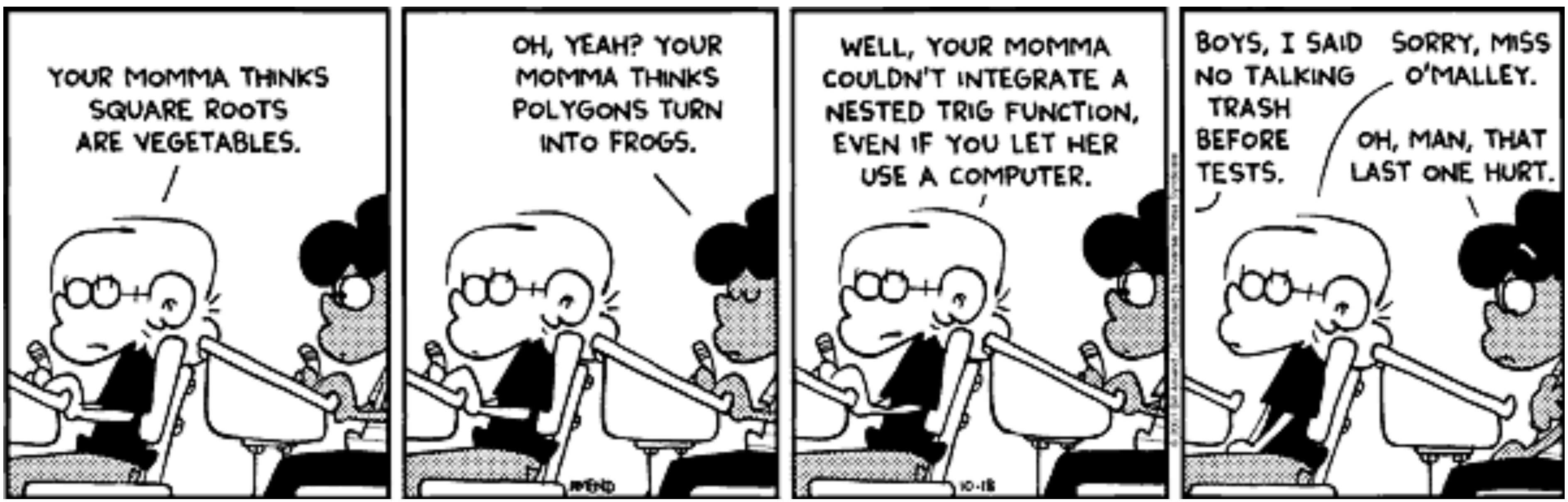
- **Theme Music:** **Take the A Train**
  Duke Ellington
- **Cartoon:** **FoxTrot**
  Bill Amend



Description of motion

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} = \frac{\text{vector displacement}}{\text{time it took to do it}}$$

$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time it took to do it}}$$

Laws that control motion

Newton 0:
An object responds to the forces it feels when it feels them.

Newton 1:
An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).

Newton 2:
An object that is acted upon by other objects changes its velocity according to the rule

$$\vec{a}_A = \vec{F}_A^{net} / m_A$$

Newton 3:
When two objects interact the forces they exert on each other are equal and opposite.

$$\vec{F}_{A\to B}^{type} = -\vec{F}_{B\to A}^{type}$$

Forces

Forces: how objects interact with each other to try to change each other's velocity.

Notation convention.

$$\vec{F}_{\text{(object causing force)}\to\text{(object feeling force)}}^{\text{type of force}}$$

Types of forces

Spring, Normal, Tension Force N, T

$$T = k\Delta L$$

Friction Force f

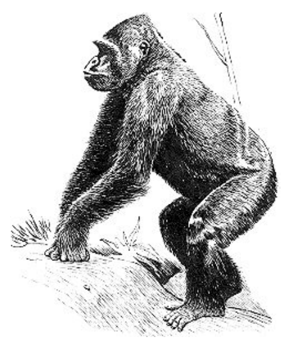
$$f \le \mu N$$

Weight Force W

$$\vec{W} = m\vec{g}$$

Electric Force

$$F_{q\to Q}^{E} = \frac{k_C qQ}{r_{qQ}^2}$$



Using them in problems

1. What objects are you interested in looking at the motion (or lack of motion) of?

2. What other objects interact with those objects?
(A System Schema might help)

3. For each object, isolate the object and see what forces act on it.
(A Free Body Diagram might help)

4. Write a Newton's second law equation for each of the objects you are considering.

5. Put in what you know about each of the forces.

Your resulting equations tell you about relations among the various variables of interest in the problem.

