

# Exam Review Slides

Foothold Ideas Since Midterm 2

# Foothold principles:

## Superposition of Mechanical waves



- *Superposition*: when two or more waves (or pulses) overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)
  - *Beats*: When sinusoidal waves of different frequencies travel in the same direction, you get variations in amplitude (when you fix either space or time) that happen at a rate that depends on the difference of the frequencies.
  - *Standing waves*: When sinusoidal waves of the same frequency travel in opposite directions, you get a stationary oscillating pattern with fixed nodes.

**Standing waves:** Sinusoidal Waves, same frequency, going in opposite directions

$$y(x, t) = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

Using trig identities (sc+cs...) we can show

$$y(x, t) = 2A \sin(kx) \cos(\omega t)$$

For each point on the string labeled “x” it oscillates with an amplitude that depends on where it is — but all parts of the string go up and down together.

# Standing Waves

- Some points in this pattern (values of  $x$  for which  $kx = n\pi$ ) are always 0. (NODES)
- To wiggle like this (all parts oscillating together in a “standing wave”) we need to have the end fixed

$$L = n \frac{\lambda}{2}$$

- We still have  $v_0 = \frac{\omega}{k}$  that is  $v_0 = \lambda f$

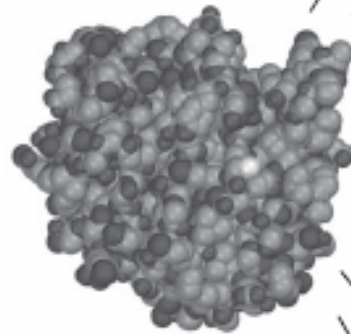
# Light

# Light: Three models

- Newton's particle model (rays)
  - Models light as bits of energy traveling very fast in straight lines. Each bit has a color. Intensity is the number of bits you get.
- Huygens's/Maxwell wave model
  - Models light as waves (transverse EM waves). Color determined by frequency, intensity by square of a total oscillating amplitude. (Allows for cancellation – interference.)
- Einstein's photon model
  - Models light as “wavicles” == quantum particles whose energy is determined by frequency and that can interfere with themselves.

# Light is not the only phenomenon in nature that requires multiple models

Proteins are an example in biology where multiple models exist



AMINO ACID SEQUENCE

...LFSIKREALSI...

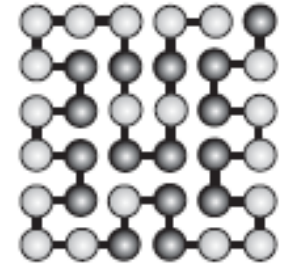
HP MODEL

...RRPHP PPRRPH...

RIBBON DIAGRAM



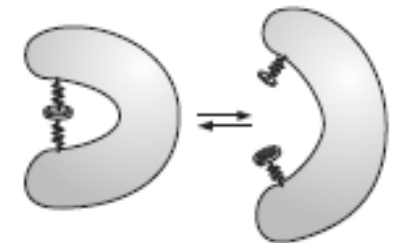
COMPACT RANDOM WALK



RECEPTOR



TWO-STATE SYSTEM



Each model highlights different properties of the protein

- Hydrophobic character
- Folding property

# Einstein's photon model



# Foothold Ideas: The Photon Model



- When it interacts with matter, light behaves as if it consisted of packets (photons) that carry both energy and momentum according to:

