Theme Music: Benny Goodman

Swing, Swing, Swing

Cartoon: Bill Watterson

Calvin & Hobbes
Mathematical structure

Express \( a = \frac{F_{\text{net}}}{m} \) in terms of derivatives.

\[
\frac{d^2 x}{dt^2} = -\omega_0^2 x
\]

Except for the constant, this is like having a functions that is its own second derivative.

\[
\frac{d^2 f}{dt^2} = -f
\]

In calculus, we learn that \( \sin(t) \) and \( \cos(t) \) work like this. How about: \( x = \cos t \)?
Interpreting the Result

- We can easily take the derivatives to show that our solution \( x(t) = A \cos(\omega_0 t) \) satisfies the N2 equations.

- What do the various terms mean?
  - \( A \) is the maximum displacement — the amplitude.
  - What is \( \omega_0 \)? If \( T \) is the period (how long it takes to go through a full oscillation) then

\[
\omega_0 t : 0 \rightarrow 2\pi \\
t : 0 \rightarrow T
\]

\[
\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}
\]
Graphs: \( \sin(\theta) \) vs \( \cos(\theta) \)

- Which is which? How can you tell?
- The two functions \( \sin \) and \( \cos \) are derivatives of each other (slopes), but one has a minus sign. Which one? How can you tell?
Graphs: $\sin(\theta)$ vs $\sin(\omega_0 t)$

- For angles, $\theta = 0$ and $\theta = 2\pi$ are the same so you only get one cycle.
- For time, $t$ can go on forever so the cycles repeat.

What does changing $\omega_0$ do to this graph?
Interpreting the Result

What about the starting point?
Using cos means you always start at a peak when $t = 0$. That might not always be true.

$$x(t) = A \cos(\omega_0(t - t_0))$$

$$= A \cos(\omega_0 t - \omega_0 t_0) = A \cos(\omega_0 t - \phi)$$
Summary with Equations: Mass on a spring

\[ a = \frac{1}{m} F_{\text{net}} \]
\[ F_{\text{net}} = -kx \]
\[ a = -\omega_0^2 x \]
\[ \omega_0^2 = \frac{k}{m} \]
\[ x(t) = A \cos(\omega_0 t + \phi) \]
\[ \omega_0 = \frac{2\pi}{T} \]

Measured from where?

Interpret!
Summary with Equations: Mass on a spring (Energy)

\[ E = \frac{1}{2} mv^2 + mgh + \frac{1}{2} k(\Delta l)^2 \]

\[ E_i = E_f \]
The Long Pendulum
Pendulum motion energy

\[ E_0 = \frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgL(1 - \cos \theta) \]

\[ \cos \theta \approx 1 - \frac{1}{2} \theta^2 \]

\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} \left[ mgL \right] \theta^2 \]

\[ \theta \approx \sin \theta = \frac{x}{L} \]

\[ E_0 \approx \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \quad k = \frac{mg}{L} \]

Same as mass on a spring!
Just with a different \( \omega_0^2 = k/m = g/L \)

What’s the period? Why doesn’t it depend on m?

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