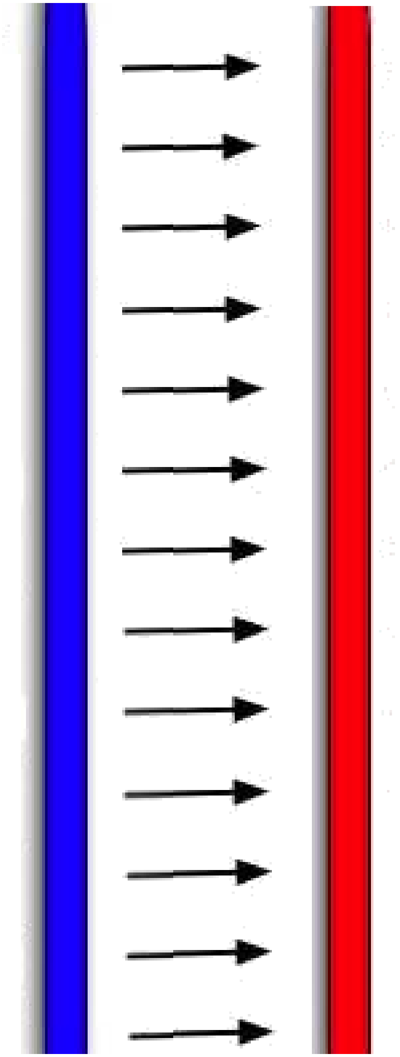
Review

Midterm II

Result



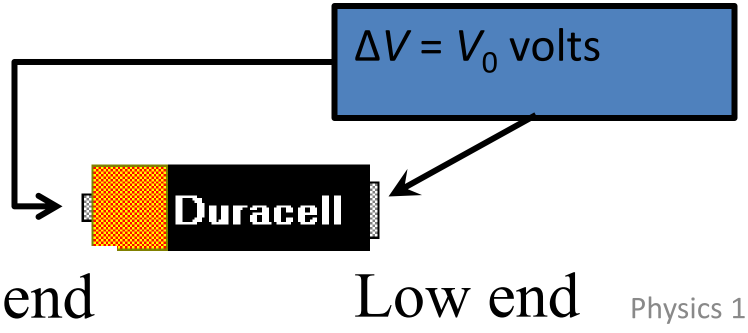
The fields of the two plates cancel each other on the outside.

The fields of the two plates add on the inside, producing double the field of a single plate.

The fields of the two plates cancel each other on the outside.

Some basic electrical ideas

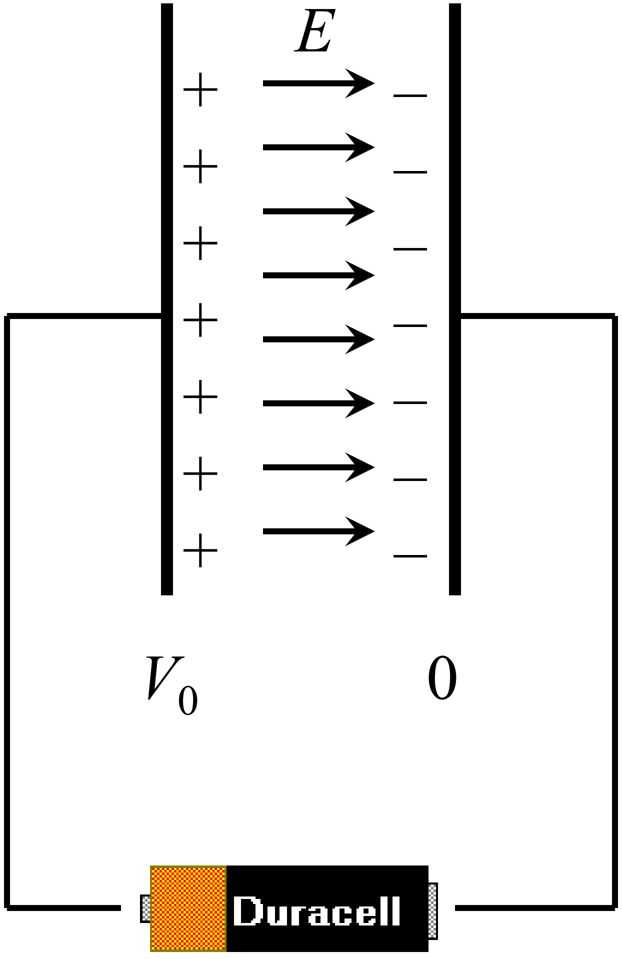
- ***Conductor*** – a material that permits some of its charges to move freely within it.
- ***Insulator*** – a material that permits some of its charges to move a little, but not freely.
- ***Battery*** – a device that creates and maintains a constant potential difference across its terminals.