$$E = hf = \frac{hc}{\lambda} \quad p = \frac{E}{c} = \frac{h}{\lambda}$$

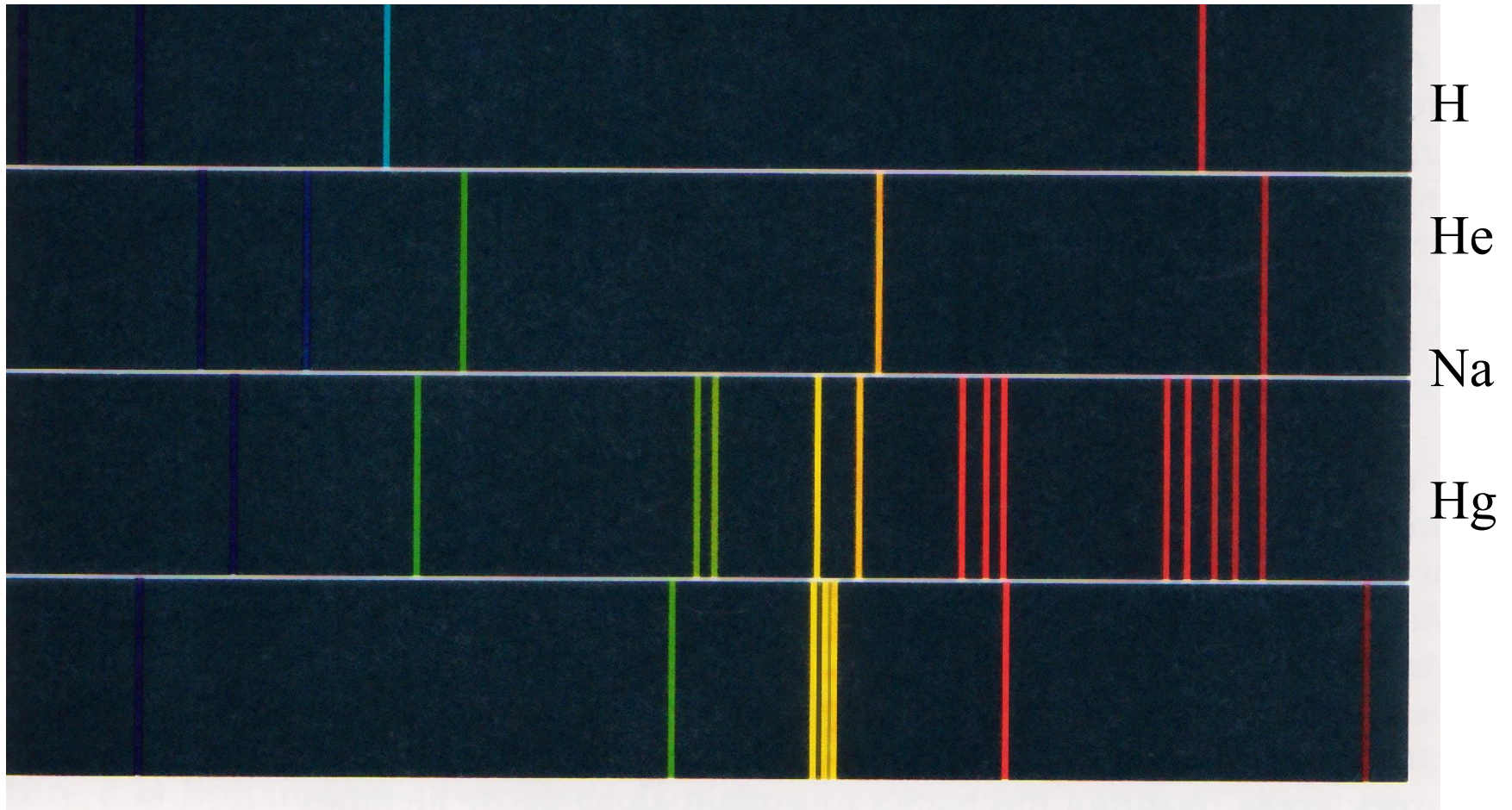
with  $hc = 1234 \text{ eV}\cdot\text{nm}$ .

- These equations are somewhat peculiar. The left side of the equations look like particle properties and the right side like wave

# Line Spectra

- When energy is added to gases of pure atoms or molecules by a spark, they give off light, but not a continuous spectrum.
- They emit light of a number of specific colors — *line spectra*.
- The positions of the lines are characteristic of the particular atoms or molecules.

# Line Spectra



# Foothold Ideas:

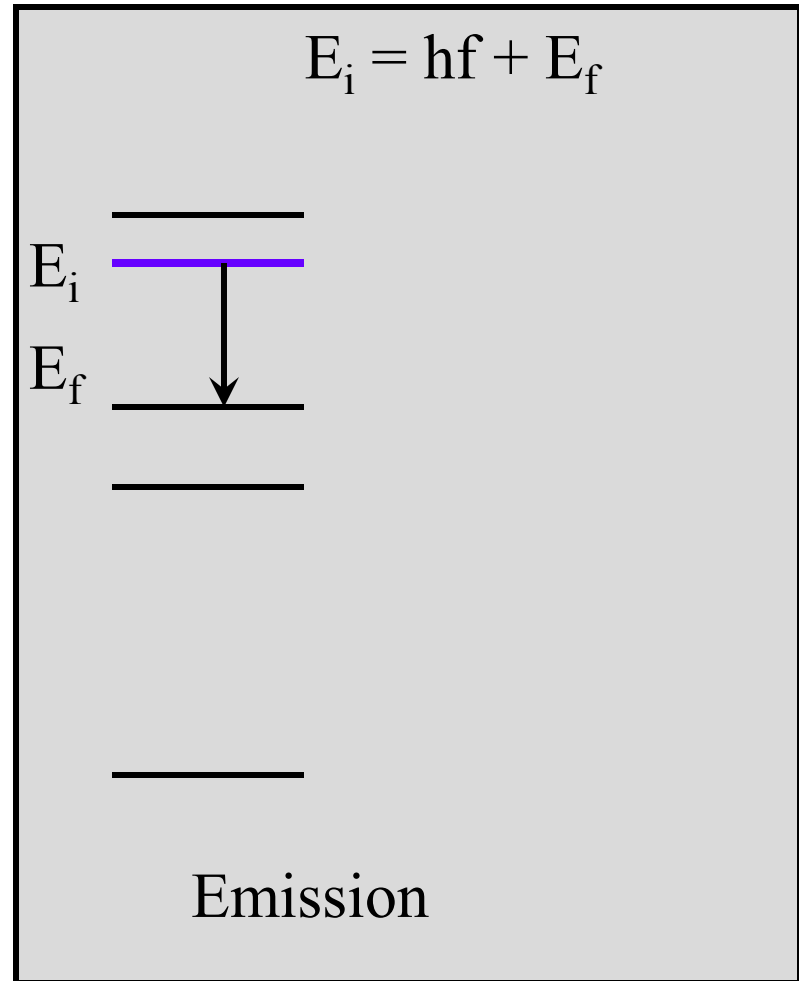
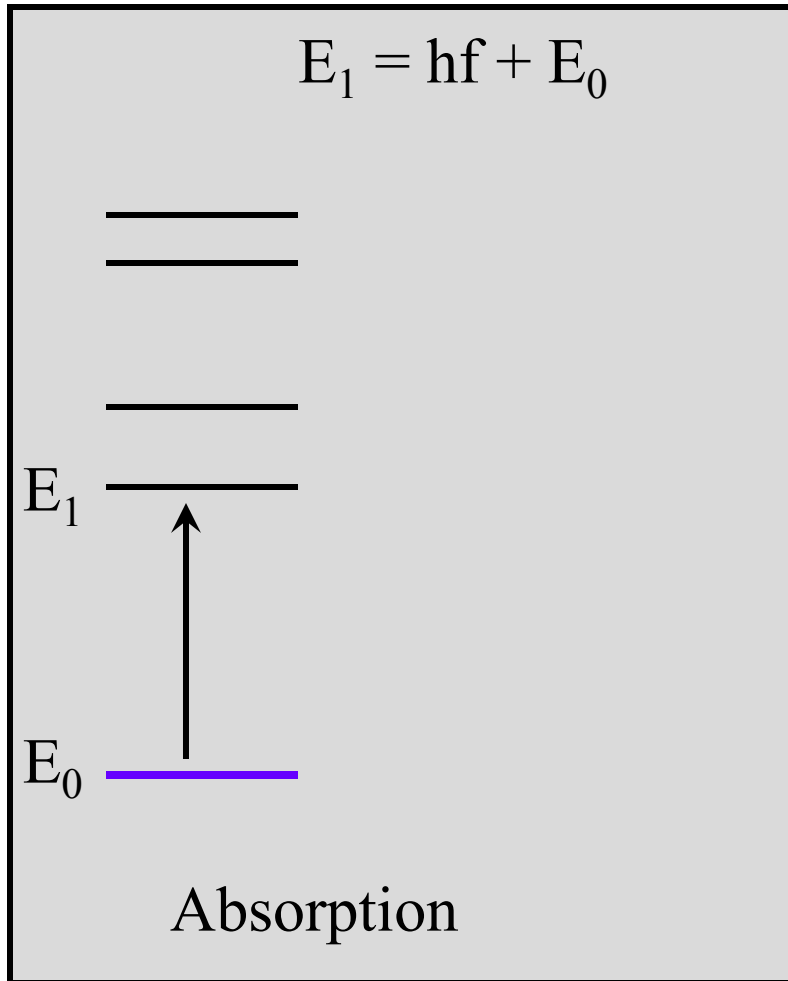
## Light interacting with Matter



