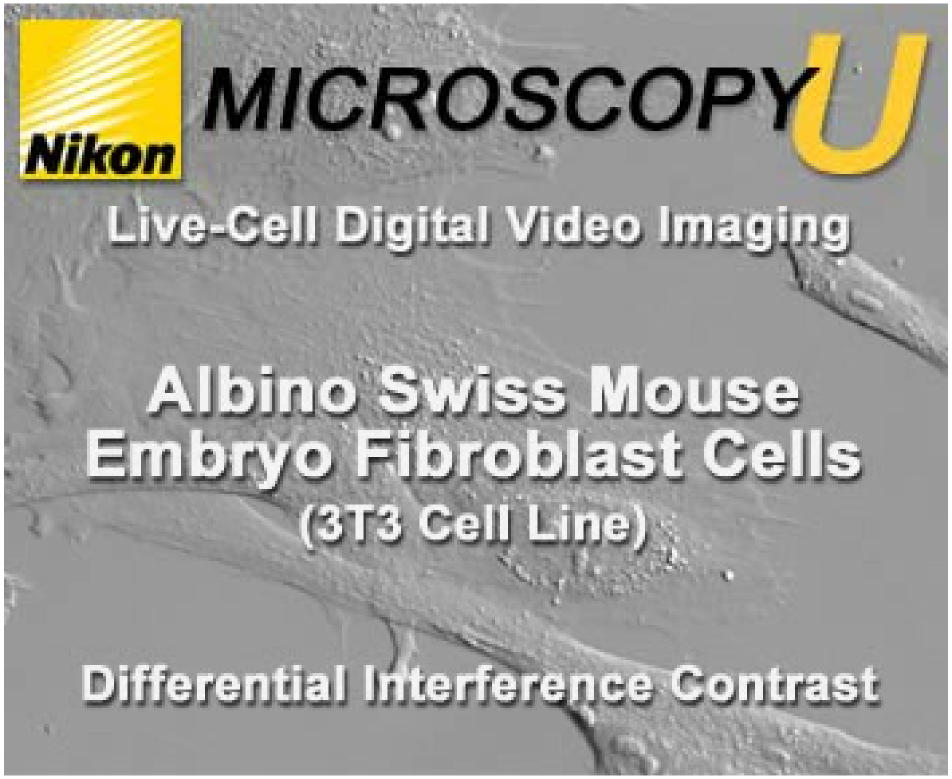
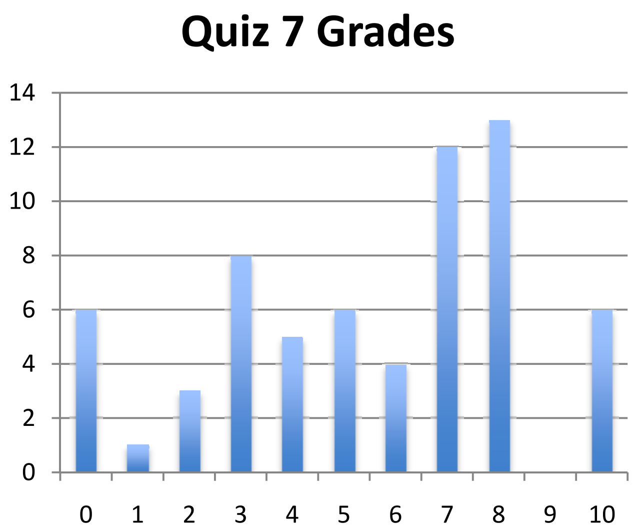
Physics 131- Fundamentals of Physics for Biologists I

Professor: Arpita Upadhyaya



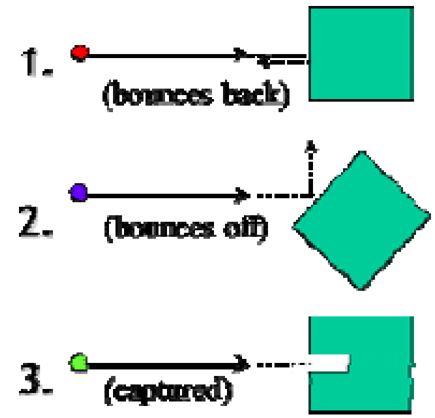
- Quiz 7 (review)
- Random Motion
- Diffusion
- Kinetic Theory
- Fluids

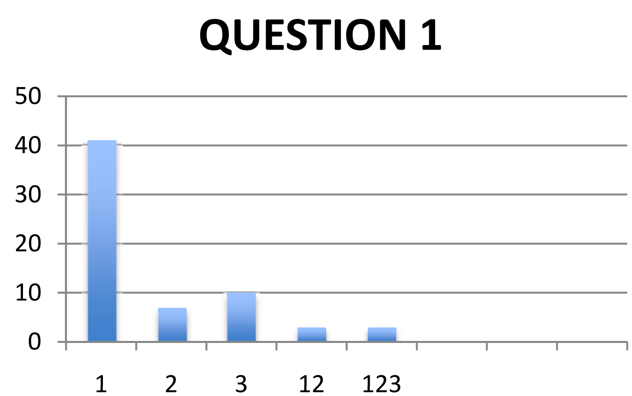
Quiz 7



Average = 5.5

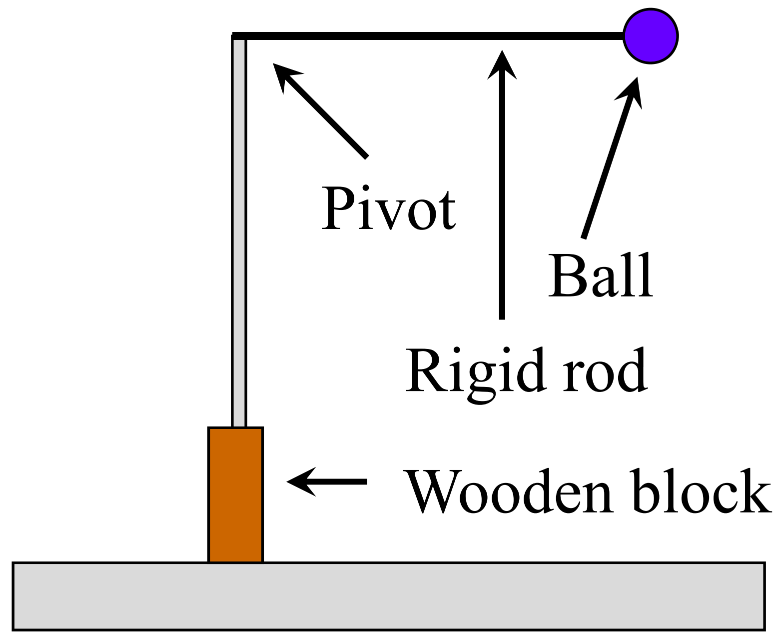
1. (2 pts) Three balls with the same mass on a table slide and hit different block. Which ball (1,2 or 3) exerts the most impulse on the block?



