The Long Pendulum

\[ T \]
\[ \theta \]

\[ mg \]
Pendulum motion energy

\[ E_0 = \frac{1}{2} mv^2 + mgh = \frac{1}{2} mv^2 + mgL(1 - \cos \theta) \]
\[ \cos \theta = 1 - \frac{1}{2} \theta^2 \]
\[ E_0 = \frac{1}{2} mv^2 + \frac{1}{2} [mgL] \theta^2 \]
\[ \sin \frac{x}{L} \]
\[ E_0 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \quad k = \frac{mg}{L} \]

Same as mass on a spring!
Just with a different \( \omega_0^2 = k/m = g/L \)

What’s the period? Why doesn’t it depend on \( m \)?
Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease
  Simplest model is viscous drag.

\[ ma = kx \quad bv \]

\[ \frac{d^2x}{dt^2} + \frac{dx}{dt} + \frac{2}{m} x = 0 = \frac{b}{m} \quad 0 = \sqrt{\frac{k}{m}} \]

- Solution:

\[ x(t) = A_0 e^{-\frac{t}{2}} \cos\left( \sqrt{\frac{k}{m}} t + \right) \]

\[ 1 = \sqrt{\frac{2}{4}} \]

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Foothold ideas: Damped oscillator 2

- Competing time constants:

\[ \frac{1}{2} = \frac{1}{\omega_0} \quad \frac{0}{2} = \frac{1}{T} \]

- If:

\[ \omega_0 > \gamma/2 \quad \text{underdamped: oscillates} \]
\[ \omega_0 = \gamma/2 \quad \text{critically damped: no oscillation, fastest decay} \]
\[ \omega_0 < \gamma/2 \quad \text{over damped: no oscillation, slower decay} \]

\[ Q = -\frac{\gamma}{2\omega_0} = -\frac{1}{T} \]

Decay time \quad Period

Tells which force dominates: restoring or damping.
Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (resonance). Otherwise, not