$\Delta V = V_0$ volts

Charging a capacitor

- What is
  the potential difference
  between the plates?
- What is the field
  around the plates?
- How much charge
  is on each plate?



Capacitor Equations

$$\Delta V = E\Delta x = Ed$$

$$E = 4\pi k_C \sigma = 4\pi k_C \frac{Q}{A} \quad \Rightarrow \quad Q = \left(\frac{A}{4\pi k_C} \right) E$$

$$Q = \left(\frac{A}{4\pi k_C d} \right) \Delta V$$

$4\pi k_c$ is often written as "$1/\varepsilon_0$"

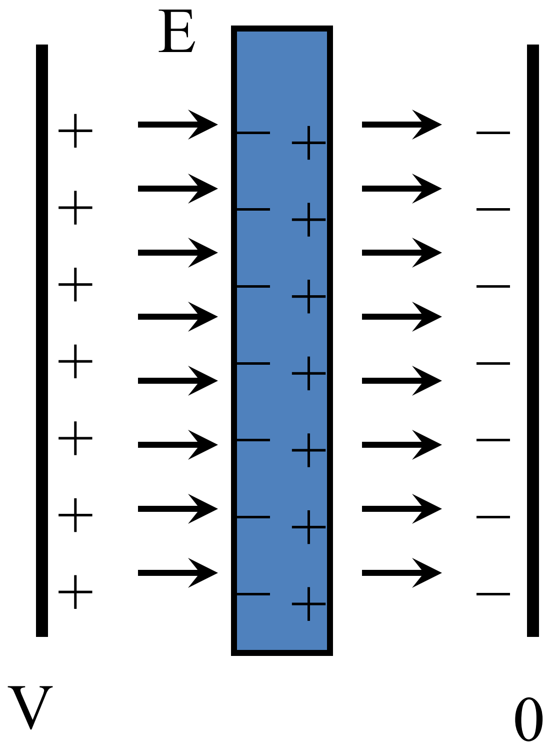
$$Q = C\Delta V$$

What does this "Q" stand for?

Conductors

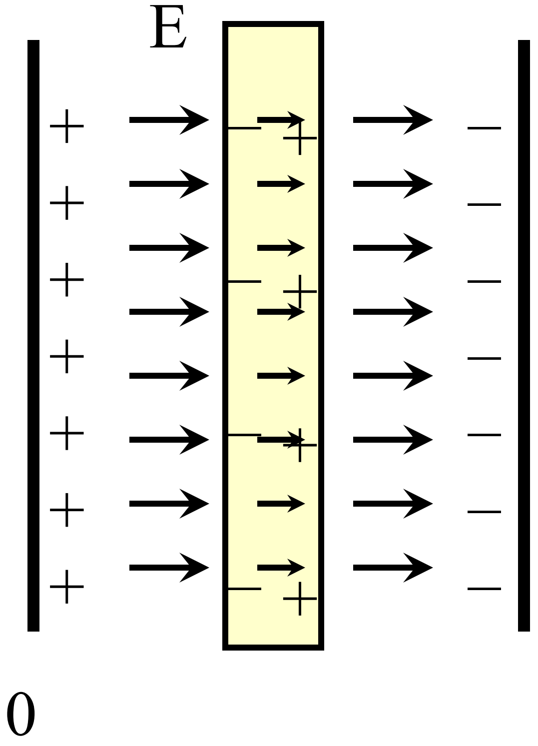
- Putting a conductor inside a capacitor eliminates the electric field inside the conductor.
- The distance, d', used to calculate the ΔV is only the place where there is an E field, so putting the conductor in reduces the ΔV for a given charge.