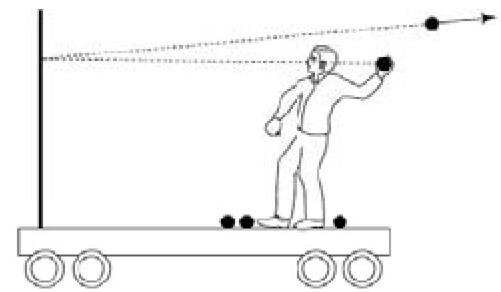


Which ball will knock the block over?

1. A superball
2. A clay ball of equal mass
3. Both
4. Neither



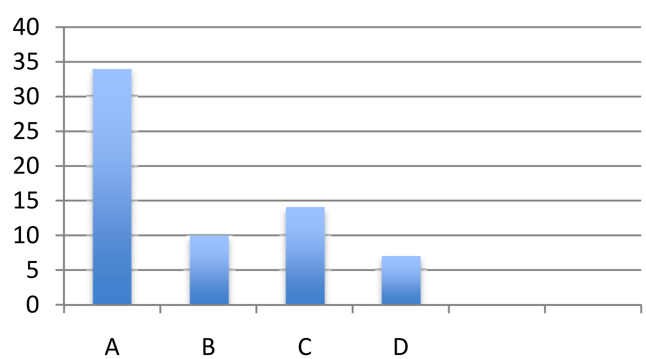
2. (2 pts) Suppose you are on a cart, initially at rest on a track with negligible friction. You throw balls at a partition that is rigidly mounted on the cart. The balls bounce straight back as shown in the figure.



Is the cart put in motion as a result?

A. Yes. Towards the left
B. Yes. Towards the right.
C. No.
D. You are not given enough information to decide.

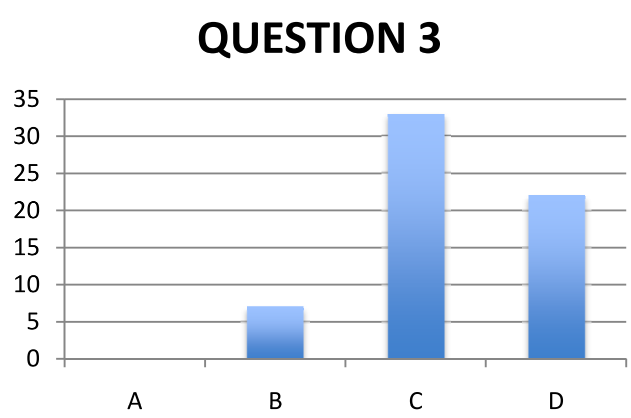
QUESTION 2



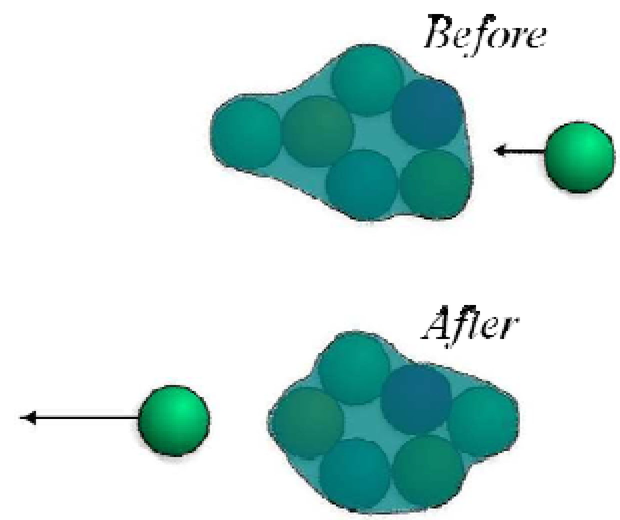
3. (3 pts) Consider two carts, of masses m and 2m, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the momentum of the light cart is

A. four times
B. twice
C. equal to
D. one-half
E. one-quarter

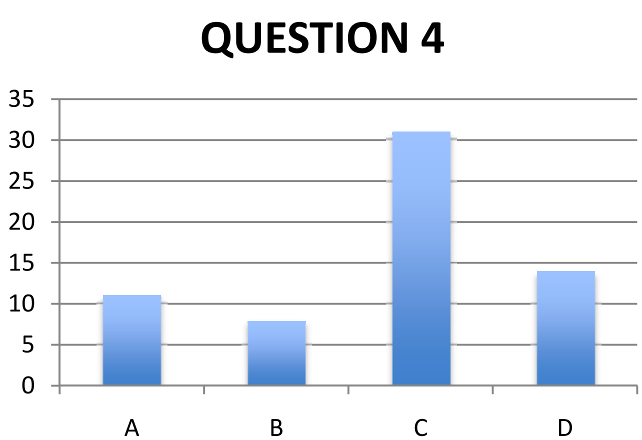
the momentum of the heavy cart.



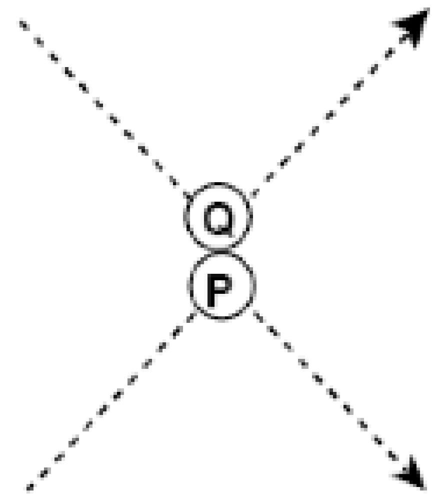
4. (3 pts) A molecular cluster at rest collides with an atom. As a result, the atom becomes strongly bound to the cluster and an identical atom (from a different part of the molecule) gets shot off with much higher speed. What can you say about the motion of the reformed cluster after the collision?



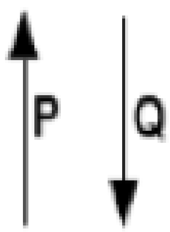
A. It will be stationary.
B. It will move to the left.
C. It will move to the right.
D. This is not really possible, despite the claim that it is.
E. You can't say anything about it from the information given.
F. Something else.



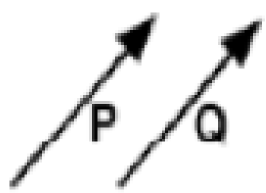
The diagram at the right depicts the path of two colliding steel balls rolling on a table. Which set of arrows best represents the direction of the change in momentum of each ball?