6. Decide what you know and what you want to find out.

7. See if your equations will let you determine the answers.

Foothold ideas: Resistive forces

- Resistive forces are contact forces acting between two touching surfaces that are parallel to the surface and tend to oppose the surfaces from sliding over each other.
- There are three types:
  - – Friction (independent of velocity)
  - – Viscosity (proportion to velocity)
  - – Drag (proportional to the square of velocity)

Foothold Ideas: Friction

- Friction is our name for the interaction between two touching surfaces that is parallel to the surface.
- It acts to oppose the <u>relative motion</u> of the surfaces. It acts as if the two surfaces stick together a bit.
- Normal forces adjust themselves in response to external forces. So does friction – up to a point.

Static

$$f_{A\to B} \le f_{A\to B}^{\max} = \mu_{AB}^{static} N_{A\to B}$$

Sliding

$$f_{A\to B} = \mu_{AB}^{kinetic} N_{A\to B}$$

$$\mu_{AB}^{kinetic} \le \mu_{AB}^{static}$$

- Friction is independent of velocity and only depends on how hard the two surfaces are being squeezed together.
- Friction can oppose motion or cause it.

Foothold Ideas: Gravity

- Every object (near the surface of the earth) feels a downward pull proportional to its mass:

$$\vec{W}_{E\rightarrow m} = m\vec{g}$$

What object causes W?

where $\vec{g}$ is referred to as *the gravitational field.*

- This is a Force even though nothing touching the object is responsible for it.
- The gravitational field has the same magnitude for all objects irrespective of their motion and at all points.
- The gravitational field always points down.
- It is measured to be $g \approx 9.8\ \mathrm{N/kg}$

Why N/kg instead of m/s^2?

Response to Gravity: Free Fall

- After an object has been released,
  - if it is dense enough so the forces from the air can be ignored
  - if nothing else is touching it

  the only force acting on it is gravity.
- The force of gravity is proportional to the mass.

$$\vec{a} = \vec{F}^{net} / m = \vec{W}_{E \to m} / m = m\vec{g} / m = \vec{g}$$

Model: Charge
A hidden property of matter

- Matter is made up of two kinds of electric matter (positive and negative) that have equal magnitude and that cancel when they are together and hide matter's electrical nature.
- Matter with an equal balance is called <u>neutral</u>.
- Like charges repel, unlike charges attract.
- The algebraic sum of postive and negative charges is a constant (i.e, N_{+}- N_{-} = const.)

Electric forces: Foothold ideas (basic)

- There are two kinds of charges: + and -.
- Charges of the same type repel each other.
- Charges of different types attract each other.
- The force between charges gets stronger as they get closer, weaker as they get farther away.
- The electric force satisfies Newton's 3rd law.

Quantifying Charge

- Need an operational definition.
- Charge is a new kind of quantity
  (to M, L, T, add Q).
- Choose our scale:
  A small object has a charge of 1 C (= 1 Coulomb) if two identical such charges held at a distance of 1 m exert forces of 9×10^9 N on each other.

- [This corresponds to choosing the constant
  $k_C = 9 \times 10^9$ N-m^2/C^2.]

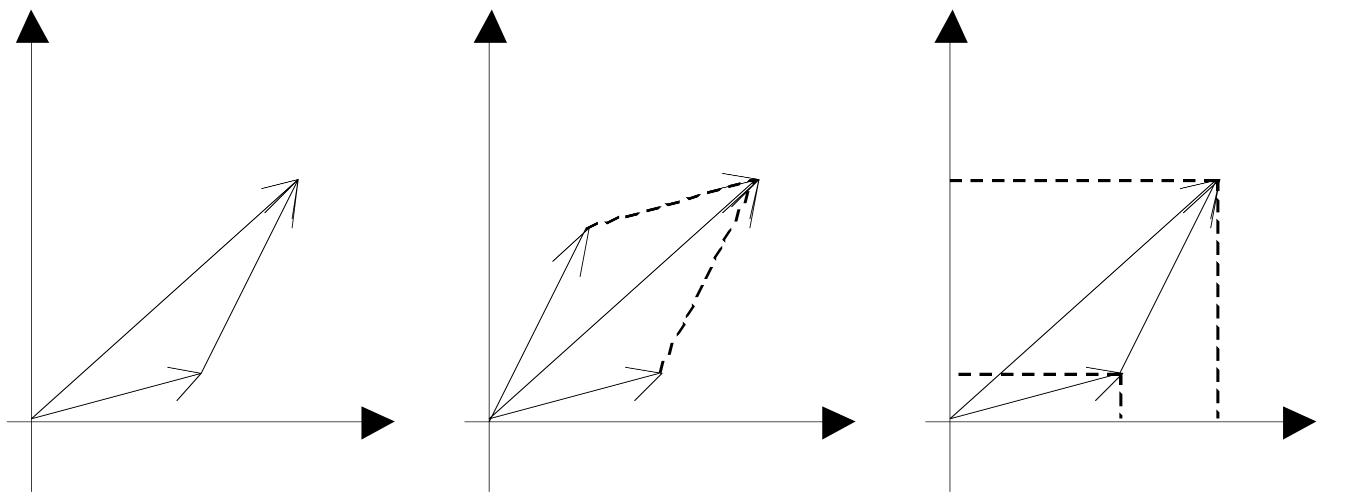
Review of Vectors (2-dimensional coordinates)