$$C = \frac{1}{4\pi k_C}\frac{A}{d'}$$



Consider what happens with an insulator

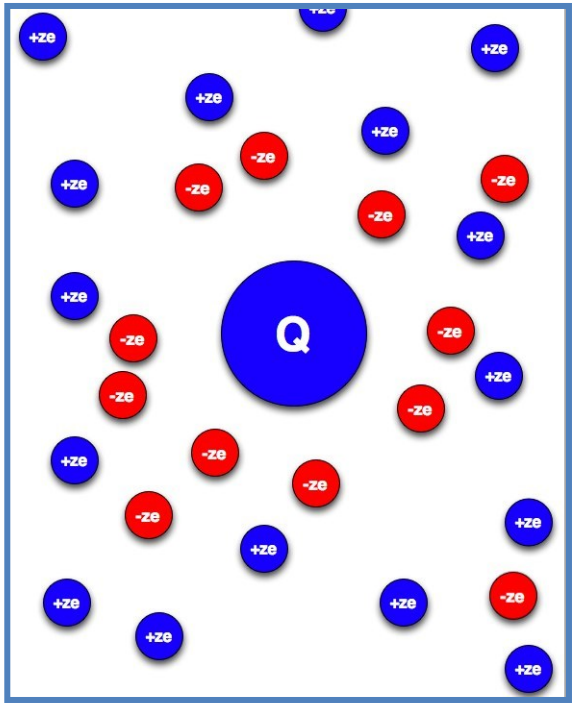
- We know that charges separate even with an insulator.
- This reduces the field inside the material, just not to 0.
- The field reduction factor is defined to be κ.



$$E_{\text{inside material}} = \frac{1}{\kappa} E_{\text{if no material were there}}$$

Debye length equations

- Charge imbedded in an ionic solution.
  - Ion charge = *ze*
  - Concentration = c_0
  - Temperature = *T*
  - Dielectric constant = κ
- The ion cloud cuts off the potential



$$\lambda_D = \sqrt{\frac{k_B T}{8\pi\left(\frac{k_C z^2 e^2}{\kappa}\right) c_0}} = \sqrt{\frac{k_B T}{2\left(\frac{z^2 e^2}{\kappa \varepsilon_0}\right) c_0}}$$

$$V(r) = \frac{k_C Q}{\kappa r} e^{-r/\lambda_D}$$

Foothold ideas: Electric charges in materials

- **Electroneutrality** – opposite charges in material attract each other strongly. Pulling them apart to create a charge unbalance costs energy.
- If a charged object is placed in an ionic solution, it tends to draw up ions of the opposite type and push away ones of the same type.
  - Result: the charge is **shielded**. As you get farther away from it the “apparent charge” gets less.
  - The scale over which this happens is called the **Debye length, λ_D.**

Foothold ideas: Currents

- Charge is moving: How much?

$$I = \frac{\Delta q}{\Delta t}$$

- How does this relate to the individual charges?

$$I = q\, n\, A\, v$$

- What pushes the charges through resistance? Electric force implies a drop in *V*!

$$F_e = qE$$

$$\Delta V = -\frac{E}{L}$$

Units

■ Current (*I*)	**Ampere** = Coulomb/sec
■ Voltage (*V*)	**Volt** = Joule/Coulomb
■ E-Field (*E*)	Newton/Coulomb = Volt/meter
■ Resistance (*R*)	**Ohm** = Volt/Ampere
■ Capacitance (*C*)	**Farad** = Volt/Coulomb
■ Power (*P*)	**Watt** = Joule/sec

Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part (*L/A*) times a part independent of the size and shape but dependent on the material. This coefficient is called the *resistivity* of the material (*ρ*).
- Its reciprocal (*g*) is called *conductivity*. *(The reciprocal of the resistance is called the conductance (G).)*

$$R = \rho \frac{L}{A} = \frac{1}{g}\frac{L}{A} = \frac{1}{G}$$

Very useful heuristic

  - Wires have very small resistance (e.g. 1 foot of 13 gauge wire has a resistance of 0.002 Ohm) This R is generally negligibly small compared to other resistances in the circuit through which current flows, so we can approximate it as zero resistance.
- The Constant Potential Corollary (CPC)
  - Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant

Foothold ideas: Kirchhoff’s principles

1. ***Flow rule***: The total amount of current flowing into any volume in an electrical network equals the amount flowing out.
2. ***Ohm’s law***: in a resistor, $\Delta V = IR$
3. ***Loop rule***: Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).