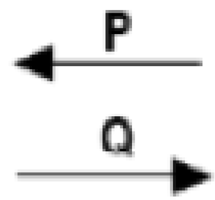
(1)



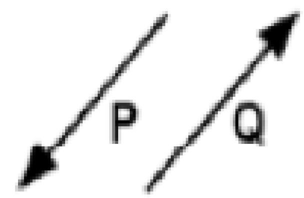
(2)



(3)



(4)



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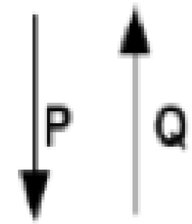
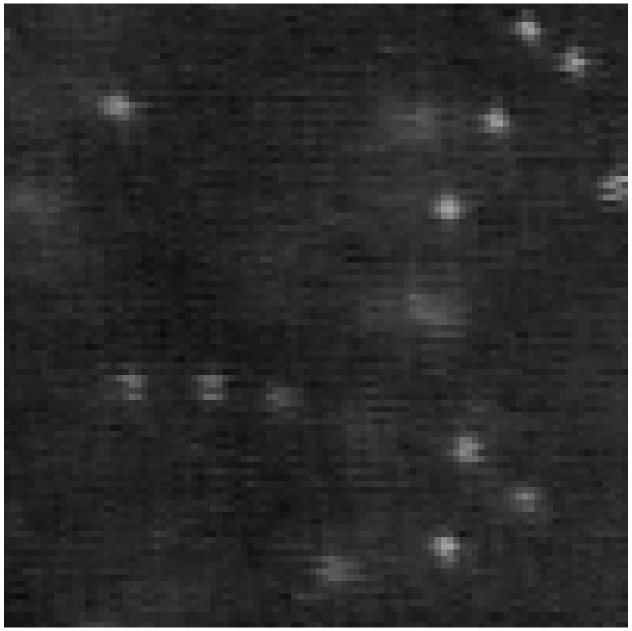
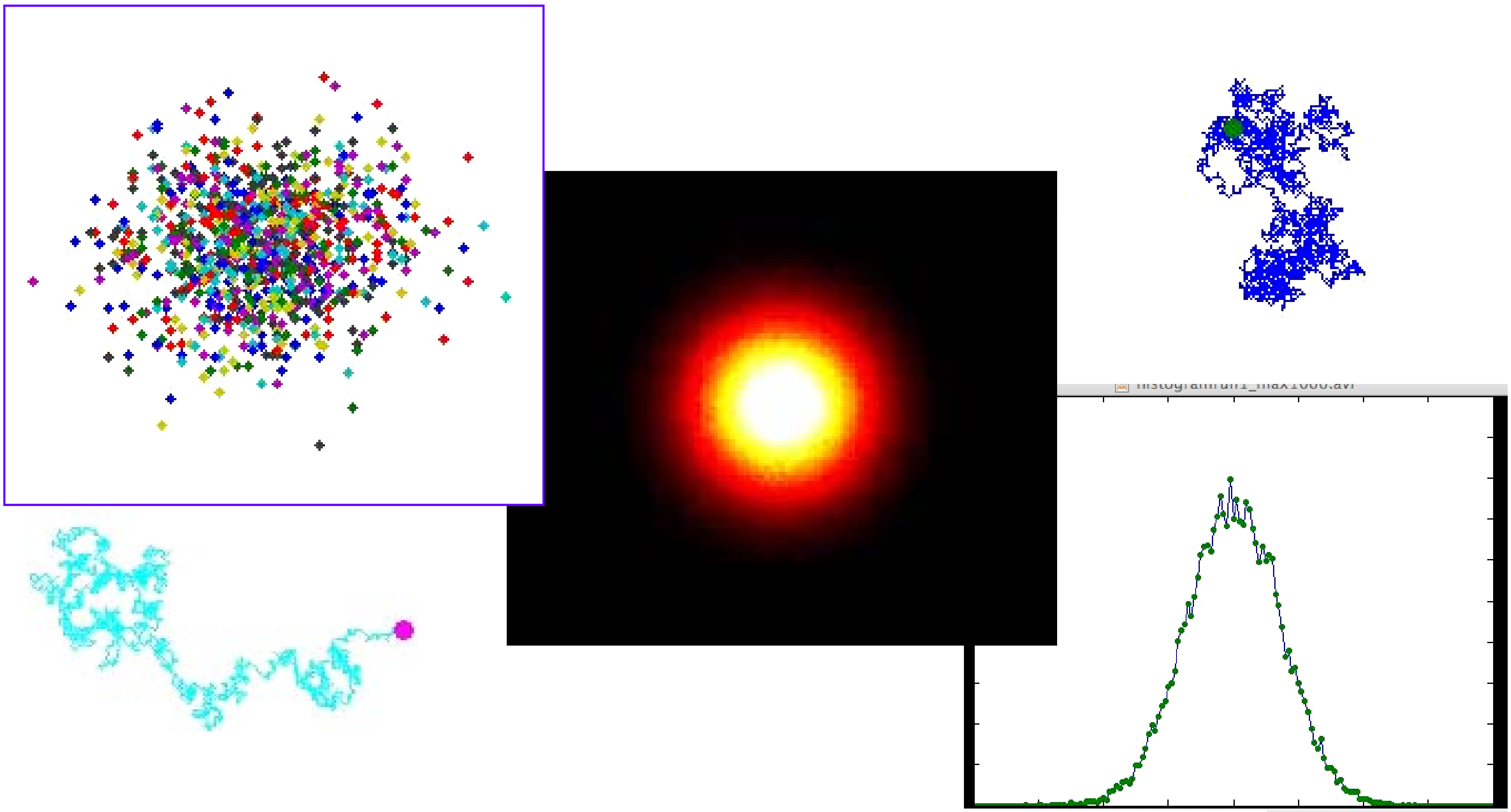




Random Motion

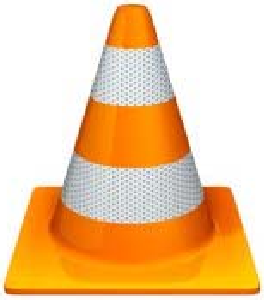
2D Simulations: Multiple representations



Simulations by Alex Morozov & Kerstin Nordstrom

2D Simulations: Multiple representations

1. Watch all the particles.
2. Look at the density of the particles
   – What do the colors represent?
3. Look at a plot of the density along a slice through the middle.
   – What it will look like and what it will do.
4. Look at the motion of individual particles.

