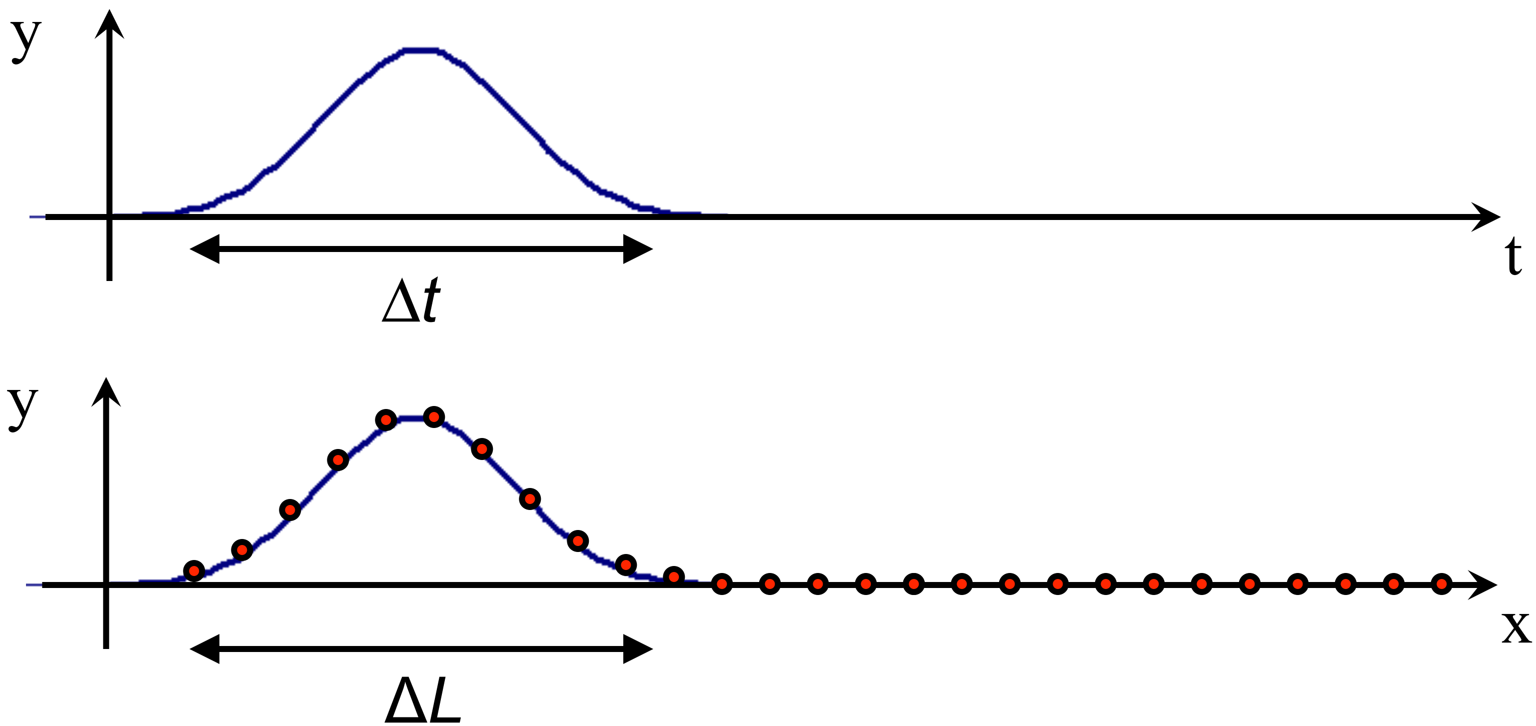
- Square brackets are used to indicate a quantities dimensions – mass ($\mathcal{M}$), length ($\mathcal{L}$), or time ($\mathcal{T}$)
  - $[m] = \mathcal{M}$
  - $[L] = \mathcal{L}$
  - $[t] = \mathcal{T}$
  - $[F] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$
- Build a velocity using mass (m), length (L), and tension (T) of the string:
  - $[v] = \mathcal{L}/\mathcal{T}$
  - $[T] = \mathcal{M}\mathcal{L}/\mathcal{T}^2$
  - $[T/m] = \mathcal{L}/\mathcal{T}^2$
  - $[TL/m] = \mathcal{L}^2/\mathcal{T}^2$

⟹

$$v_0^2 = \frac{TL}{m}$$

or, using $\mu = m/L$ $\quad v_0 = \sqrt{\frac{T}{\mu}}$

What controls the widths of the pulses in time and space?

