- **<u>Review Materials Midterm 1</u>**
  - **Intro Materials**
  - **Thermodynamics**
  - **Electrostatic Charges**

Intro Material

Foothold principles: Newton's Laws

- Newton 0:
  - An object responds **only** to the forces it feels and only at the instant it feels them.
- Newton 1:
  - An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).
- Newton 2:
  - An object that is acted upon by other objects changes its velocity according to the rule

$$\vec{a}_A = \vec{F}_A^{net} / m_A$$

- Newton 3:
  - When two objects interact the forces they exert on each other are equal and opposite.

$$\vec{F}_{A\to B}^{type} = -\vec{F}_{B\to A}^{type}$$

Foothold ideas: Kinetic Energy and Work

- Newton's laws tell us how velocity changes. The Work-Energy theorem tells us how speed (independent of direction) changes.
- Kinetic energy = $\frac{1}{2}mv^2$
- Work done by a force = $F_x \Delta x$ or $F_{||} \Delta r$ (part of force || to displacement)
- Work-energy theorem: $\Delta(\frac{1}{2}mv^2) = F_{||}^{net}\ \Delta r$ (small step)

  $\Delta(\frac{1}{2}mv^2) = \int_i^f F_{||}^{net}\ dr$ (any size step)

Foothold ideas: Potential Energy

- The work done by some forces only depends on the change in position. Then it can be written

$$\vec{F}\cdot\Delta\vec{r} = -\Delta U$$

U is called a *potential energy*.

- For gravity, $U_{gravity} = mgh$

  For a spring, $U_{spring} = \frac{1}{2} kx^2$

  For electric force, $U_{electric} = k_C Q_1 Q_2 / r_{12}$

- Potential to force:

$$\vec{F} = -\frac{\Delta U}{\Delta\vec{r}} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla}U$$

The force associated with a PE at a given place points "downhill" – in the direction where the PE falls the fastest.

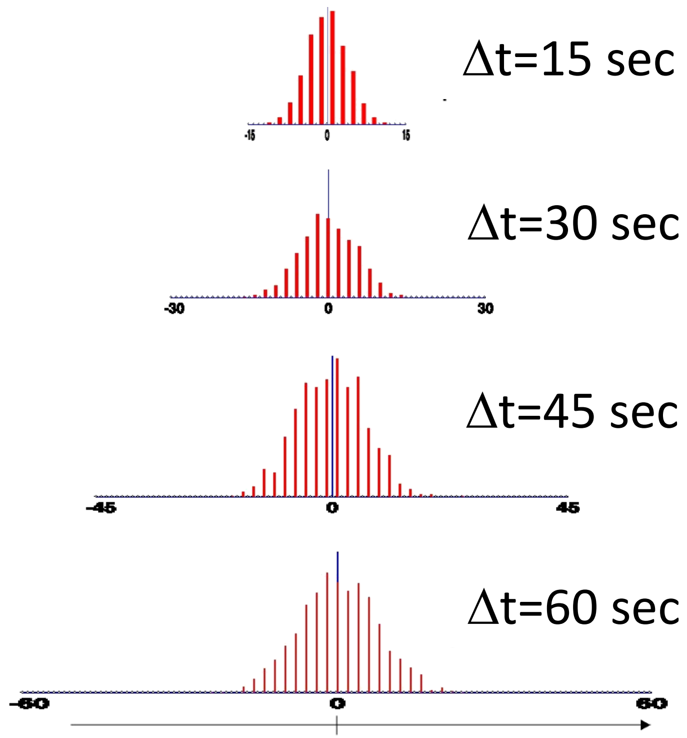
Foothold ideas: Random walk in 1D

- In random motion, the distribution of distances moved in a time Δt is predicable. This phenomenon is called *diffusion*
- The square of the average distance traveled during random motion will grow with time:

$$\left\langle (\Delta x)^2 \right\rangle = 2D\Delta t$$

- *D* is called *the diffusion constant* and has dimensionality [*D*] = L^2/T

Distribution of Distances



Distance Travelled

Thermodynamics

Foothold ideas: Energy

- Kinds of energy
  - Kinetic
  - Potential
  - Thermal
  - Chemical

- First law of thermodynamics
  - Conservation of total energy

Energy of System

Thermal energy
Entering system

Work done
on system

$$\Delta E = Q + W$$

Foothold ideas: Enthalpy

Internal Energy

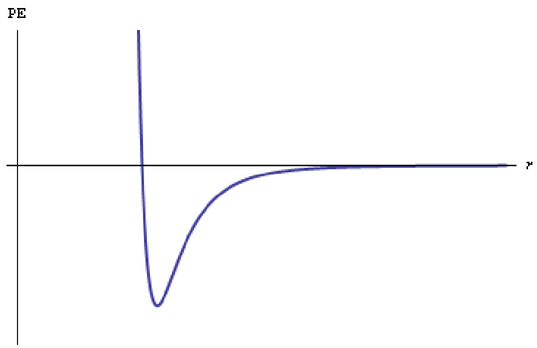
Work done on system

Energy needed to add internal energy at constant pressure (Enthalpy)

$$\Delta H = \Delta U + p\Delta V$$

Foothold ideas:
Inter-atomic potentials

- The interaction between neutral atoms includes an attraction that arises from the fluctuating charge distribution in each atom; the PE behaves like $1/r^6$.
- When the atoms are pressed close, they repel each other strongly
- Two commonly used models are:
  - The Lennard-Jones potential (A/r^{12}-B/r^6)
  - The Morse potential (exponentials)



Foothold ideas: Microstate and macrostates

- A *microstate* is a specific distribution of energy telling how much is in each DoF.
- A *macrostate* is a statement about some average properties of a state (pressure, temperature, density,...).
  - A given macrostate corresponds to many microstates.
- If the system is sufficiently random, each microstate is equally probable. As a result, the probability of seeing a given macrostate depends on how many microstates it corresponds to.

Foothold ideas:
Thermal Equilibrium & Equipartition

- ***Degrees of freedom are "bins"*** where internal energy resides in a system.
- ***Equipartition*** – In equilibrium, the same average energy in each Degree of Freedom (bin).
- ***Thermodynamic equilibrium is dynamic*** – Energy moves from bin to bin, changes keep happening in each bin but cancel out.

Foothold ideas: Entropy

- Entropy – an extensive measure of how well energy is spread in a system.
- Entropy measures
  - The number of microstates in a given macrostate

$$S = k_B \ln(W)$$

  - The amount that the energy of a system is spread among the various degrees of freedom
- Change in entropy upon heat flow

$$\Delta S = \frac{Q}{T}$$

Foothold ideas:
The Second Law of Thermodynamics

- Systems spontaneously move toward the thermodynamic (macro)state that correspond to the largest possible number of particle arrangements (microstates).
  - The 2nd law is probabilistic. Systems show fluctuations – violations that get proportionately smaller as N gets large.
- Systems that are not in thermodynamic equilibrium will spontaneously transform so as to increase the entropy.
  - The entropy of any particular system can decrease as long as the entropy of the rest of the universe increases more.
- The universe tends towards states of increasing chaos and uniformity. (Is this contradictory?)

Foothold ideas: Transforming energy

- Internal energy: thermal plus chemical $$DU$$
- Enthalpy: internal plus amount needed to make space at constant *p* $$DH = DU + pDV$$
- Gibbs free energy: enthalpy minus amount associated with raising entropy of the rest of the universe due to energy dumped $$DG = DH - T DS$$
- A process will go spontaneously if $\Delta G < 0$.

Foothold ideas:
Energy changes in a **process**

- Internal energy:
  thermal and chemical
  $$\Delta U$$
- Enthalpy:
  internal plus amount needed
  to make space at constant *p*
  $$\Delta H = \Delta U + p\Delta V$$
- Gibbs free energy:
  enthalpy minus amount associated with raising
  entropy of the rest of the universe due to
  energy dumped
  $$\Delta G = \Delta H - T\Delta S$$
- A process will go spontaneously if $\Delta G < 0$.

Foothold ideas: Exponents and logarithms

- Power law: $f(x) = x^2 \quad g(x) = Ax^7$

  a variable raised to a fixed power.

- Exponential: $f(x) = e^x \quad g(N) = 2^N \quad h(z) = 10^z$

  a fixed constant raised to a variable power.