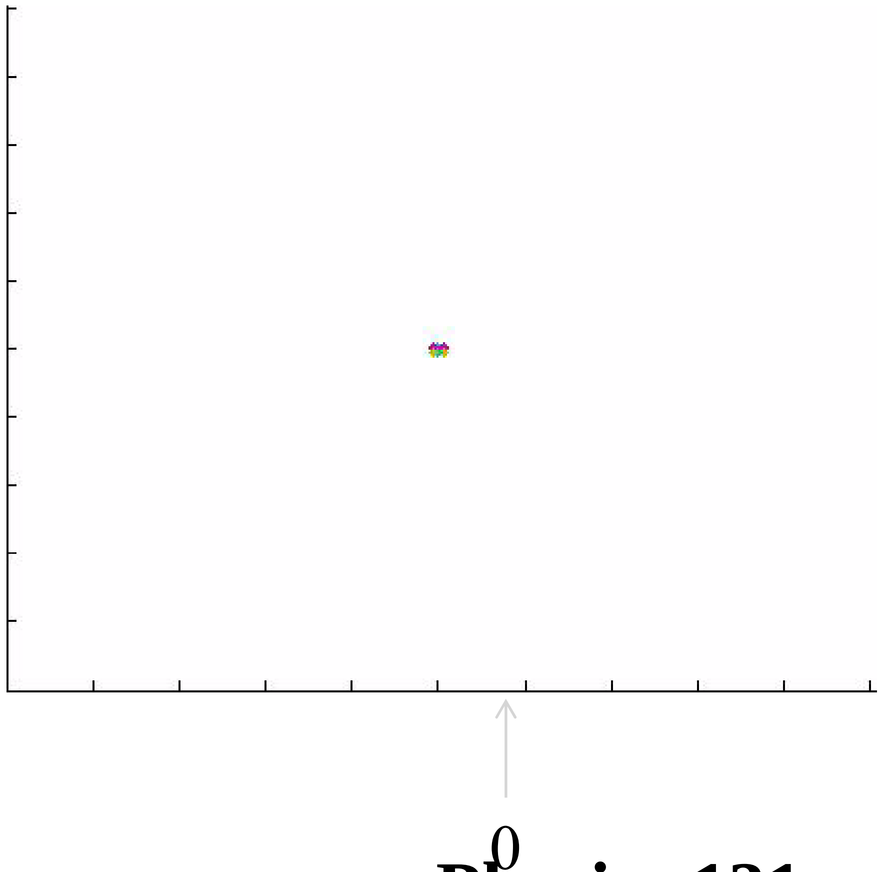


Start 200 random walkers in two dimension near 0

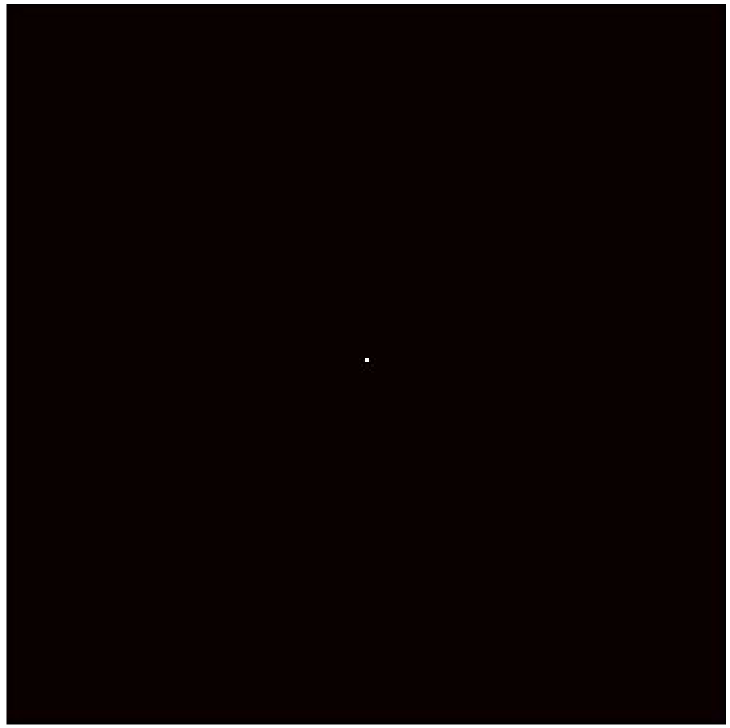
Sketch what you expect to see &answer clicker question!

Whiteboard, TA & LA

1. Particles will form a “wave” – a ragged ring of particles moving outward.
2. They will be mostly stay near 0 no matter how long you wait.
3. Many particles will remain near zero, but they will gradually spread out.

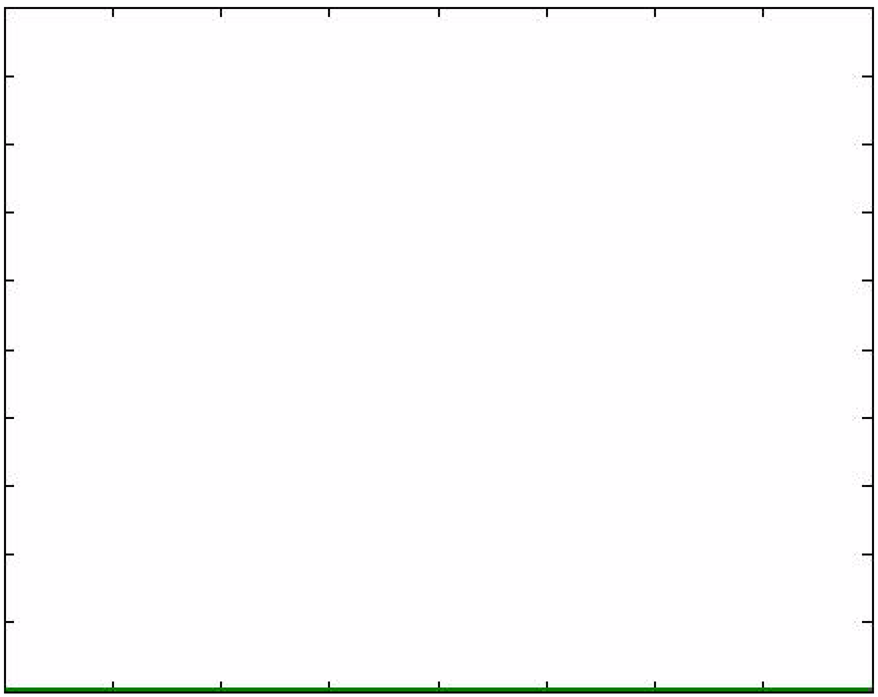


A simulation of many random walkers



Alex Morozov &
Kerstin Nordstrom

Density as a function of position



Gaussian Distribution

Measure width at half the peak height. Measure the full width.