Electric Power

- The rate at which electric energy is depleted from a battery or dissipated (into heat or light) in a resistor is

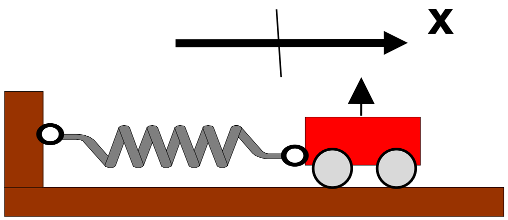
$$Power = I\Delta V$$

Nernst Equation

- Diffusion: Concentration gradient in the presence of ion channel -> ions flow to equilibrate concentration
- Electrostatic potential: only one ion species can flow -> electrostatic potential builds up -> makes it less likely for ions to keep flowing across channel

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_2}{c_1}\right)$$

Doing the Math: The Equation of Motion



- Newton's equation for the cart is

$$a = {F_{net}}\big/{m} = {-kA(t)}\big/{m} = -\left(\frac{k}{m}\right)A(t)$$

- What kind of a quantity is k/m? (i.e. what is its "Dimension"

$$\left[\frac{k}{m}\right] =$$

Interpreting the Result

- We'll leave it to our friends in math to show that these results actually satisfy the N2 equations.
- What do the various terms mean?
  - A_{max} is the maximum displacement — the *amplitude* of the oscillation.
  - What is ω_0? If T is the *period* (how long it takes to go through a full oscillation) then

$$\omega_0 t : 0 \rightarrow 2\pi$$

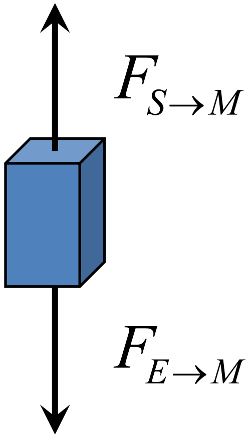
$$t \quad : 0 \rightarrow T$$

$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

Summary with Equations: Mass on a spring

$$a = \frac{1}{m} F^{net} \qquad F^{net} = -kx$$

Measured from where?

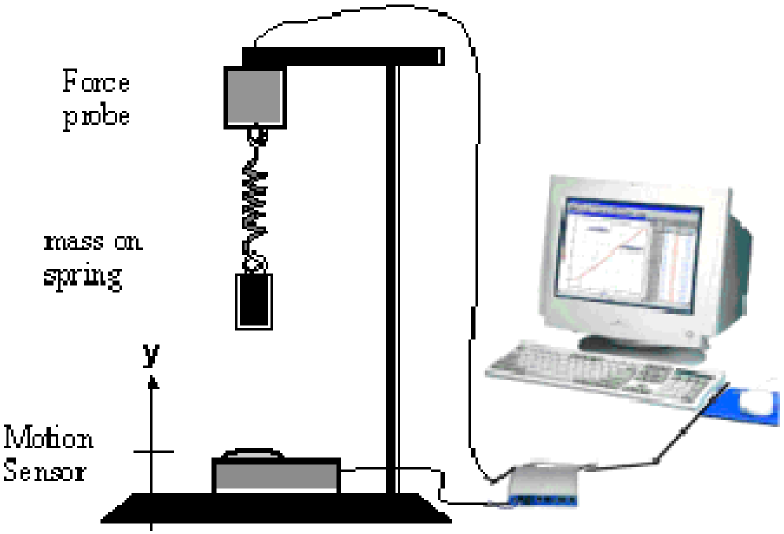


$$a = -\omega_0^2 x \qquad \omega_0^2 = \frac{k}{m}$$

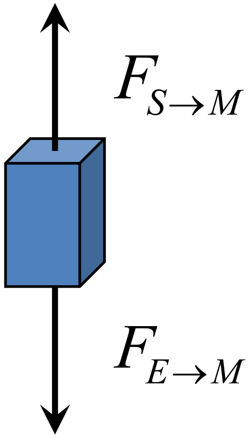
$$A(t) = A_0 \cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