- Logarithm: the inverse of the exponential.

$$\log\left(e^x\right) = x \qquad \log\left(e^x e^y\right) = \log\left(e^x\right) + \log\left(e^y\right) = x + y$$

$$\log\left(x^2\right) = 2\log\left(x\right) \qquad \log\left(xy\right) = \log\left(x\right) + \log\left(y\right)$$

Logs convert multiplying to adding!

Foothold ideas: Energy distribution

- Due to the randomness of thermal collisions, eve in (local) thermal equilibrium a range of energy is found in each degree of freedom.
- The probability of finding an energy *E* is proportional to the Boltzmann factor
-
$$P(E)\ \mu\ e^{-E/k_BT} \quad \text{(for one DoF)}$$
$$P(E)\ \mu\ e^{-E/RT} \quad \text{(for one mole)}$$
- At 300 K, $k_BT \sim 1/40$ eV
  $N_Ak_BT = RT \sim 2.4$ kJ/mol

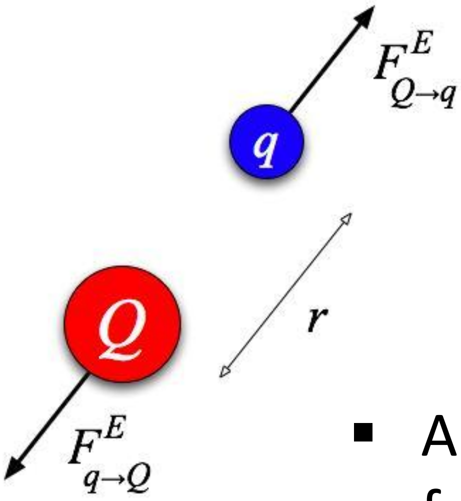
Electric Charges

Model: Charge
A hidden property of matter

- Matter is made up of two kinds of electric charges (positive and negative) that have equal magnitude and that cancel when they are together and hide matter’s electrical nature.
- Like charges repel, unlike charges attract.
- The net charge (postive minus negative charges) is a constant
- Matter with an equal balance is called <u>neutral</u>.

Can Charges Move?

- Insulators
  - Charges are bound and cannot move around freely.
  - Excess charge tends to just sit there.
- Conductors
  - Charges can move around throughout the object.
  - Excess charge redistributes itself or flows off
    - Solid: Electrons move
    - Fluid: Charged atoms move
- Unbalanced charges attract neutral matter (polarization)



Foothold idea: Coulomb’s Law

- All objects attract or repel each other with a force whose magnitude is given by

$$\vec{F}_{q\to Q} = \frac{k_C qQ}{r_{qQ}^2}\hat{r}_{q\to Q}$$

$$k_C = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

Foothold ideas: Energies between charge clusters

- Atoms and molecules are made up of charges.
- The potential energy between two charges is

$$U_{12}^{elec} = \frac{k_C Q_1 Q_2}{r_{12}}$$

No vectors!

- The potential energy between many charges is

$$U_{12\ldots N}^{elec} = \sum_{i<j=1}^{N} \frac{k_C Q_i Q_j}{r_{ij}}$$

Just add up all pairs!

Foothold idea: Electric Forces and Fields

When we focus our attention on the electric force on a particular object with charge q_0 (a "test charge") we see the force it feels depends on q_0.

Define quantity that does not depend on charge of test object "test" charge -> **Electric Field E**

$$\vec{F}_{q_0}^{E\,net} = \frac{k_C q_0 q_1}{r_{01}^2}\hat{r}_{1\to 0} + \frac{k_C q_0 q_2}{r_{02}^2}\hat{r}_{2\to 0} + \frac{k_C q_0 q_3}{r_{03}^2}\hat{r}_{3\to 0} + \ldots \frac{k_C q_0 q_N}{r_{0N}^2}\hat{r}_{N\to 0}$$

$$\vec{F}_{q_0}^{E\,net} = q_0 \vec{E}(\vec{r}_0)$$

$$\vec{E}(\vec{r}_0) = \frac{k_C q_1}{r_{01}^2}\hat{r}_{1\to 0} + \frac{k_C q_2}{r_{02}^2}\hat{r}_{2\to 0} + \frac{k_C q_3}{r_{03}^2}\hat{r}_{3\to 0} + \ldots \frac{k_C q_N}{r_{0N}^2}\hat{r}_{N\to 0}$$