Since we only have a fixed number of walkers, the line is choppy

The variability is equal to
e.g. if on average we have 100 particles in a bin, variability will be 10

Units analysis

- Think about what particles are doing. What do you think *D* depends on?
- Units may help you a lot here. [*D*] = L/T^2.
- Think about the "speed" of diffusion: what variables (position, velocity, ease of motion) would change the time scale? (Make diffusion faster, e.g.)

$$\left\langle (\Delta r)^2 \right\rangle = 6Dt$$

Whiteboard, TA & LA

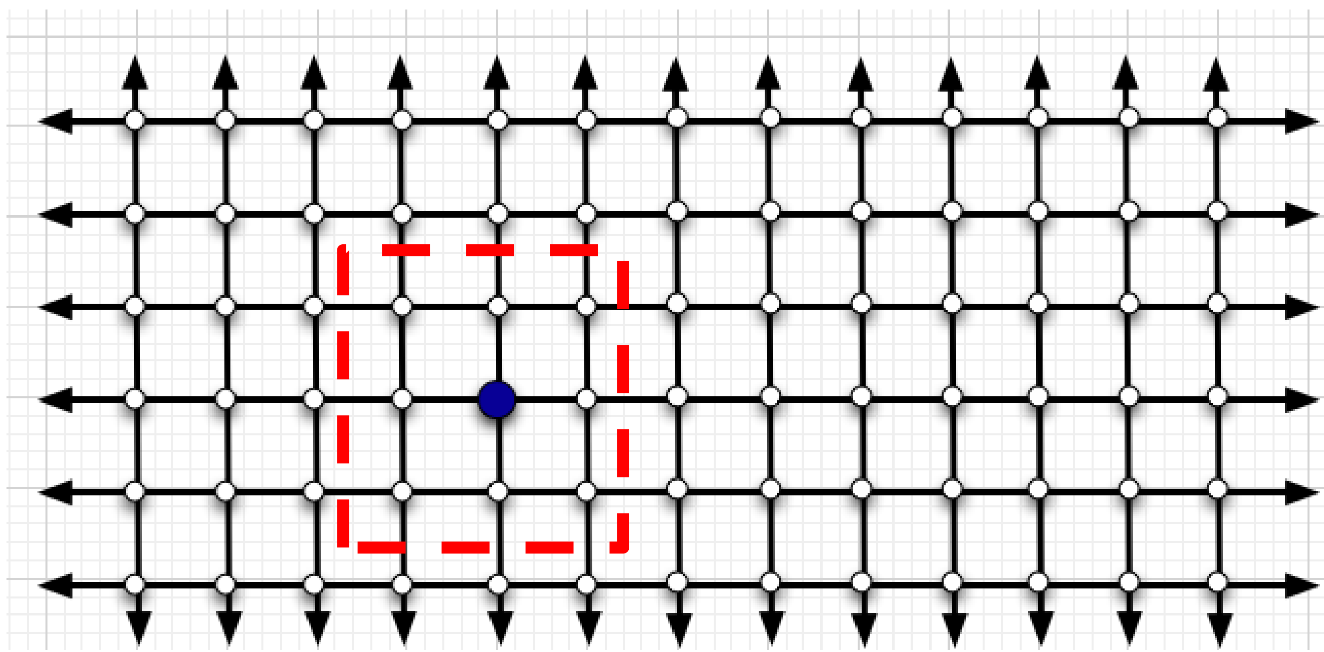
I wait a certain amount of time, *t,* and get a distribution with width associated with Δx. If I wait four times longer, the width of the distribution increases by what factor?

A. 1
B. 2
C. ½
D. 4
E. 1/4

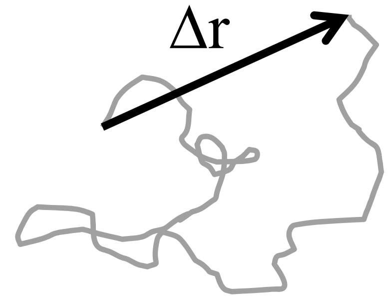
Trajectory of individual particles

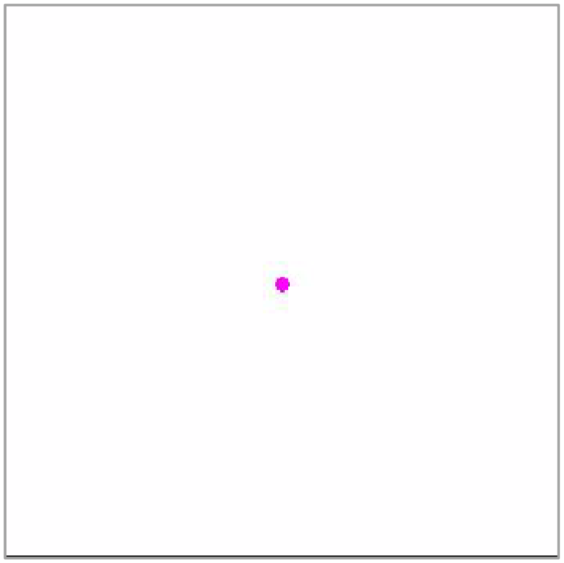
Compared to a 1D random walk if the walker can in addition also take a step in the second dimension following the same rules, the following is true about the Diffusion Constant D and the distance squared traveled $<\Delta r^2>$

1. $<\Delta r^2>$ is larger by factor 2
2. $<\Delta r^2>$ is larger by factor $\sqrt{2}$
3. $<\Delta r^2>$ is the same
4. $<\Delta r^2>$ is smaller by factor $\sqrt{2}$
5. $<\Delta r^2>$ is smaller by factor 2