- Atoms and molecules naturally exist in states having specified energies. EM radiation can be absorbed or emitted by these atoms and molecules.
- When light interacts with matter, both energy and momentum are conserved.
- The energy of radiation either emitted or absorbed therefore corresponds to the difference of the energies of states.

# Energy Level Diagrams

E



# Newton's Particle Model (Rays)

# Foothold Ideas 1:

## Light as Rays - **The Physics**



- Through empty space (or ~air) light travels in straight lines.
- Each point on an object scatters light, spraying it off in all directions.
- A polished surface reflects rays back again according to the rule: *The angle of incidence equals the angle of reflection.*

# Foothold Ideas 2:

## Light as Rays - **the perception**



- We only see something when light coming from it enters our eyes.
- Our eyes identify a point as being on an object when rays traced back converge at that point.

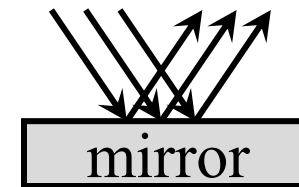


# Foothold Ideas 3:

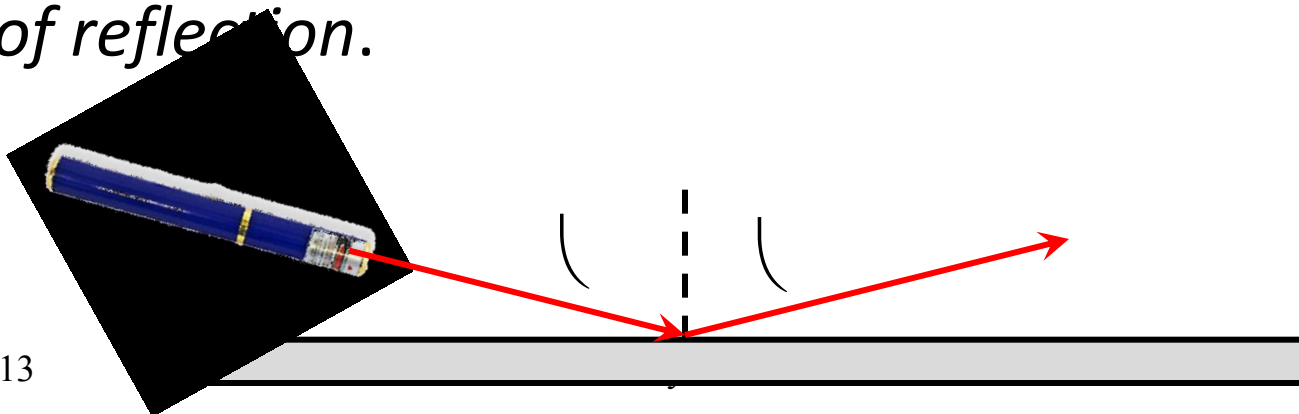
## Mirrors



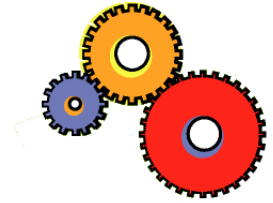
- For most objects, light scatters in all directions. For some objects (mirrors) light scatters from them in controlled directions.



