Theme Music: Elvis Presley

All Shook Up

Cartoon: Scott & Borgman

Zits

I’ve been reading about the chaos theory.

Yeah?

It claims that a single butterfly merely flapping its wings could cause a storm on the other side of the world.

Hmm.

BURRP!

Do you believe it?

Seems unlikely.
Summary with Equations: Mass on a spring

\[ a = \frac{1}{m} F^{net} \]
\[ F^{net} = -kx \]

\[ a = -\omega_0^2 x \]
\[ \omega_0^2 = \frac{k}{m} \]

\[ x(t) = A \cos(\omega_0 t + \phi) \]
\[ \omega_0 = \frac{2\pi}{T} \]

Measured from where?

Interpret!
Summary with Equations: Mass on a spring (Energy)

\[ E = \frac{1}{2} mv^2 + mgh + \frac{1}{2} k(\Delta l)^2 \]

\[ E_i = E_f \]
The Long Pendulum

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Physics 132
Pendulum motion energy

\[ E_0 = \frac{1}{2}mv^2 + mgh = \frac{1}{2}mv^2 + mgL(1 - \cos \theta) \]

\[ \cos \theta \approx 1 - \frac{1}{2} \theta^2 \]

\[ E_0 \approx \frac{1}{2}mv^2 + \frac{1}{2}[mgL]\theta^2 \]

\[ \theta \approx \sin \theta = \frac{x}{L} \]

\[ E_0 \approx \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad k = \frac{mg}{L} \]

Same as mass on a spring!
Just with a different \( \omega_0^2 = k/m = g/L \)

What’s the period? Why doesn’t it depend on m?
Foothold ideas:
Damped oscillator 1

- Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

\[ ma = -kx - bv \]

\[ \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \quad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}} \]

- Solution:

\[ x(t) = A_0 e^{-\frac{\gamma t}{2}} \cos(\omega_1 t + \phi) \]

\[ \omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} \]
Foothold ideas: Damped oscillator 2

- Competing time constants:
  \[ \frac{\gamma}{2} = \frac{1}{\tau} \quad \frac{\omega_0}{2\pi} = \frac{1}{T} \]

  - Decay time
  - Period

- If:
  - \( \omega_0 > \gamma/2 \) underdamped: oscillates
  - \( \omega_0 = \gamma/2 \) critically damped: no oscillation, fastest decay
  - \( \omega_0 < \gamma/2 \) over damped: no oscillation, slower decay

\[ Q = \frac{\omega_0}{\gamma} = \pi \frac{\tau}{T} \]

Tells which force dominates: restoring or damping.
Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (resonance). Otherwise, not much.