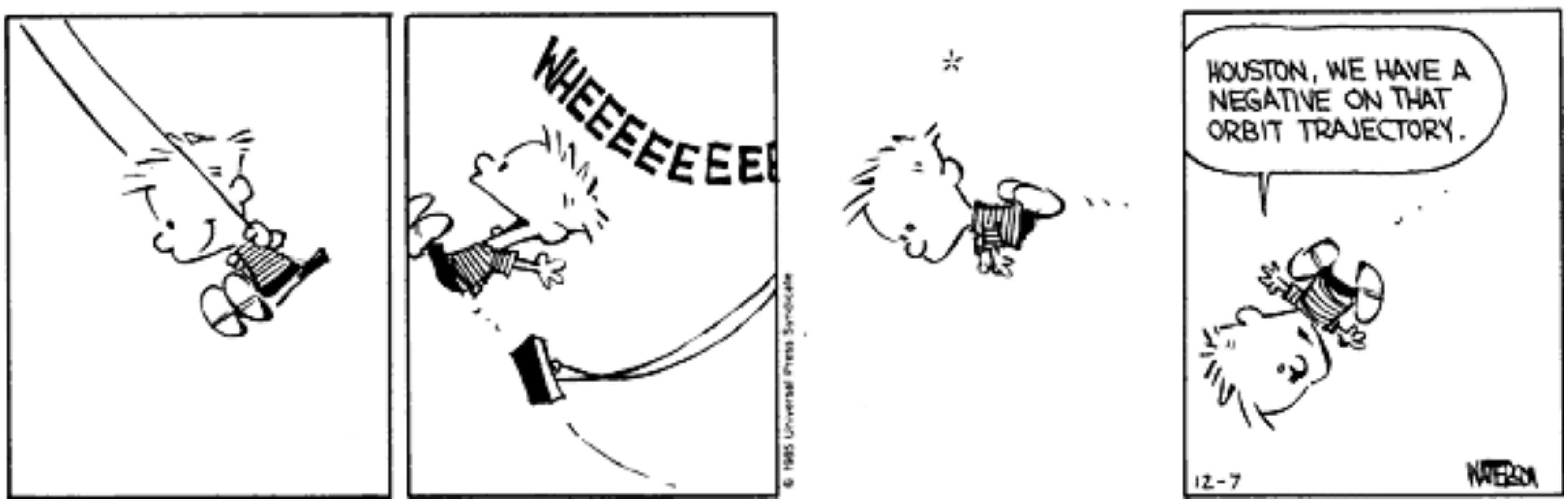
- **<u>Theme Music:</u> Benny Goodman**

  Swing, Swing, Swing

- **<u>Cartoon:</u> Bill Watterson**

  Calvin & Hobbes



Mathematical structure

- Express $a = F^{net}/m$ in terms of derivatives.

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

- Except for the constant, this is like having a functions that is its own second derivative.

$$\frac{d^2 f}{dt^2} = -f$$

- In calculus, we learn that sin(t) and cos(t) work like this. How about: $\boxed{x = \cos t}$?

Interpreting the Result

- We can easily take the derivatives to show that our solution $x(t) = A\cos(\omega_0 t)$ satisfies the N2 equations
- What do the various terms mean?
  - A is the maximum displacement — the *amplitude*.
  - What is ω_0? If T is the *period* (how long it takes to go through a full oscillation) then