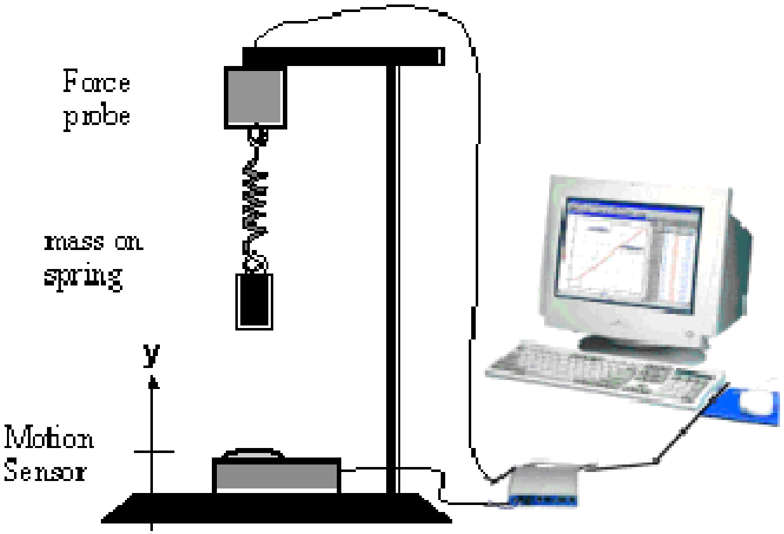
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k\left(\Delta l\right)^2$$

$$E_i = E_f$$



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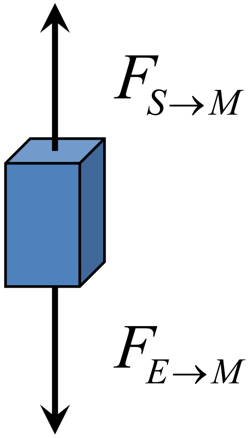

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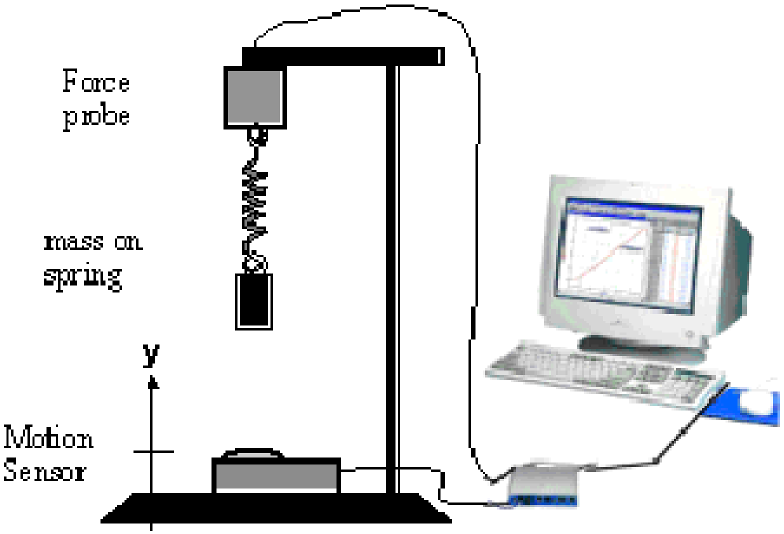

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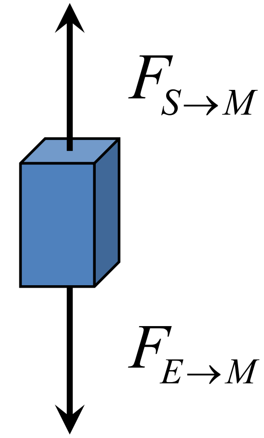