- A polished surface reflects rays back again according to the rule: *The angle of incidence equals the angle of reflection.*

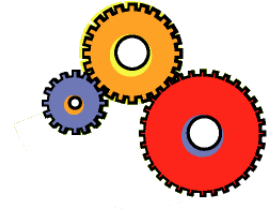


# Kinds of Images: Virtual



- In the case of the previous slide, the rays seen by the eye do not actually meet at a point – but the brain, only knowing the direction of the ray, assumes it came directly from an object.
- When the rays seen by the eye do not meet, but the brain assumes they do, the image is called *virtual*.
- If a screen is put at the position of the virtual image, there are no rays there so nothing will be seen on the screen.

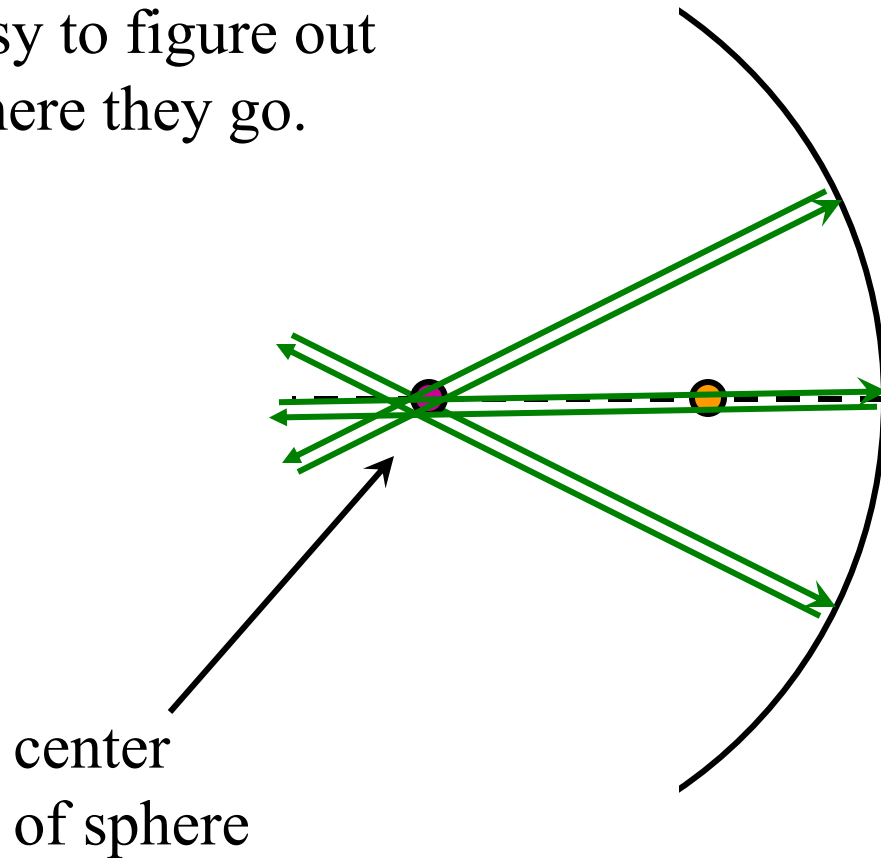
# Kinds of Images: Real



- In the case of the previous slide, the rays seen by the eye do in fact converge at a point.
- When the rays seen by the eye do meet, the image is called *real*.
- If a screen is put at the real image, the rays will scatter in all directions and an image can be seen on the screen, just as if it were a real object.

# A Spherical Mirror: Central Rays

A few rays are easy to figure out where they go.



All rays satisfy the “angle of incidence = angle of reflection” measured to the normal to the surface

All rays through the center strike the mirror perpendicular to the surface and bounce back along their incoming path.

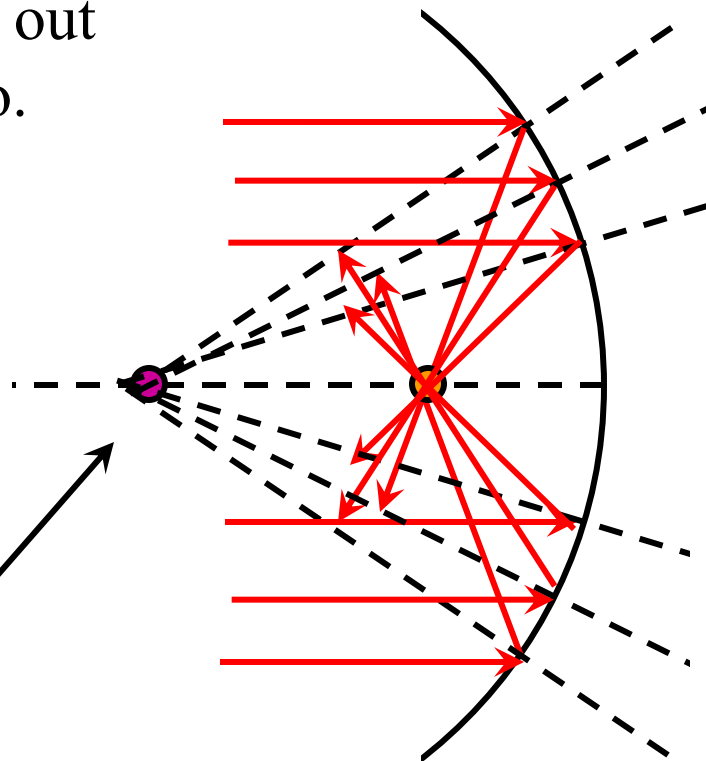
# A Spherical Mirror: Parallel Rays

A few rays are easy to figure out where they go.