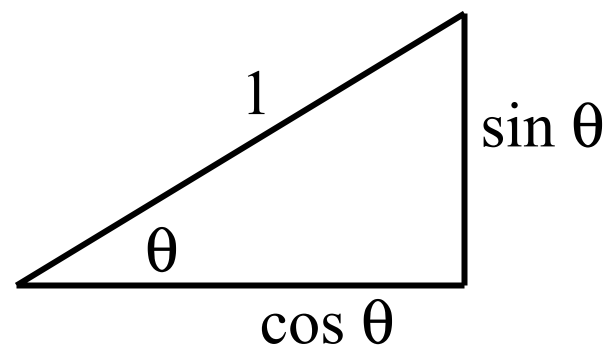
$$\omega_0 t : 0 \to 2\pi$$

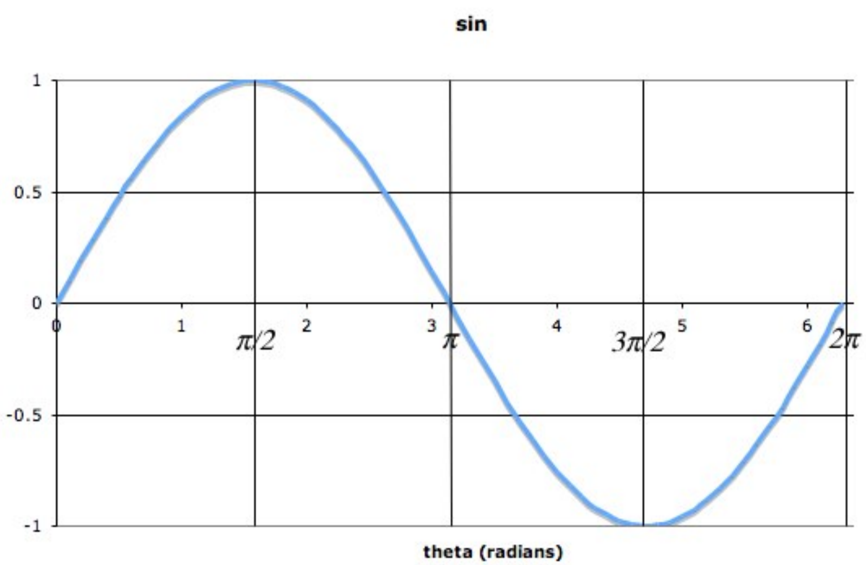
$$t \quad : 0 \to T$$

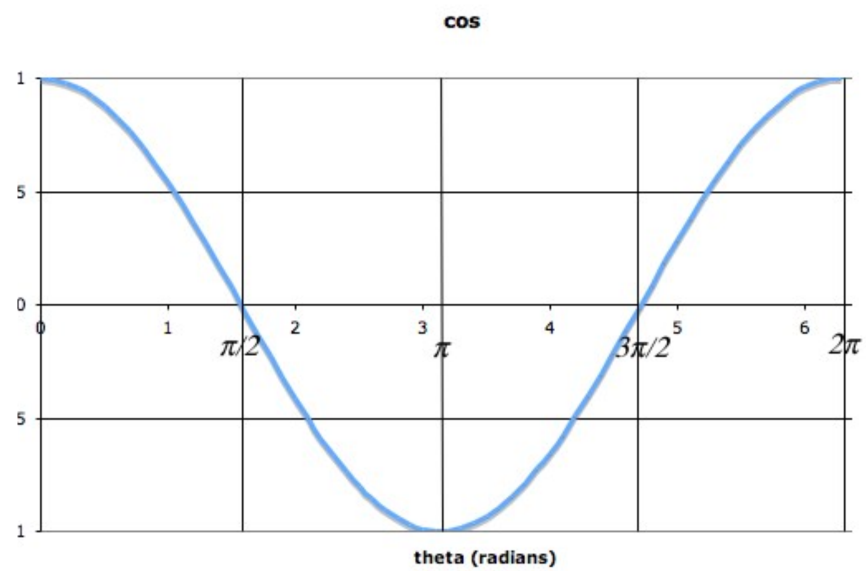
$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

Graphs: sin(θ) vs cos(θ)

- Which is which? How can you tell?
- The two functions sin and cos are derivatives of each other (slopes), but one has a minus sign.
  Which one?
  How can you tell?





