Foothold principles: Newton’s Laws

- **Newton 0:**
  - An object responds **only** to the forces it feels and only at the instant it feels them.

- **Newton 1:**
  - An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).

- **Newton 2:**
  - An object that is acted upon by other objects changes its velocity according to the rule
    \[
    \ddot{a}_A = \frac{\vec{F}^{\text{net}}_A}{m_A}
    \]

- **Newton 3:**
  - When two objects interact the forces they exert on each other are equal and opposite.
    \[
    \vec{F}_{A \rightarrow B}^{\text{type}} = -\vec{F}_{B \rightarrow A}^{\text{type}}
    \]
Foothold ideas: Kinetic Energy and Work

- Newton’s laws tell us how velocity changes. The Work-Energy theorem tells us how speed (independent of direction) changes.
- Kinetic energy = \( \frac{1}{2}mv^2 \)
- Work done by a force = \( F_x \Delta x \) or \( F\parallel \Delta r \) (part of force \( \parallel \) to displacement)
- Work-energy theorem: \( \Delta (\frac{1}{2}mv^2) = F_{\parallel} \Delta r \) (small step)
  \[
  \Delta (\frac{1}{2}mv^2) = \int F_{\parallel} dr \quad \text{(any size step)}
  \]

Foothold ideas: Potential Energy

- The work done by some forces only depends on the change in position. Then it can be written
  \[ U \] is called a potential energy.
  \[ \vec{F} \cdot \Delta \vec{r} = -\Delta U \]
- For gravity, \( U_{\text{gravity}} = mgh \)
  For a spring, \( U_{\text{spring}} = \frac{1}{2}kx^2 \)
  For electric force, \( U_{\text{electric}} = kQ_1Q_2/r_{12} \)
- Potential to force: \( \vec{F} = -\frac{\Delta U}{\Delta \vec{r}} = -\left( \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right) = -\nabla U \)

The force associated with a PE at a given place points “downhill” – in the direction where the PE falls the fastest.
Foothold ideas: Energy

- Kinds of energy (macro)
  - Kinetic
  - Potential
  - Thermal
  - Chemical
- Kinds of energy (micro)?
- First law of thermodynamics
  - Conservation of total energy

\[ \Delta U = Q - W \]
\[ \Delta H = \Delta U + p \Delta V \]

Foothold ideas: Inter-atomic interactions

- The interaction between atoms arises from the combination of the electrical forces of its components (electrons and nuclei).
  - It can be quite complex and involve electron sharing and chemical bonds.
  - The complexity arises from the quantum character of electrons.
- Despite this complexity, a simple potential model summarizes many features of a two-atom interaction.
Foothold ideas:
Inter-atomic potentials

- The interaction between neutral atoms includes an attraction at long-range that arises from the fluctuating charge distribution in each atom; the PE behaves like $1/r^6$.
- When the atoms are pressed close, they repel each other strongly; both because the +nuclei repel and because of the Pauli principle (two electrons cannot be in the same state).
- Two commonly used models are:
  - The Lennard-Jones potential ($A/r^{12} - B/r^6$)
  - The Morse potential (exponentials)

Foothold principles:
Randomness

- Matter is made of of molecules in constant motion and interaction. This motion moves stuff around.
- If the distribution of a chemical is non-uniform, the randomness of molecular motion will tend to result in molecules moving from more dense regions to less.
- This is not directed but is an emergent phenomenon arising from the combination of random motion and non-uniform concentration.
Foothold ideas: Thermal Equilibrium & Equipartition

- Degrees of freedom – where energy can reside in a system.
- Thermodynamic equilibrium is dynamic. Changes keep happening, but equal amounts in both directions.
- Equipartition – At equilibrium, the same energy density in all space and in all DoFs.

Foothold ideas: Microstate and macrostates

- A microstate is a specific distribution of energy telling how much is in each DoF.
- A macrostate is a statement about some average properties of a state (pressure, temperature, density,...).
  - A given macrostate corresponds to many microstates.
- If the system is sufficiently random, each microstate is equally probable. As a result, the probability of seeing a given macrostate depends on how many microstates it corresponds to.
Foothold ideas:
Thermal Equilibrium & Equipartition

- **Degrees of freedom** – where energy can reside in a system.
- **Thermodynamic equilibrium is dynamic** – Changes keep happening, but equal amounts in both directions.
- **Equipartition** – At equilibrium, the same energy density in all space and in all DoFs.

Foothold ideas:
Entropy

- **Entropy** – an extensive measure of how well energy is spread in an object.
- **Entropy measures** –
  - The number of microstates in a given macrostate
  \[ S = k_B \ln(W) \]
  - The amount that the energy of a system is spread among the various degrees of freedom
- **Change in entropy** upon heat flow
  \[ \Delta S = \frac{Q}{T} \]
Foothold ideas:
The Second Law of Thermodynamics

- Systems composed of a large number of particles spontaneously move toward the thermodynamic (macro) state that correspond to the largest possible number of particle arrangements (microstates).
  - The 2nd law is probabilistic. Systems show fluctuations – violations that get proportionately smaller as N gets large.
- Systems that are not in thermodynamic equilibrium will spontaneously transform so as to increase the entropy.
  - The entropy of any particular system can decrease as long as the entropy of the rest of the universe increases more.
- The universe tends towards states of increasing chaos and uniformity. (Is this contradictory?)