All rays satisfy the “angle of incidence = angle of reflection” measured to the normal to the surface

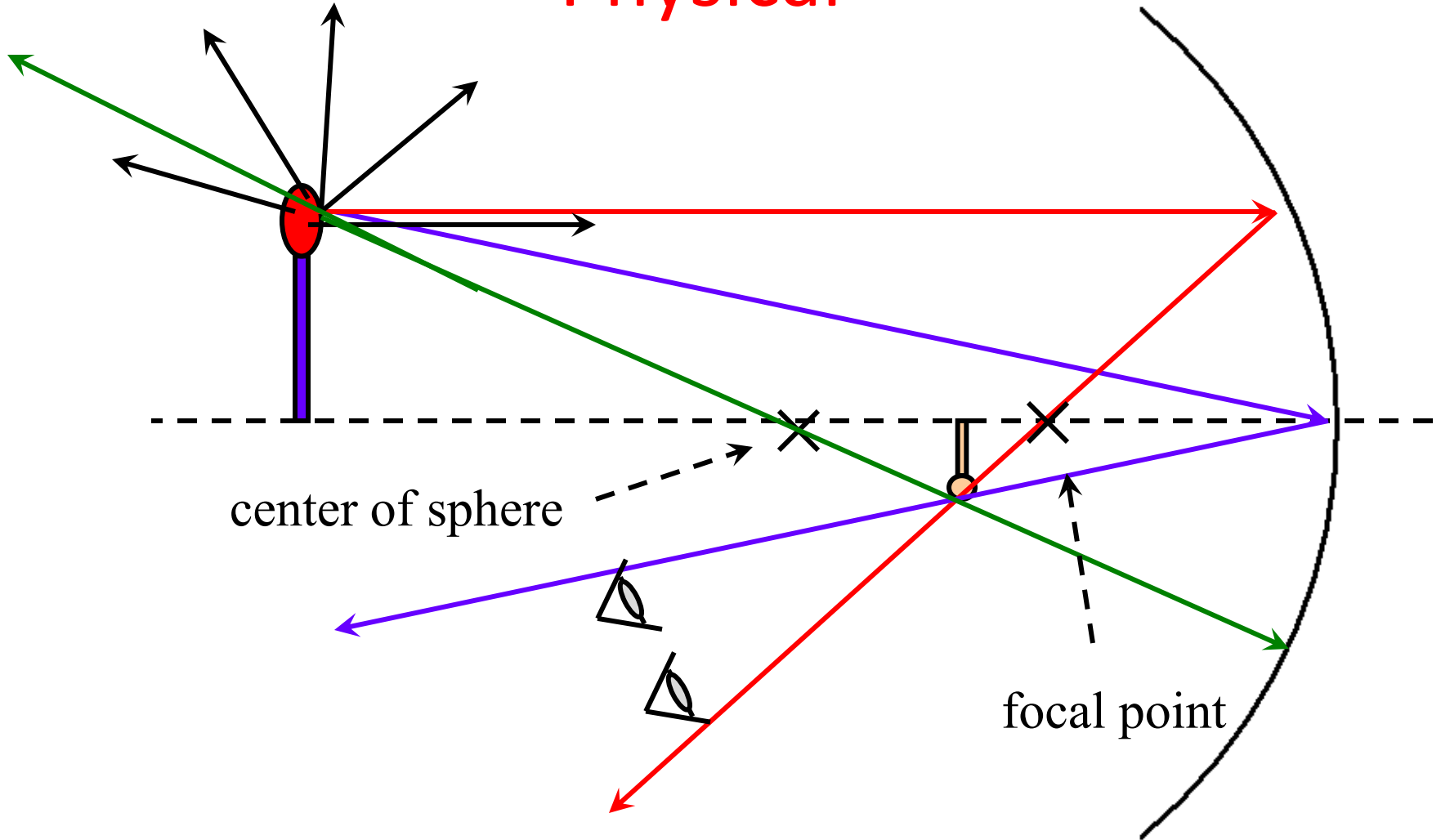
All rays parallel to and near an axis of the sphere reflect through a single point on the axis (the focal point)

center of sphere



# Images in a Spherical Mirror: 1

## Physical

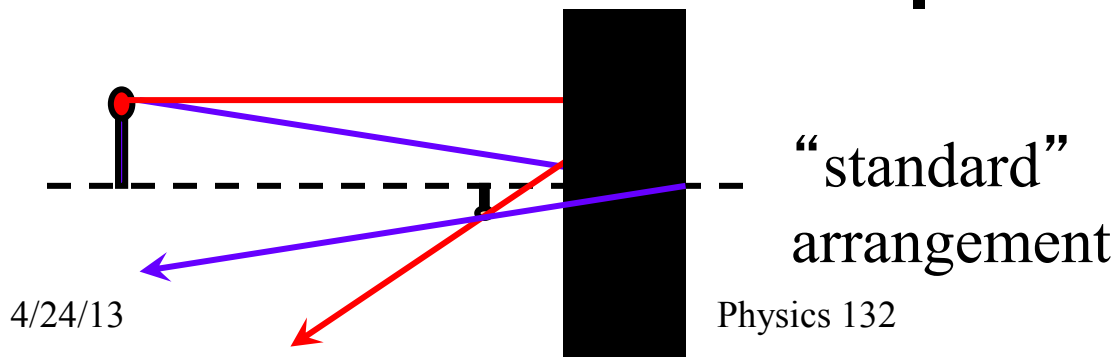


# Unifying Equation for Mirrors

- If we treat our mirror quantities as “signed” and let the signs carry directional information, we can unify all the situations in a single set of equations.