Random Motion in two dimensions





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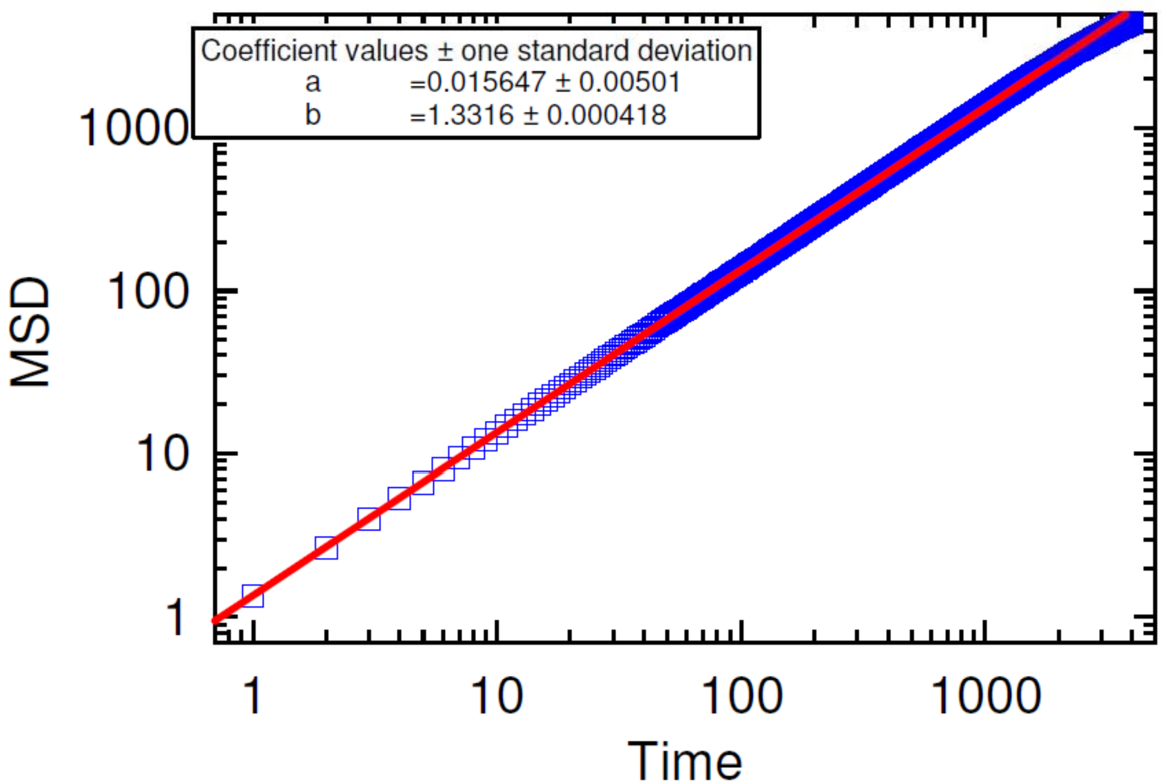
If I wait four times as long, the trajectory is on average longer by a factor ___?

If I wait four times as long, the distance between start and end point Δr is on average longer by a factor _____?

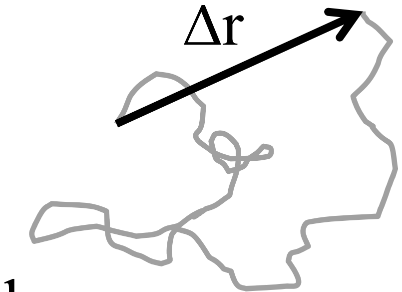
$$\left\langle \left(\Delta r\right)^2 \right\rangle = 4D\Delta t$$

D is called *the* ***diffusion constant*** and has dimensionality [*D*] = L^2/T



Coefficient values ± one standard deviation
a =0.015647 ± 0.00501
b =1.3316 ± 0.000418
1000
100
10
1
MSD
1
10
100
1000
Time

Random walk in 2D



- As a result of random motion, an initially localized distribution will spread out, getting wider and wider. This phenomenon is called *diffusion*
- The square of the average distance traveled during random motion will grow with time.

- 1 Dimension $$\left\langle (\Delta x)^2 \right\rangle = 2D\Delta t$$
- 2 Dimensions $$\left\langle (\Delta r)^2 \right\rangle = \left\langle (\Delta x)^2 + (\Delta y)^2 \right\rangle = 4D\Delta t$$
- 3 Dimensions $$\left\langle (\Delta r)^2 \right\rangle = \left\langle (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \right\rangle = 6D\Delta t$$

- *D* is called *the diffusion constant* and has dimensionality $[D] = L^2/T$

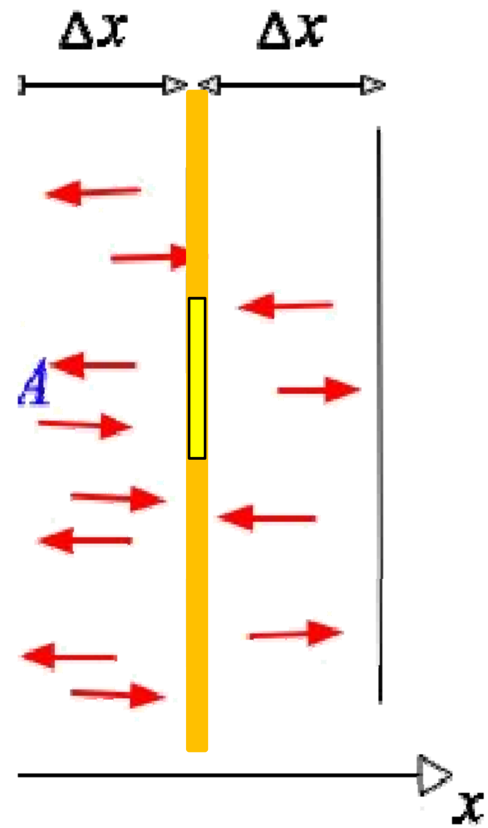
For what biological systems or situations might a 1D, 2D, or 3D random model of diffusion be relevant?

Whiteboard, TA & LA

Sodium ions are at different densities on the inside and outside of a cell. Assume each ion moves randomly as a result of collisions with other atoms and molecules.

A small patch of membrane (area A) is shown in yellow. There are more ions on the left than on the right.

What do you expect is true about the ions on the left side of the membrane?

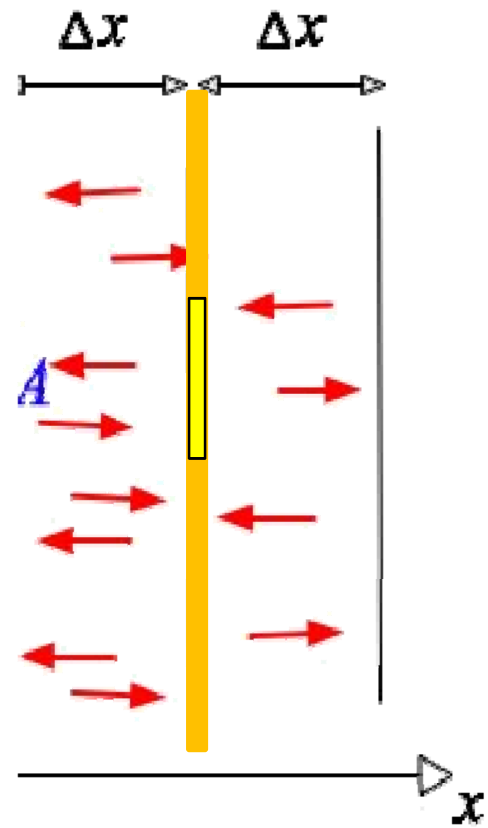


A. More go to the right
B. More go to the left
C. Equal amount goes left and right
D. There is not enough information to tell

Sodium ions are at different densities on the inside and outside of a cell. Assume each ion moves randomly as a result of collisions with other atoms and molecules.