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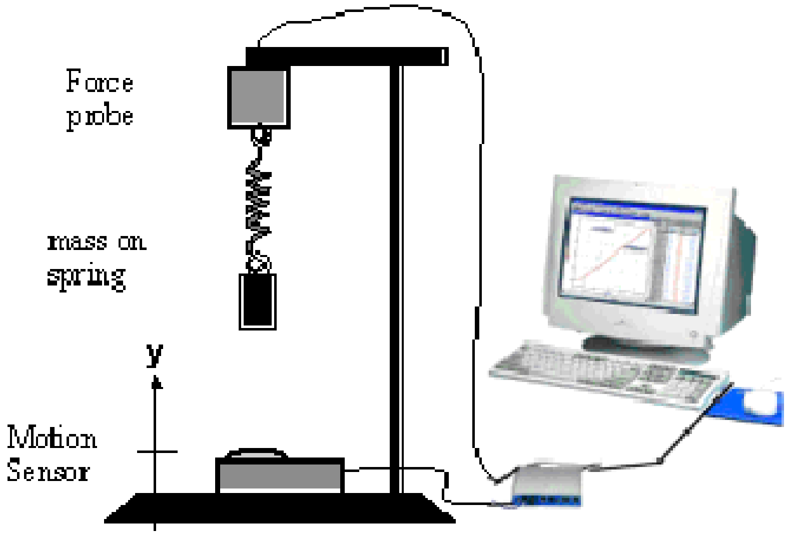


Pendulum motion energy

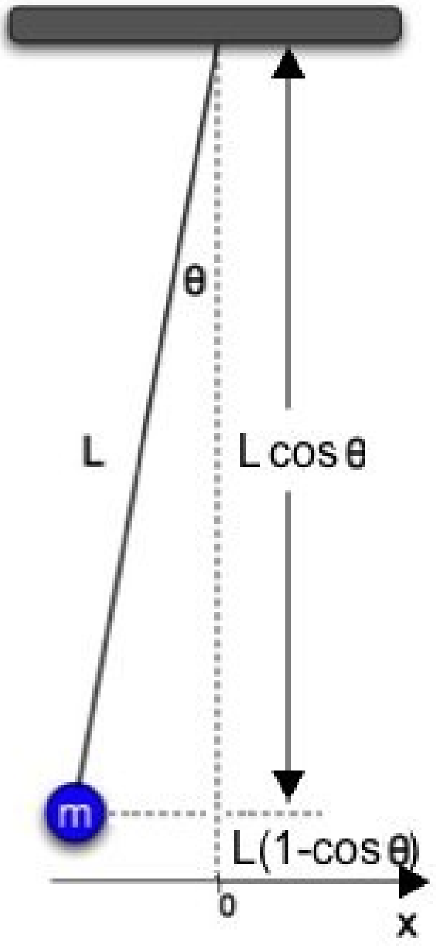
$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1-\cos\theta)$$

$$\cos\theta \square 1 - \tfrac{1}{2}\theta^2$$

$$E_0 \square \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left[mgL\right]\theta^2$$

$$\theta \square \sin\theta = \frac{x}{L}$$

$$E_0 \square \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



Same as mass on a spring!

Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease
  Simplest model is viscous drag.

$$ma = -kx - bv$$

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \qquad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

- Solution:

$$x(t) = A_0 e^{-\gamma t/2} \cos(\omega_1 t + \phi)$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

Foothold ideas: Damped oscillator 2

- Competing time constants:

$$\frac{\gamma}{2} = \frac{1}{\tau} \qquad \frac{\omega_0}{2\pi} = \frac{1}{T} \qquad Q = \frac{\omega_0}{\gamma} = \pi\frac{\tau}{T}$$

Decay time

Period

Tells which force dominates: restoring or damping.