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o} \quad \frac{h'}{h} = \frac{i}{o} \quad f = R/2$$

$h > 0$	$h' < 0$	$i > 0$	$i < 0$	}	$f < 0$
$o > 0$	$o < 0$				
$h < 0$	$h' > 0$			}	$f > 0$

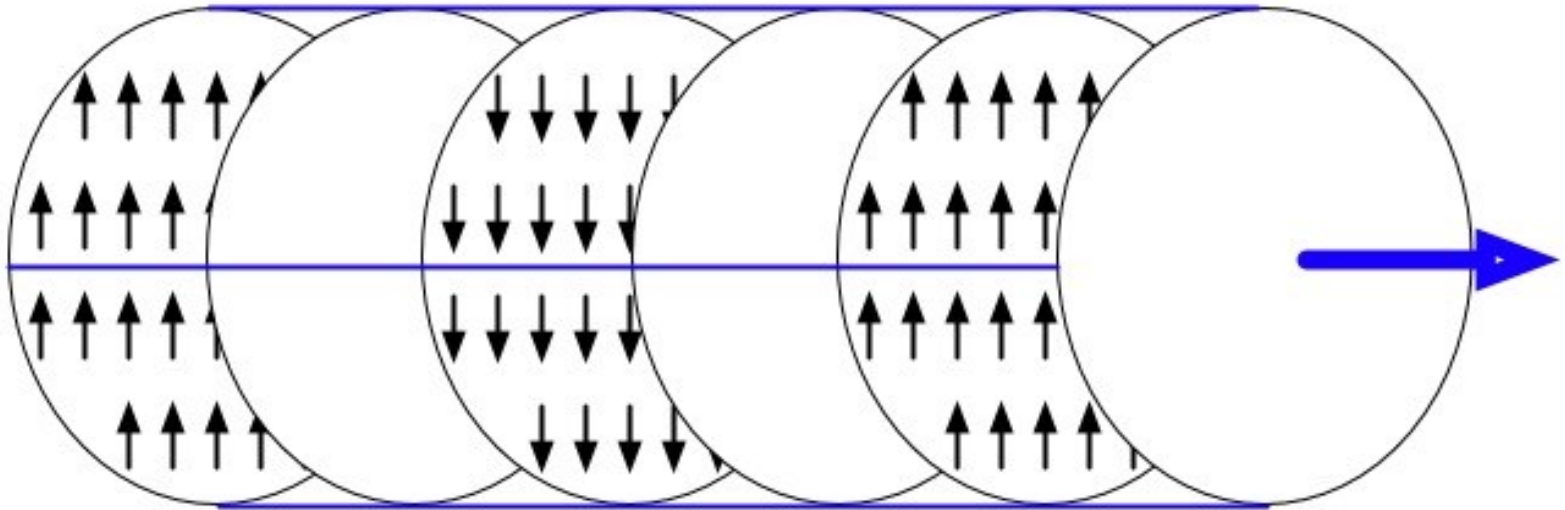


# Huygen's/Maxwell Wave Model



# The third model for light: Electromagnetic wave

- Light is an oscillating electromagnetic wave. (Long story)
- A “close-up” of a ray: a plane wave



$$\vec{E}(x, y, z, t) = \vec{E}_0 \sin(kx - \omega t)$$

# Foothold wave ideas: Huygens' Model



- The critical structure for waves are the lines or surfaces of equal phase: wavefronts.
- Each point on the surface of a wavefront acts as a point source for outgoing spherical waves (wavelets).
- The sum of the wavelets produces a new wavefront.
- The waves are slower in a denser medium.
- We can even make rays – sort of.

# Phase difference and path difference

- Our two waves from different sources have a phase difference,  $\phi_1 - \phi_2$  because we are different distances from the two sources.  
 $y = A \sin(kr_1 - \omega t) + A \sin(kr_2 - \omega t)$   
 $y = A \sin(\phi_1 - \omega t) + A \sin(\phi_2 - \omega t)$

- The phase difference depends on the path difference:

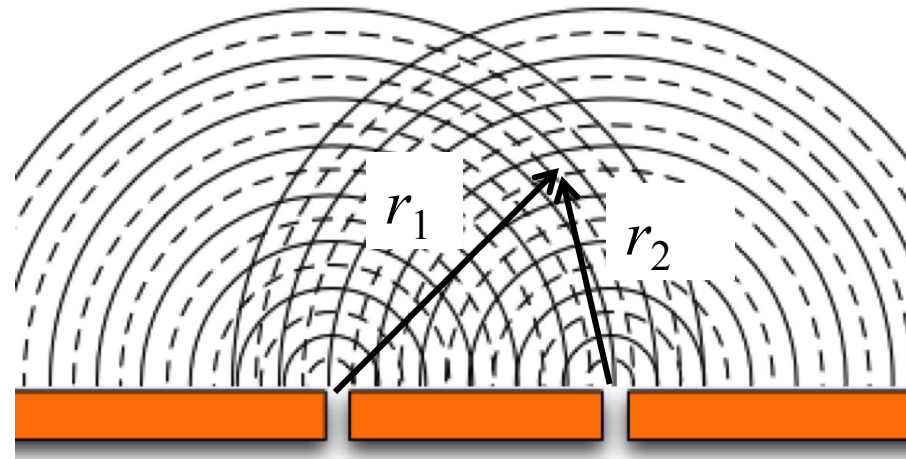
$$\phi_1 - \phi_2 = kr_1 - kr_2 = k(r_1 - r_2) = k\Delta r = 2\pi \frac{\Delta r}{\lambda}$$