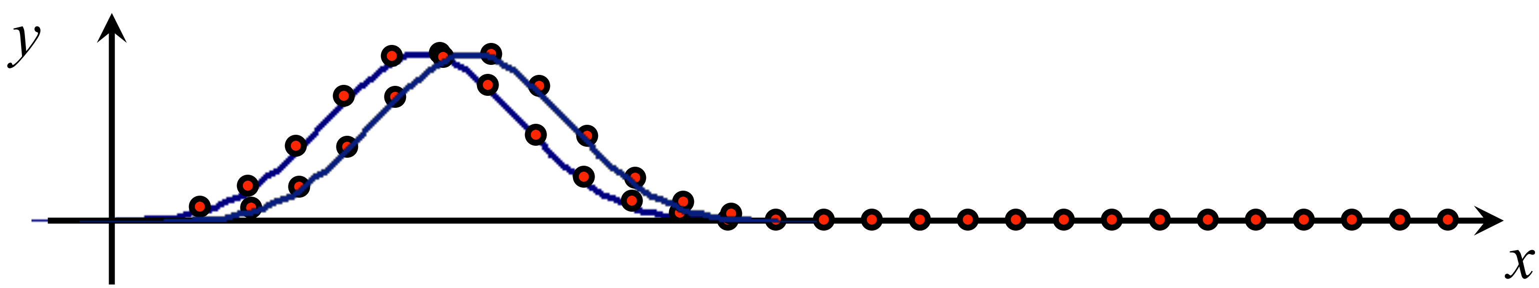


Width of a pulse

- The amount of time the demonstrator's hand was displaced up and down determines the time width of the t-pulse, Δt.
- The speed of the signal propagation on the string controls the width of the x-pulse, ΔL.
  - The leading edge takes off with some speed, v_0.
  - The pulse is over when the trailing edge is done.
  - The width is determined by "how far the leading edge got to" before the displacement was over.

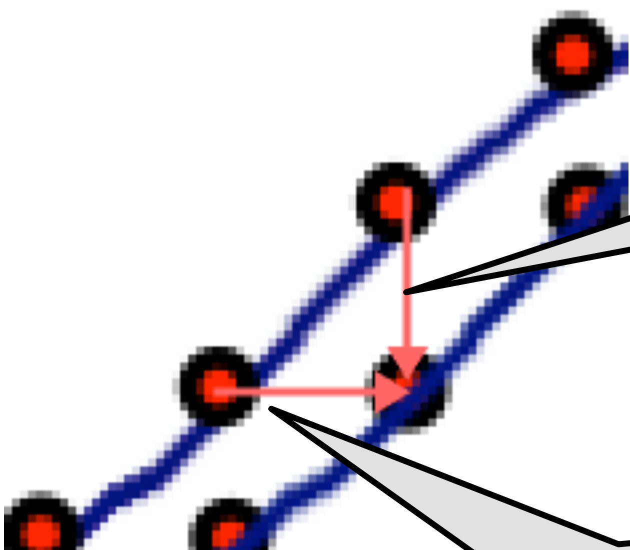
$$\Delta L = v_0 \Delta t$$

What controls the speed of the beads?



Speed of a bead

- The speed the bead moves depends on how fast the pulse is moving and how far it needs to travel to stay on the string.



dy = how far bead moves in time dt

dx = how far pulse moves in time dt

slope of pulse

$$v(x,t) = \frac{dy(x,t)}{dt} = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{dy}{dx}\right)v_0$$

speed of bead

speed of pulse

Doing the math:
Displacements on an elastic string / spring

- Each bit of the string can move up or down (perpendicular to its length).
- To describe the motion of the string we need to describe the motion of each bit of the string at every instant of time.
- We therefore need to tell both which bit and when in order to specify a displacement.

$$y_i = f_i(t) \qquad\qquad y = f(x,t)$$