E is defined everywhere in space not just in places where charges are present

Foothold ideas: Electrostatic Potential energy and Electrostatic Potential

- Again we focus our attention on a test charge!
- Usual definition of “electrostatic potential energy”: How much does the energy of our system change if we add the test charge

It’s really a change in potential energy!

$$U_{q_0}^{elec}(\vec{r}_0) = \frac{k_C q_0 q_1}{r_{01}} + \frac{k_C q_0 q_2}{r_{02}} + \ldots + \frac{k_C q_0 q_N}{r_{0N}} = \sum_{i=1}^{N} \frac{k_C q_0 q_i}{r_{0i}}$$

- We ignore the electrostatic potential energies of all other pairs (since we assume the other charges do not move)
- We can pull the test charge magnitude out of the equation and obtain en **electrostatic potential**

$$V(\vec{r}_0) = \frac{U_{q_0}^{elec}(\vec{r}_0)}{q_0} = \frac{k_C q_1}{r_{01}} + \frac{k_C q_2}{r_{02}} + \ldots + \frac{k_C q_N}{r_{0N}} = \sum_{i=1}^{N} \frac{k_C q_i}{r_{0i}}$$

Forces and Fields

$$\vec{F}_q = \sum_{i=1}^{N} \frac{k_C q Q_i}{r_{iq}^2} \hat{r}_{iq}$$

$$\vec{E} = \frac{\vec{F}_q}{q}$$

Potential Energy and Potential

$$\Delta U_q^{elec} = \sum_{i=1}^{N} \frac{k_C q Q_i}{r_{iq}}$$

$$V = \frac{\Delta U_q^{elec}}{q}$$

Foothold idea:
Fields

- *Test particle*
  - We pay attention to what force it feels.
    We assume it does not have any affect on the source particles.
- *Source particles*
  - We pay attention to the forces they exert and assume they do not move.
- *Physical field*
  - We consider what force a test particle would feel if it were at a particular point in space and divide by its coupling strength to the force. This gives a vector at each point in space.

$$\vec{g} = \frac{1}{m}\vec{W}_{E\to m} \qquad \vec{E} = \frac{1}{q}\vec{F}_{\text{all charges}\to q} \qquad V = \frac{1}{q}U^{elec}_{\text{all charges}\to \text{q}}$$

Foothold ideas: Capacitors

$$DV = EDx = Ed$$

$$E = 4pk_C s = 4pk_C \frac{Q}{A} \Rightarrow Q = \left(\frac{A}{4pk_C}\right)E$$

$$Q = \left(\frac{A}{4pk_C d}\right)DV$$

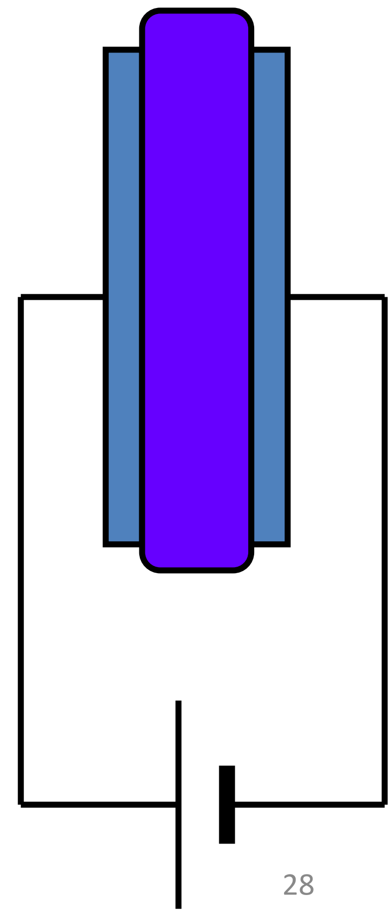
$$Q = CDV$$

What does this "Q" stand for?

$$C = \frac{ke_0 A}{d}$$

If plates are separated by a material

$$\text{Energy stored } = \tfrac{1}{2} Q DV$$



Units

- Gravitational field
  units of g = Newtons/kg
- Electric field
  units of E = Newtons/C
- Electric potential
  units of V = Joules/C = Volts
- Energy = qV so $e\Delta V$ = the energy gained by an electron (charge $e = 1.6 \times 10^{-19}$ C) in moving through a change of ΔV volts.
  1 eV = 1.6×10^{-19} J