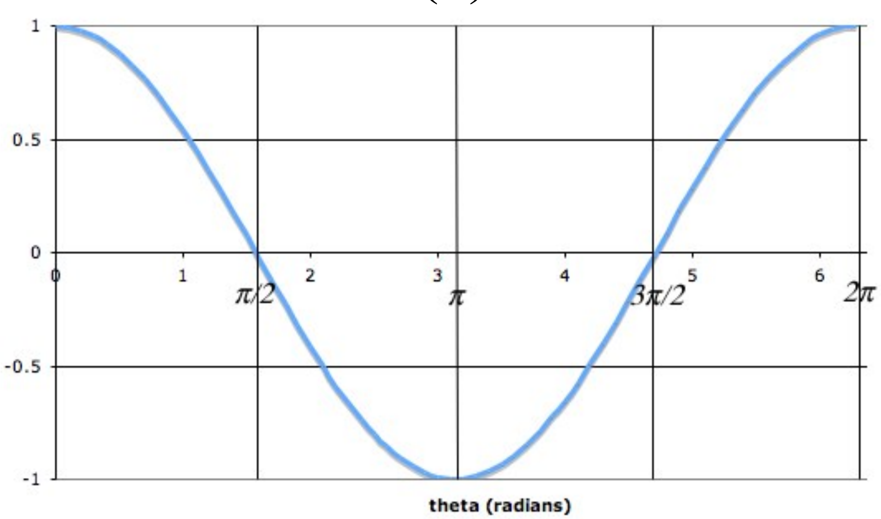


Graphs: $\sin(\theta)$ vs $\sin(\omega_0 t)$

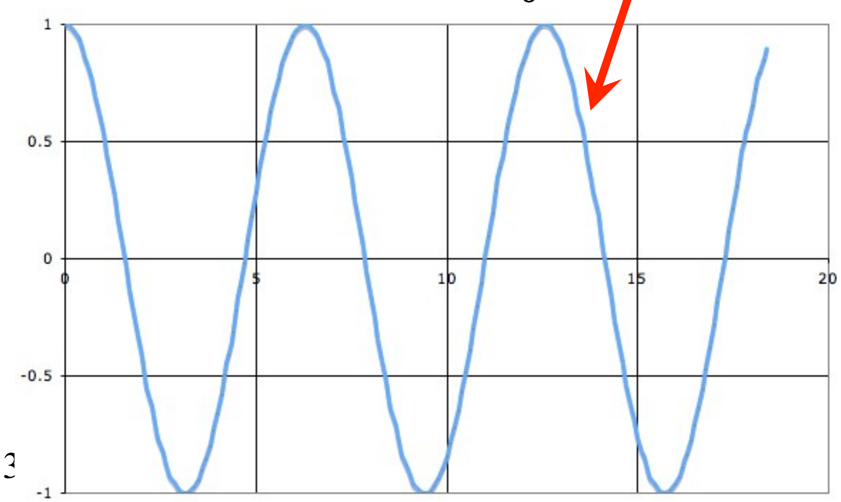
- For angles, $\theta = 0$ and $\theta = 2\pi$ are the same so you only get one cycle.
- For time, t can go on forever so the cycles repeat.

What does changing ω_0 do to this graph?

$\cos(\theta)$

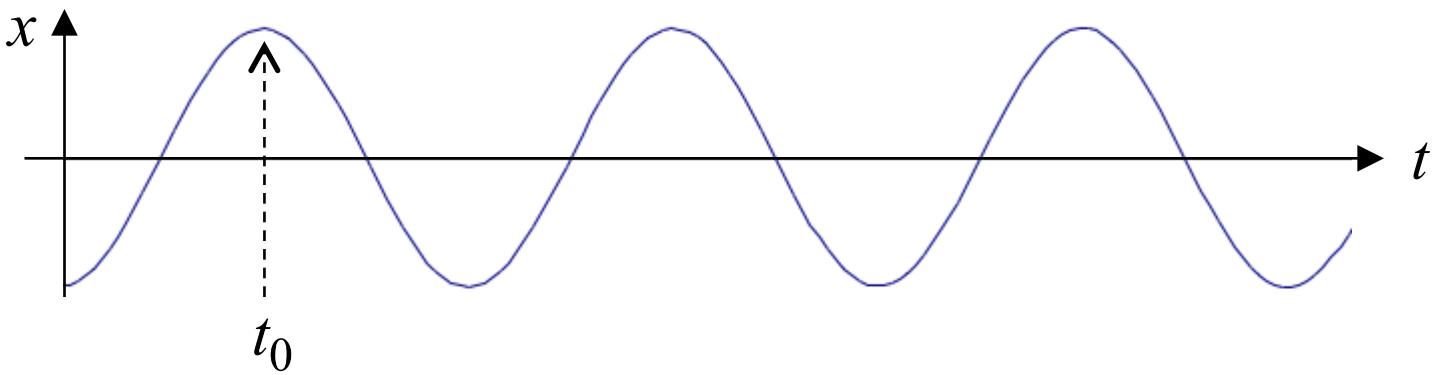


$\cos(\omega_0 t)$



Interpreting the Result

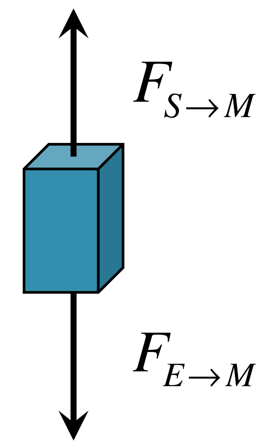
- What about the starting point?
  Using cos means you always start at a peak when $t = 0$. That might not always be true.



