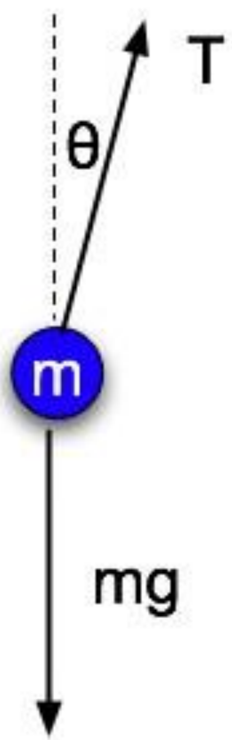
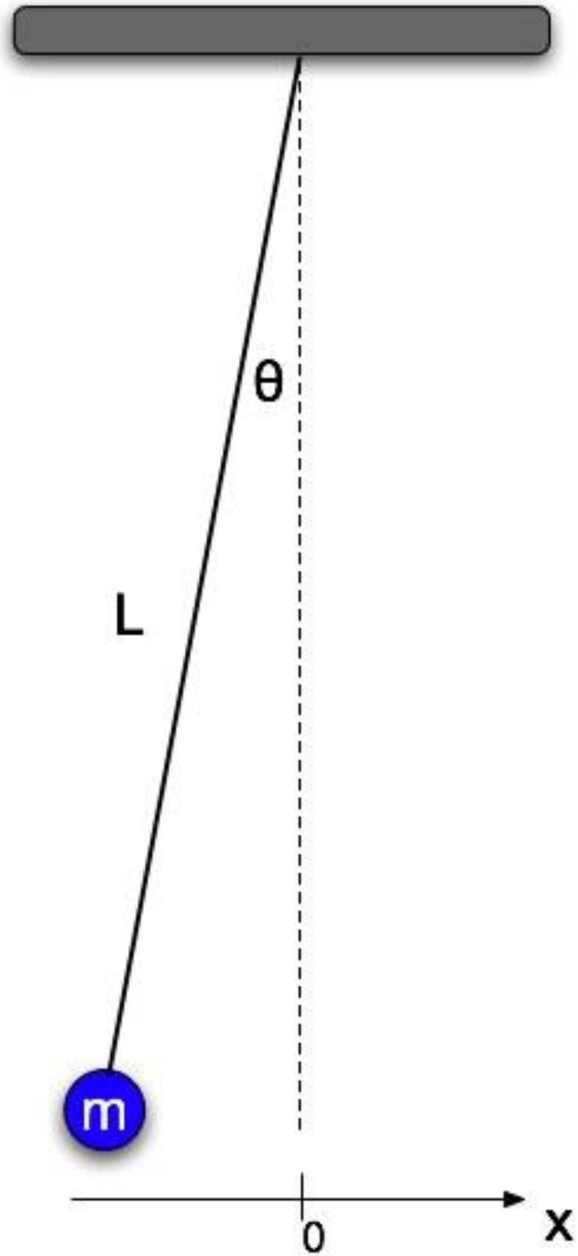
The Long Pendulum





Pendulum motion energy

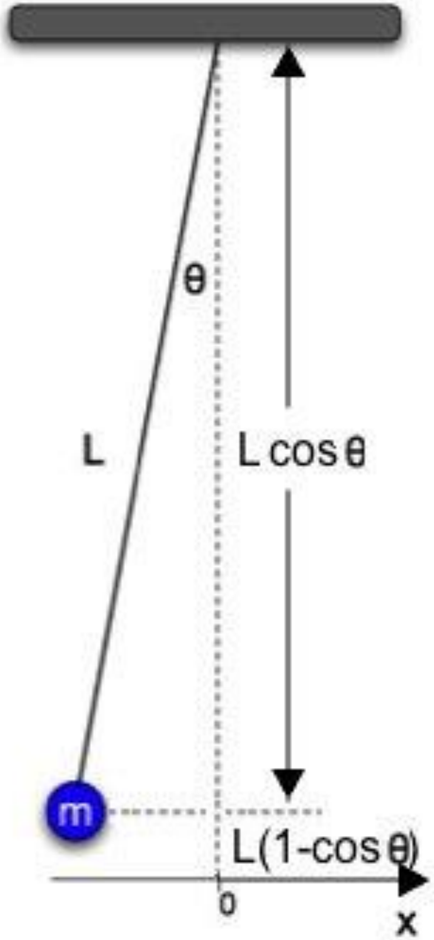
$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1 - \cos\theta)$$

$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left[mgL\right]\theta^2$$

$$\theta \approx \sin\theta = \frac{x}{L}$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



Same as mass on a spring!
Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease
  Simplest model is viscous drag.

$$ma = -kx - bv$$

$$\frac{d^2x}{dt^2} + g\frac{dx}{dt} + w_0^2 x = 0 \qquad g = \frac{b}{m} \qquad w_0 = \sqrt{\frac{k}{m}}$$

- Solution:

$$x(t) = A_0 e^{-gt/2}\cos\left(w_1 t + f\right)$$

$$w_1 = \sqrt{w_0^2 - \frac{g^2}{4}}$$

Foothold ideas: Damped oscillator 2

- Competing time constants:

$$\frac{g}{2} = \frac{1}{t} \qquad \frac{w_0}{2p} = \frac{1}{T} \qquad Q = \frac{w_0}{g} = p\frac{t}{T}$$

Decay time

Period

Tells which force dominates: restoring or damping.

- If:

$\omega_0 > \gamma/2$ underdamped: oscillates

$\omega_0 = \gamma/2$ critically damped: no oscillation, fastest decay

$\omega_0 < \gamma/2$ over damped: no oscillation, slower decay

Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not