Distribution of the mass of the Earth is not uniform. For example, the mass is greater at the equator and less at the poles. This affects the tidal force acting on objects in the Earth's gravitational field.

\[ F_{\text{net}} = -kx \]

\[ a = \frac{1}{m} F_{\text{net}} \]

\[ a = -\omega_0^2 x \]

\[ \omega_0^2 = \frac{k}{m} \]

\[ x(t) = A \cos(\omega_0 t + \phi) \]

\[ \omega_0 = \frac{2\pi}{T} \]

Interpret! Measured from where?
Summary with Equations: Mass on a spring (Energy)

\[ E = \frac{1}{2} mv^2 + mgh + \frac{1}{2} k(\Delta l)^2 \]

\[ E_i = E_f \]

The Long Pendulum

\[ m \]

\[ mg \]

\[ \theta \]

\[ T \]

\[ L \]

\[ x \]
Pendulum motion energy

\[ E_0 = \frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgL(1 - \cos \theta) \]
\[ \cos \theta \approx 1 - \frac{1}{2} \theta^2 \]
\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} [mgL] \theta^2 \]
\[ \theta \approx \sin \theta = \frac{x}{L} \]
\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \quad k = \frac{mg}{L} \]

Same as mass on a spring!
Just with a different \( \omega_0^2 = k/m = g/L \)

What’s the period? Why doesn’t it depend on \( m \)?

Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease.
  Simplest model is viscous drag.
  \[ ma = -kx - bv \]
  \[ \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \quad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}} \]

- Solution:
  \[ x(t) = A_0 e^{-\gamma t/2} \cos(\omega_1 t + \phi) \]
  \[ \omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} \]
Foothold ideas: Damped oscillator 2

- Competing time constants:
  \[
  \frac{\gamma}{2} = \frac{1}{\tau} \quad \frac{\omega_0}{2\pi} = \frac{1}{T} \quad Q = \frac{\omega_0}{\gamma} = \pi \frac{\tau}{T}
  \]
  - \textbf{Decay time}
  - \textbf{Period}
  - Tells which force dominates: restoring or damping.

- If:
  \begin{align*}
  \omega_0 &> \gamma/2 \quad \text{underdamped: oscillates} \\
  \omega_0 &= \gamma/2 \quad \text{critically damped: no oscillation, fastest decay} \\
  \omega_0 &< \gamma/2 \quad \text{over damped: no oscillation, slower decay}
  \end{align*}

Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (\textbf{resonance}). Otherwise, not much.