$$x(t) = A\cos(\omega_0(t - t_0))$$

$$= A\cos(\omega_0 t - \omega_0 t_0) = A\cos(\omega_0 t - \phi)$$

Summary with Equations: Mass on a spring



$$a = \frac{1}{m}F^{net} \qquad F^{net} = -kx$$

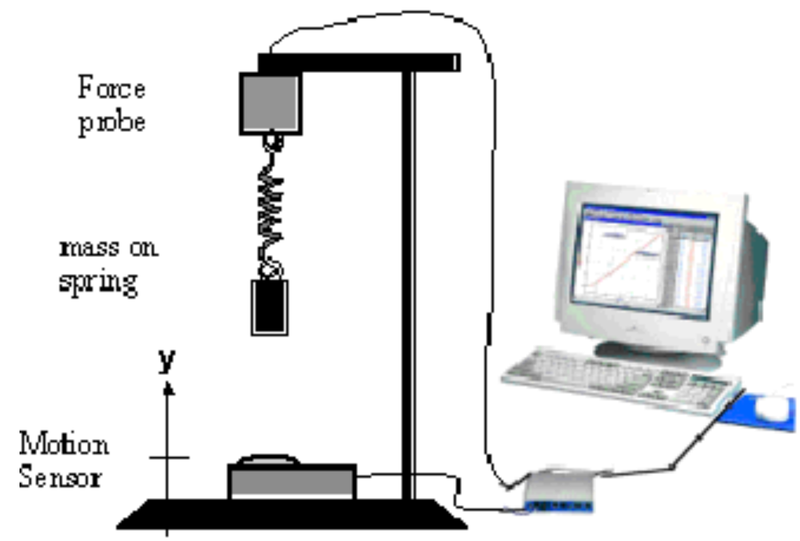
Measured from where?

$$a = -\omega_0^2 x \qquad \omega_0^2 = \frac{k}{m}$$

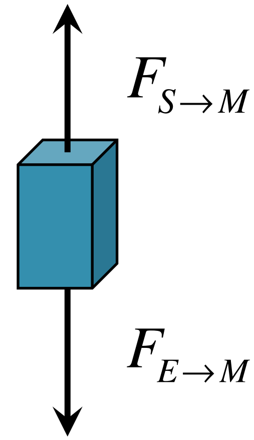
$$x(t) = A\cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



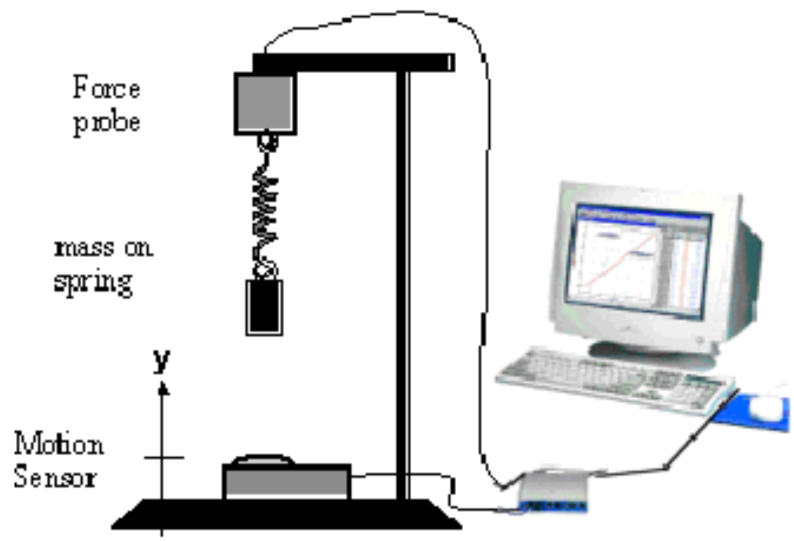
Summary with Equations: Mass on a spring (Energy)



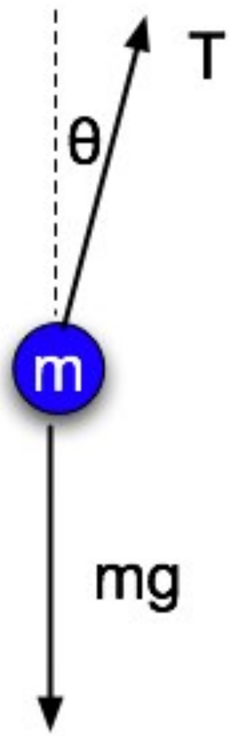
Measured from where?