# Superposition from two sources

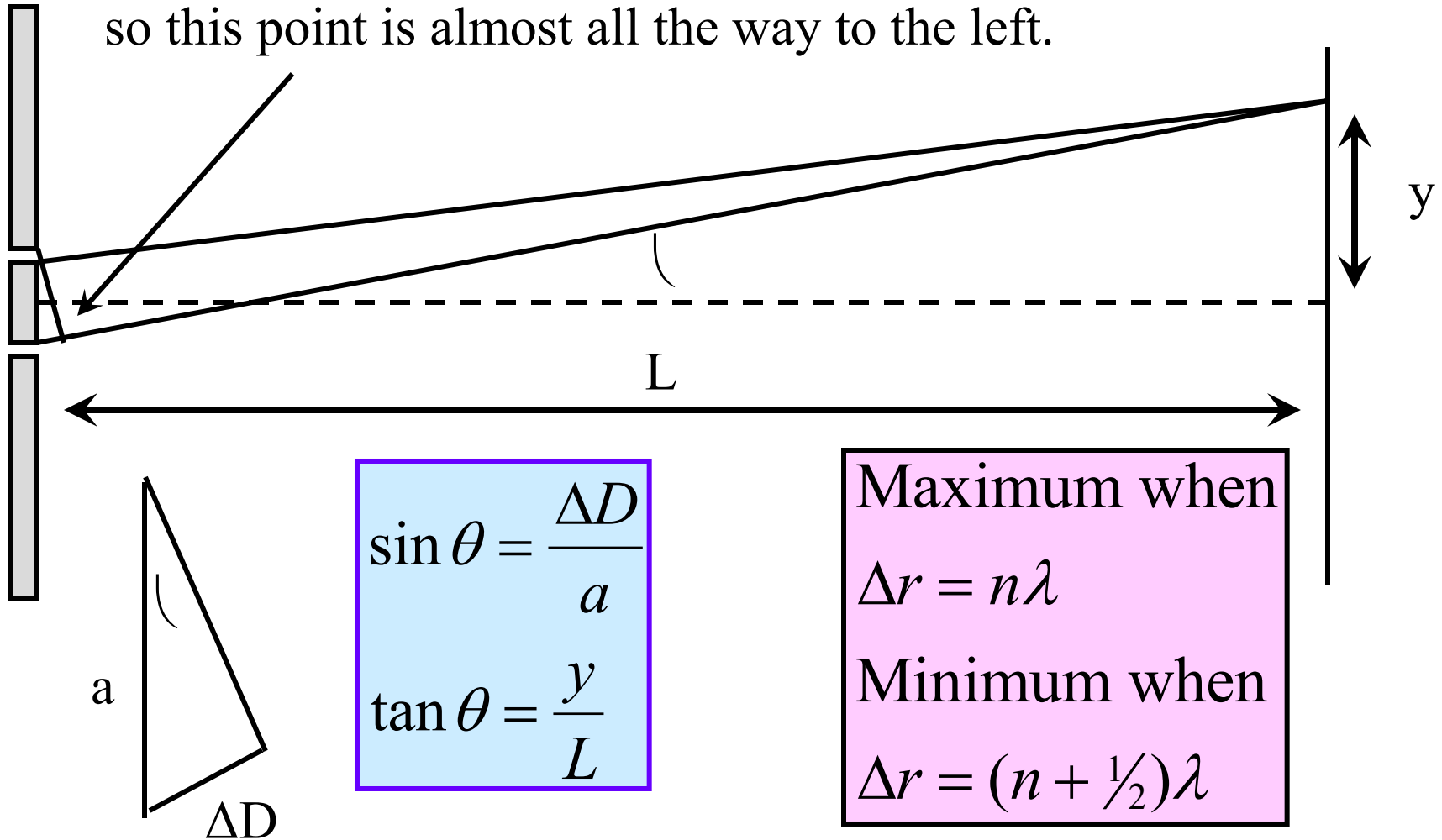
- If we are at a particular point in space and two traveling waves,  $y = A \sin(kr - \omega t)$ , reach us coming from different starting points, we are at different “ $r$ ” values for the two waves.
- The result looks like the sum of two waves with different phases:

$$y = A \sin(kr_1 - \omega t) + A \sin(kr_2 - \omega t)$$

$$y = A \sin(\phi_1 - \omega t) + A \sin(\phi_2 - \omega t)$$

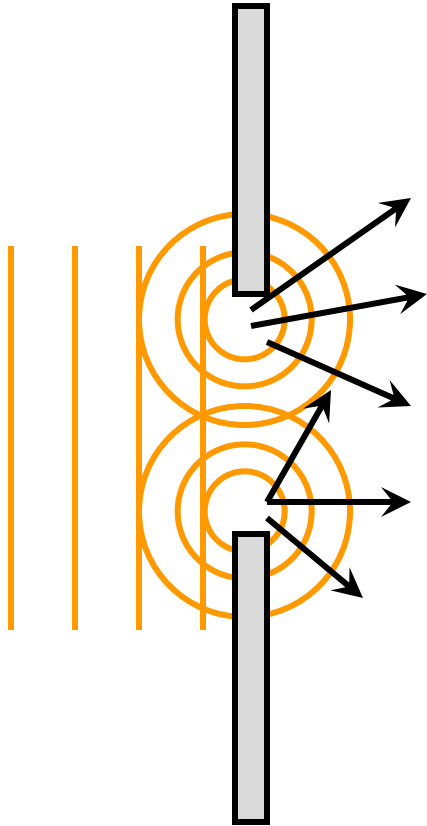


Slits are really much, much closer than shown so this point is almost all the way to the left.

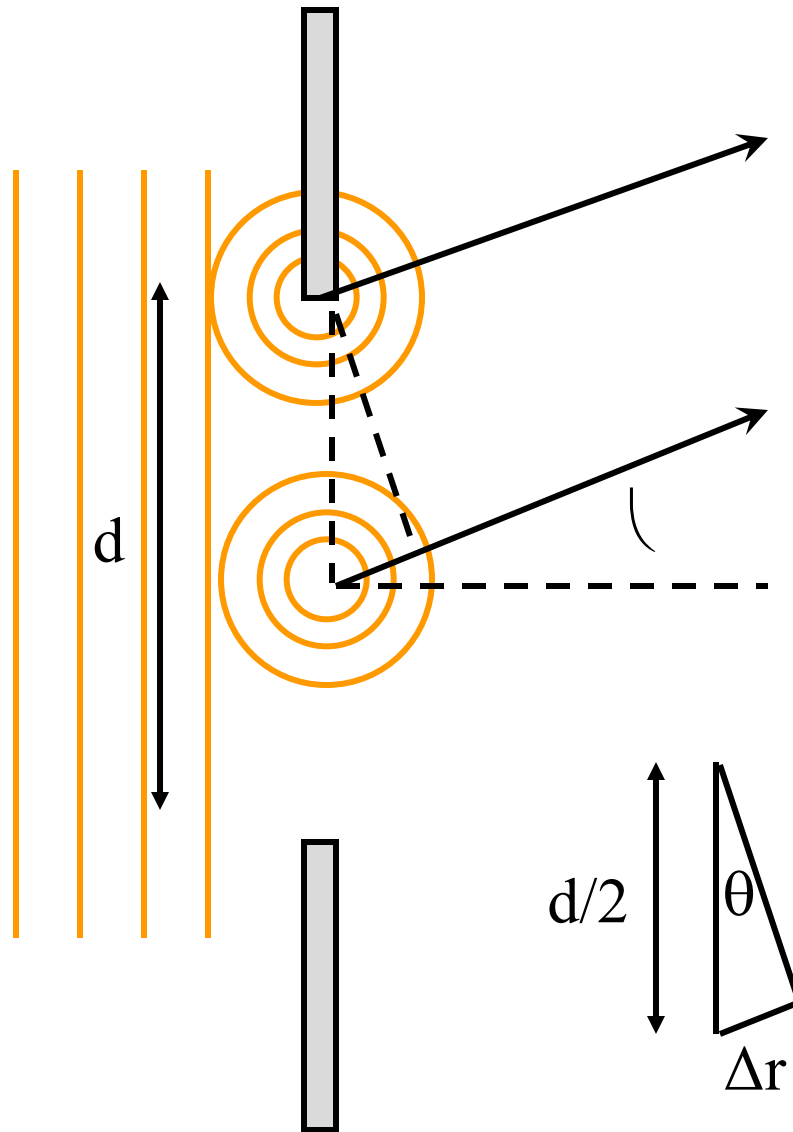


For small angles,  $\sin \theta \sim \theta$ ,  $\tan \theta \sim \theta \Rightarrow \frac{\Delta r}{a} = \frac{y}{L} \Rightarrow y = \Delta r \left( \frac{L}{a} \right)$

# Diffraction



- Every bit of the interior of the slit acts as a source of outgoing spherical Huygens' wavelets.
- The outgoing wavelets from one part of the slit can interfere with the wavelets from another part of the slit.



When the distance traveled by the wavelet from the middle of the slit is half a wavelength greater than the distance traveled by the wavelet from the top of the slit every wavelet from the top half of the slit has a canceling wavelet from the bottom half of the slit.

The result is no intensity at that angle.

# Connecting the wave and photon model of light



# Foothold Ideas: The Probability Framework



- It's clear that both the wave model and the photon have an element of truth. Here's the way we reconcile it:
  - *Maxwell's equations and the wave theory of light yield a function – the electric field – whose square (the intensity of the light) is proportional to the probability of finding a photon.*
  - *No theory of the exact propagation of individual photons exist. This is the best we can do: a theory of the probability function for photons.*

# A useful analogy: Connecting the wave vs particle model for electrons



- Quantum mechanics gives us a wave function of an electron, whose square gives us the probability of finding an electron
  - *Schrödinger's equation is the wave theory of matter. Its solution yields the wave function whose square is proportional to the probability of finding an electron.*
  - *No theory of the exact propagation of individual electrons exist. This is the best we can do: a theory of the probability function for electrons.*