A small patch of membrane (area A) is shown in yellow. There are more ions are on the left than on the right.

What do you expect is true about the ions on the **right side** of the membrane?



A. More go to the right

B. More go to the left

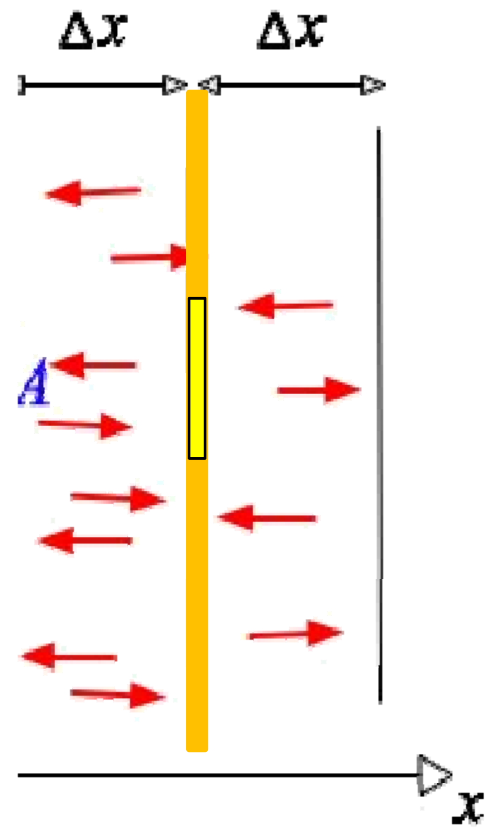
C. Equal amount goes left and right

D. There is not enough information to tell

Sodium ions are at different densities on the inside and outside of a cell. Assume each ion moves randomly as a result of collisions with other atoms and molecules.

A small patch of membrane (area A) is shown in yellow. There are more ions are on the left than on the right.

If the membrane allows ions to pass through what do you expect will be true?



A. There'll be a net flow of ions to the right

B. There'll be a net flow of ions to the left

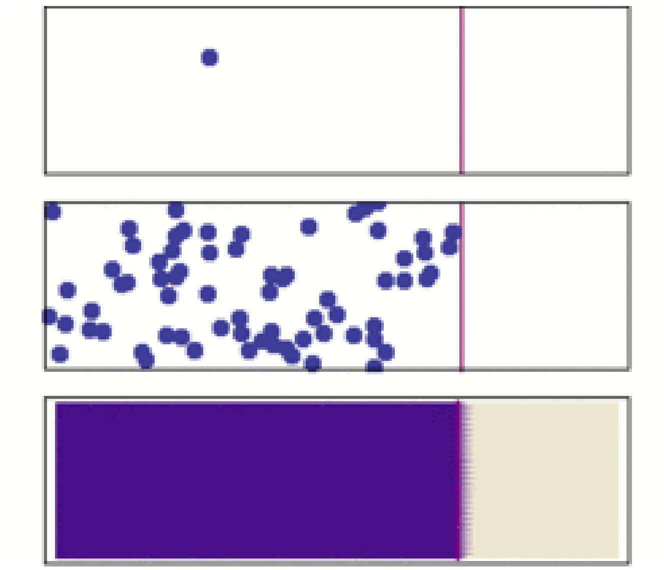
C. There'll be no net flow. Equal amounts will go left and right.

D. There is not enough information to tell

Foothold principles: Randomness

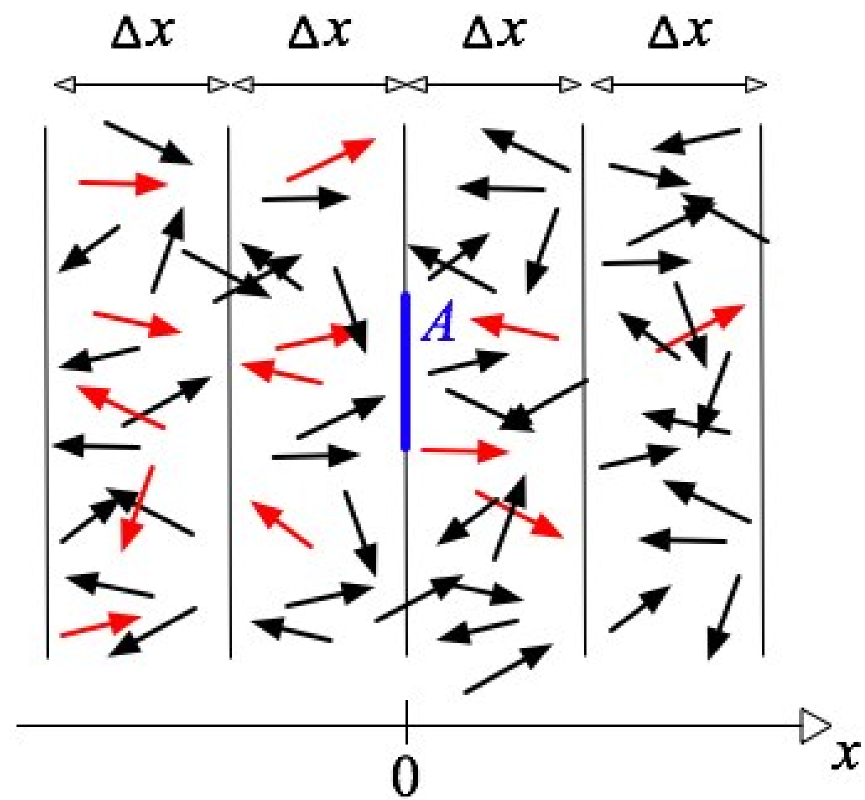
- Matter is made of molecules in constant motion and interaction. This moves things around.
- If the **distribution** of a chemical is non-uniform, the randomness of molecular motion will tend to result in molecules moving from more dense regions to less.
- This is **not** directed! It is an **emergent** phenomenon arising from the combination of random motion and non-uniform concentration.