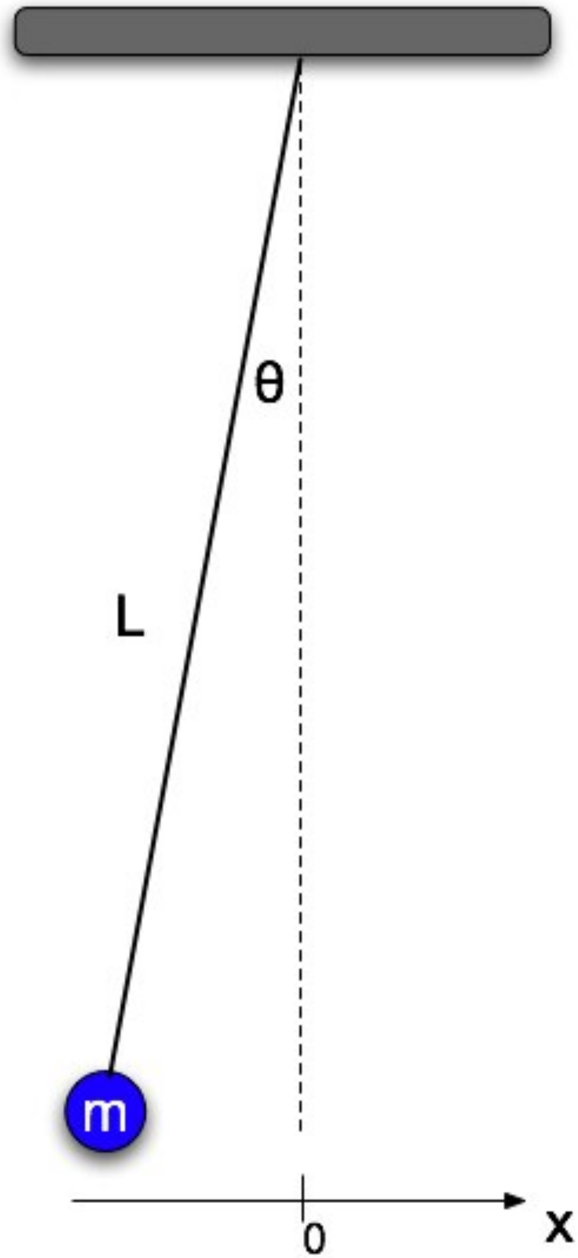
$$E = \tfrac{1}{2} m v^2 + m g h + \tfrac{1}{2} k \left(\Delta l \right)^2$$

$$E_i = E_f$$



The Long Pendulum





Pendulum motion energy

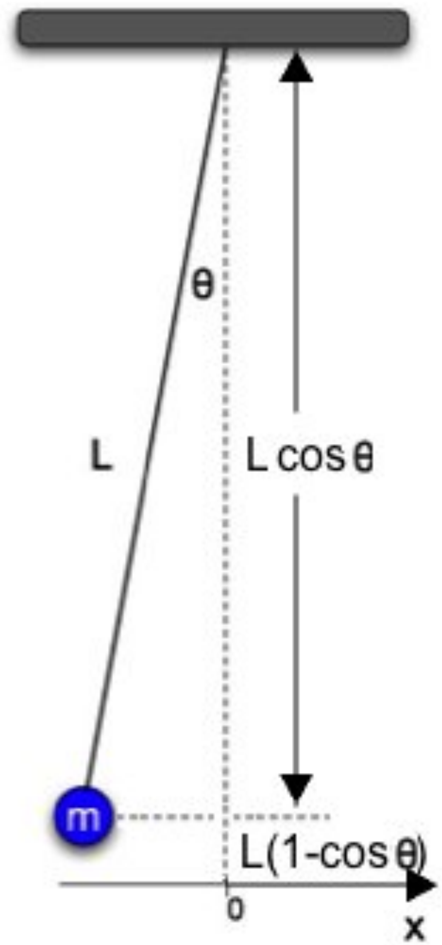
$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1-\cos\theta)$$

$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}[mgL]\theta^2$$

$$\theta \approx \sin\theta = \frac{x}{L}$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



Same as mass on a spring!

Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?