Foothold ideas:
Transforming energy

- Internal energy: $\Delta U$
  - thermal plus chemical
- Enthalpy: $\Delta H = \Delta U + p\Delta V$
  - internal plus amount needed to make space at constant $p$
- Gibbs free energy: $\Delta G = \Delta H - T\Delta S$
  - enthalpy minus amount associated with raising entropy of the rest of the universe due to energy dumped
- A process will go spontaneously if $\Delta G < 0.$
ΔG = ΔH - TΔS

The sign of the Gibbs Free Energy change indicates spontaneity!

ΔG < 0 → ΔS_{total} > 0 → spontaneous

ΔG > 0 → ΔS_{total} < 0 → not spontaneous

Foothold ideas:
Energy distribution

- Due to the randomness of thermal collisions, even in (local) thermal equilibrium a range of energy is found in each degree of freedom.
- The probability of finding an energy E is proportional to the Boltzmann factor
  $$P(E) \propto e^{-E/k_B T}$$  (for one DoF)
  $$P(E) \propto e^{-E/RT}$$  (for one mole)
- At 300 K, $k_B T \sim 1/40$ eV
  $$N_A k_B T = RT \sim 2.4$ kJ/mol
Foothold ideas:
Charge – A hidden property of matter

• Matter is made up of two kinds of electrical matter (positive and negative) that usually cancel very precisely.
• Like charges repel, unlike charges attract.
• Bringing an unbalanced charge up to neutral matter polarizes it, so both kinds of charge attract neutral matter.
• The total amount of charge (pos – neg) is constant.

Foothold ideas:
Conductors and Insulators

• Insulators
  – In some matter, the charges they contain are bound and cannot move around freely.
  – Excess charge put onto this kind of matter tends to just sit there (like spreading peanut butter).

• Conductors
  – In some matter, charges in it can move around throughout the object.
  – Excess charge put onto this kind of matter redistributes itself or flows off (if there is a conducting path to ground).
Foothold idea: Coulomb’s Law

• All objects attract each other with a force whose magnitude is given by

\[ \mathbf{F}_{q \to Q} = -\mathbf{F}_{Q \to q} = \frac{k_C q Q}{r_{qQ}^2} \hat{r}_{q \to Q} \]

• \( k_C \) is put in to make the units come out right.

\[ k_C = 9 \times 10^9 \text{ N-m}^2 / \text{C}^2 \]

Foothold ideas: Energies between charge clusters

• Atoms and molecules are made up of charges.

• The potential energy between two charges is

\[ U_{12}^{\text{elec}} = \frac{k_C Q_1 Q_2}{r_{12}} \]

• The potential energy between many charges is

\[ U_{12 \ldots N}^{\text{elec}} = \sum_{i<j=1}^{N} \frac{k_C Q_i Q_j}{r_{ij}} \]

No vectors! Just add up all pairs!
Foothold idea: Fields

- **Test particle**
  - We pay attention to what force it feels.
  - We assume it does not have any affect on the source particles.

- **Source particles**
  - We pay attention to the forces they exert and assume they do not move.

- **Physical field**
  - We consider what force a test particle would feel if it were at a particular point in space and divide by its coupling strength to the force. This gives a vector at each point in space.

\[
\begin{align*}
\vec{g} &= \frac{1}{m} \vec{W}_{E \rightarrow m} \\
\vec{E} &= \frac{1}{q} \vec{F}_{\text{all charges} \rightarrow q} \\
V &= \frac{1}{q} U_{\text{all charges} \rightarrow q}
\end{align*}
\]

Foothold ideas: Electric potential energy and potential

- The potential energy between two charges is

\[
U_{12}^{\text{elec}} = \frac{k_e Q_1 Q_2}{r_{12}}
\]

- The potential energy of many charges is

\[
U_{12...N}^{\text{elec}} = \sum_{i<j=1}^{N} \frac{k_e Q_i Q_j}{r_{ij}}
\]

- The potential energy added by adding a test charge \( q \) is

\[
\Delta U_q^{\text{elec}} = \sum_{i=1}^{N} \frac{k_e q Q_i}{r_{iq}} = qV
\]

\( = \text{the voltage at the position of the test charge} \)
Units

- Gravitational field
  units of $g = \text{Newtons/kg}$

- Electric field
  units of $E = \text{Newtons/C}$

- Electric potential
  units of $V = \text{Joules/C} = \text{Volts}$

- Energy = $qV$ so $e\Delta V$ = the energy gained by an electron (charge $e = 1.6 \times 10^{-19} \text{C}$) in moving through a change of $\Delta V$ volts. $1 \text{eV} = 1.6 \times 10^{-19} \text{J}$

Foothold ideas: Electric charges in materials

- The electric field inside the body of a static conductor (no moving charges) is zero.
- The entire body of a static conductor (no charges moving through it) is at the same potential.
- The average electric field in an insulator is reduced (due to the polarization of the material by the field) by a factor that is a property of the material: the dielectric constant, $\kappa$. ($\kappa$ is the ratio of two fields, it is dimensionless.)
Foothold ideas:
Capacitors

\[ \Delta V = E \Delta x = Ed \]

\[ E = 4\pi k_c \sigma = 4\pi k_c \frac{Q}{A} \Rightarrow Q = \left( \frac{A}{4\pi k_c} \right) E \]

\[ Q = \left( \frac{A}{4\pi k_c d} \right) \Delta V \]

\[ Q = C \Delta V \]

\[ C = \kappa \varepsilon_0 \frac{A}{d} \]

Energy stored = \( \frac{1}{2} Q \Delta V \)

What does this "Q" stand for?

If plates are separated by a material