- If:

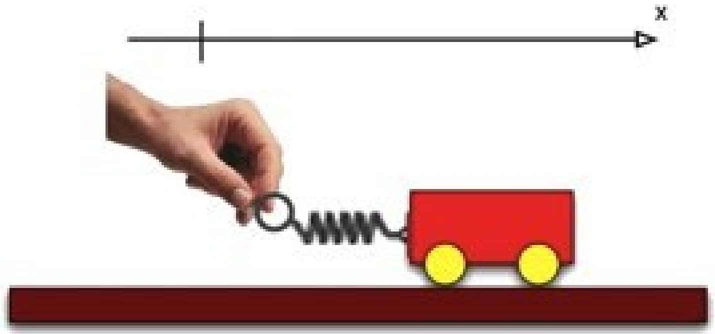
$\omega_0 > \gamma/2$ underdamped: oscillates

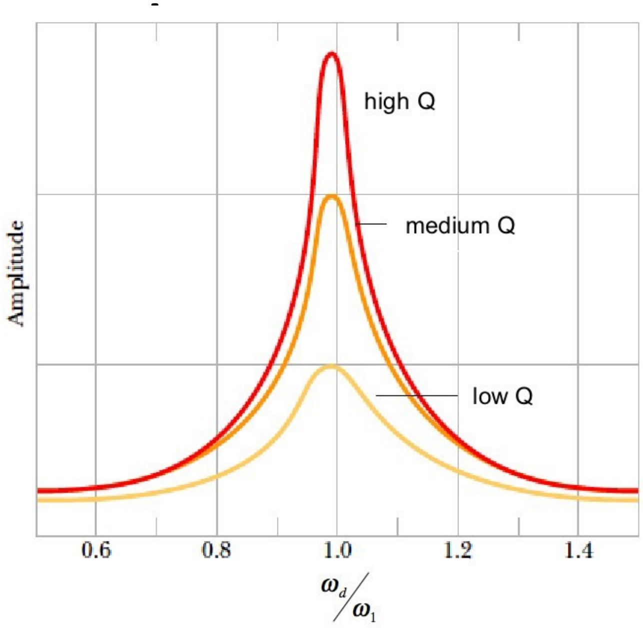
$\omega_0 = \gamma/2$ critically damped: no oscillation, fastest decay

$\omega_0 < \gamma/2$ over damped: no oscillation, slower decay

Foothold ideas:
Driven oscillator

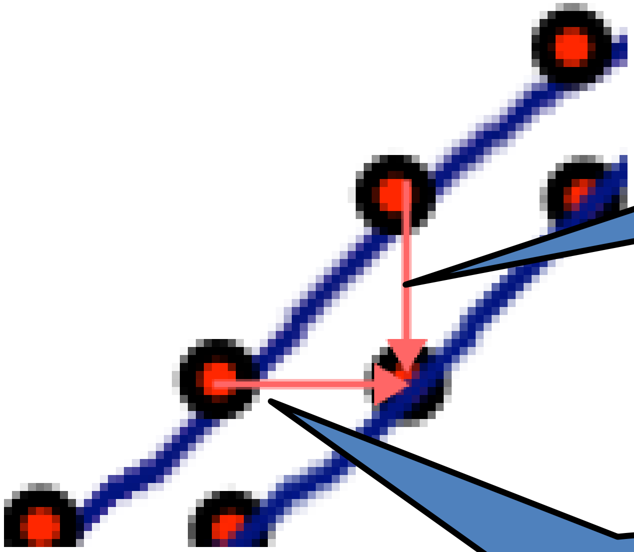
- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not





Speed of a bead

- The speed the bead moves depends on how fast the pulse is moving and how far it needs to travel to stay on the string.



dy = how far bead moves in time dt

dx = how far pulse moves in time dt

slope of pulse

$$v(x,t) = \frac{dy(x,t)}{dt} = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{dy}{dx}\right) v_0$$

speed of bead

speed of pulse

Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)

$$v_0 = \sqrt{T/\mu}$$

v_0 = speed of pulse
T = tension of spring
μ = mass density of spring (M/L)

- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and pattern speed.

The math

- We express the position of a bit of string at a particular time by labeling which bit of string by its *x* position, at x at time *t* the position of the string is *y*(*x,t*).
- Since subtracting a *d* from the argument of a function ($f(x) \to f(x-d)$) shifts the graph of the function to the right by an amount *d*, if we want to set the graph of a shape *f*(*x*) into motion at a constant speed, we just need to set $d = v_0 t$ and take

$$f(x) \to f(x - v_0 t)$$