Diffusion (simulation)

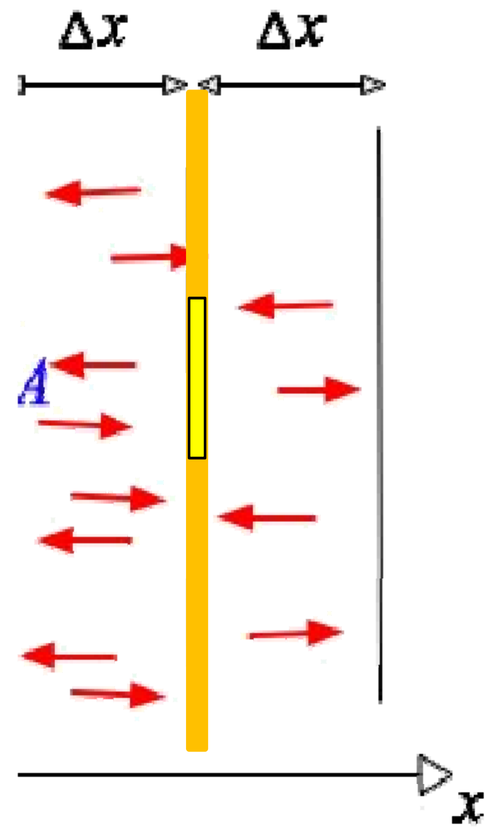


Diffusion:
Fick's law (1D analysis)

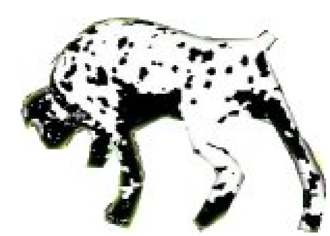
- Uniform fluid (black) containing (red) molecules with a varying concentration.
- Fluid molecules jiggle the (red) molecules around.



Fick's law



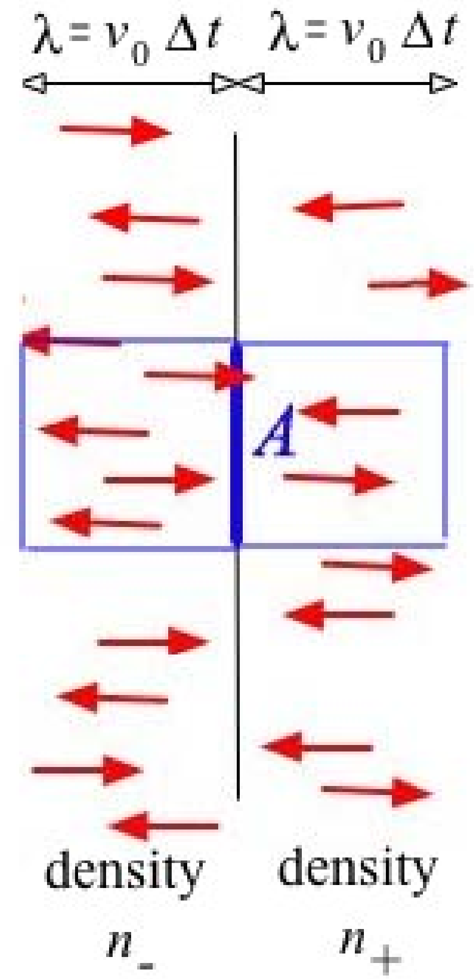
- What can the total flow depend on?
- It depends on the difference between the concentration on either side of the barrier: *dn/dx;* what else?
- The average speed of the particles $\langle v_0 \rangle$ which we'll just call v_0
- How far they get before hitting other particles λ ("mean free path")
- The size of the opening *A*



Whiteboard, TA & LA

Fick's law

- *J*: "flux" = number of particles **per unit area per unit time;**
- Total flow is flux times area times amount of time
- Break up our volume into equal "bins" of width "mean free path"
- $\Delta n/\Delta x = n_+ - n_-$, $\langle v_0 \rangle$, λ, A, Δt



How many cross A in a time Δt?

- **Total** number hitting A from left bin

$$\frac{1}{2}(n_-)Volume = \frac{1}{2}(n_-)A\lambda$$

- …from right…

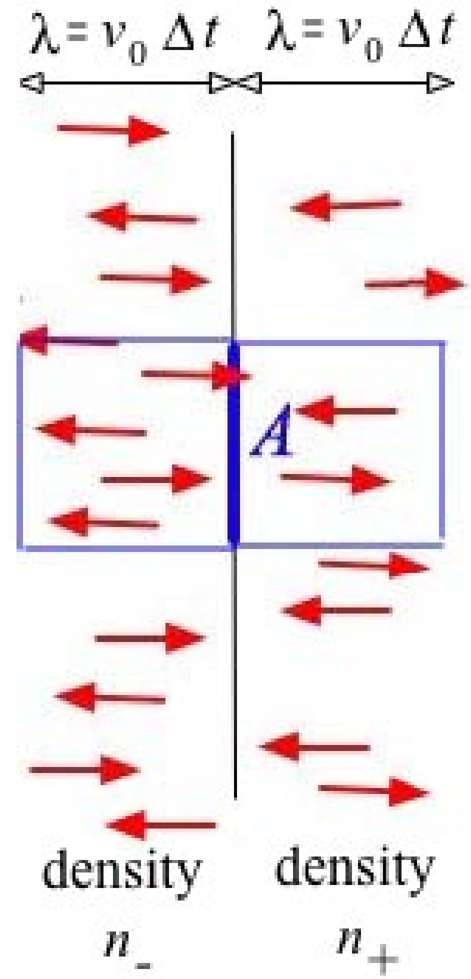
$$\frac{1}{2}(n_+)A\lambda$$

- Net flow across A in the $+dx$ direction

$$\frac{1}{2}(n_- - n_+)A\lambda = -\frac{1}{2}\left(\frac{\Delta n}{\Delta x}\Delta x\right)A\lambda$$

- So the total flow in Δt becomes:

$$JA\Delta t = -\frac{1}{2}\frac{\Delta n}{\Delta x}\frac{\Delta x}{\Delta t}A\lambda\Delta t$$



Whiteboard, TA & LA

Fick's law

- 1D result

$$JA\Delta t = -\frac{1}{2}\frac{dn}{dx}\frac{dx}{dt}A\lambda\Delta t$$

$$J = -\frac{1}{2}\frac{dn}{dx}v_0\lambda$$

$$\boxed{J = -D\frac{dn}{dx}} \qquad D = \frac{1}{2}\lambda v_0$$

Not the specific trajectory for individual molecules

Tells us how a *collection* of molecules distributes based on "mean free path" and average speed

Fick's law

- 1D result

$$J = -D\frac{dn}{dx} \quad D = \tfrac{1}{2}\lambda v_0$$

- For all directions (not just 1D) Fick's law becomes

$$\vec{J} = -D\vec{\nabla}n$$

The gradient

- If we want to take the derivative of a function of one variable, $y = df/dx$, it's straightforward.
- If we have a function of three variables – $f(x,y,z)$ – what do we do?
- The gradient is the **vector derivative.**
  To get it at a point (x,y,z)
  - Find the direction in which f is changing the fastest.
  - Take the derivative by looking at the rate of change in that direction.
  - Put a vector in that direction with its magnitude equal to the <u>maximum</u> rate of change.
  - The result is the vector called $\vec{\nabla} f$