- We have 2 directions to specify. We must
  - Choose a reference point (origin)
  - Pick 2 perpendicular axes (x and y)
  - Choose a scale
- We specify our x and y directions by drawing little arrows of unit length in their positive direction. $\hat{i},\ \hat{j}$
- A force vector is written

$$\vec{F} = F_x\hat{i} + F_y\hat{j} = \left(F_x, F_y\right)$$

Adding Vectors: Methods

- There are 3 mathematical ways to add vectors



head to tail | parallelogram rule | add components (may use trig)

Foothold idea: Coulomb's Law

- Point charges attract each other with a force whose magnitude is given by

$$\vec{F}_{q\to Q} = -\vec{F}_{Q\to q} = \frac{k_C qQ}{r_{qQ}^2}\hat{r}_{q\to Q}$$

- k_C is put in to make the units come out right.

$$k_C = 9\times 10^9 \text{ N-m}^2/\text{C}^2$$

Adding forces for many charges!

$$\vec{F}_q = \vec{F}_{Q_1 \to q} + \vec{F}_{Q_2 \to q} + \vec{F}_{Q_3 \to q} + \vec{F}_{Q_4 \to q} + \ldots$$

$$\vec{F}_q = \frac{k_C q Q_1}{r_1^2}\widehat{r}_1 + \frac{k_C q Q_2}{r_2^2}\widehat{r}_2 + \frac{k_C q Q_3}{r_3^2}\widehat{r}_3 + \frac{k_C q Q_4}{r_4^2}\widehat{r}_4 + \ldots$$

where

$r_1 =$ distance from Q_1 to q

$r_2 =$ distance from Q_2 to q

...

$\widehat{r}_1 =$ direction from Q_1 to q (mag. 1, no units!)

$\widehat{r}_2 =$ direction from Q_2 to q (mag. 1, no units!)

Foothold ideas: Momentum

- We define the <u>momentum</u> of an object, A:

$$\vec{p}_A = m_A \vec{v}_A$$

- This is a way of defining "the amount of motion" an object has.
- Our "delta" form of N2 becomes

$$\left\langle \vec{F}_A^{net} \right\rangle = m_A \frac{\Delta \vec{v}_A}{\Delta t} = m_A \left\langle \vec{a}_A \right\rangle$$

which we can rewrite as

$$\left\langle \vec{F}_A^{net} \right\rangle = \frac{\Delta\left(m_A \vec{v}_A\right)}{\Delta t} = \frac{\Delta \vec{p}_A}{\Delta t}$$

Foothold idea:
The Impulse-Momentum Theorem

- Newton 2

$$\vec{a}_A = \vec{F}_A^{net} \Big/ m_A$$

- Put in definition of *a*

$$\frac{d\vec{v}_A}{dt} = \frac{\vec{F}_A^{net}}{m_A}$$

- Multiply up by Δt

$$m_A \Delta\vec{v}_A = \left\langle \vec{F}_A^{net} \right\rangle \Delta t$$

- Define Impulse

$$\vec{\mathscr{I}}_A^{net} = \left\langle \vec{F}_A^{net} \right\rangle \Delta t$$

- Combine to get Impulse-Momentum Theorem for any object A

$$\Delta\vec{p}_A = \vec{\mathscr{I}}_A^{net}$$

Foothold idea:
Momentum Conservation: 1

- If two objects, A and B, interact with each other and with other ("external") objects, By the IMT

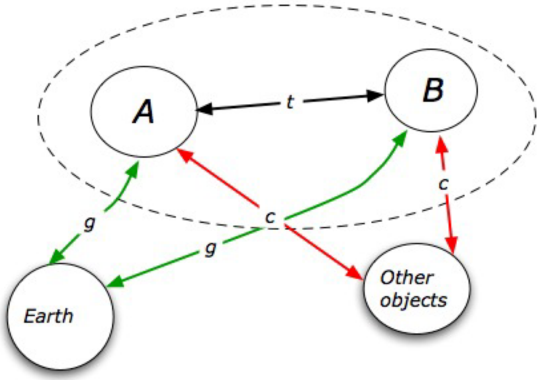
$$m_A\,\Delta\vec{v}_A = (\vec{F}_A^{ext} + \vec{F}_{B\to A})\Delta t$$

$$m_B\,\Delta\vec{v}_B = (\vec{F}_B^{ext} + \vec{F}_{A\to B})\Delta t$$

- Adding:

$$m_A\,\Delta\vec{v}_A + m_B\,\Delta\vec{v}_B = \left[\vec{F}_A^{ext} + \vec{F}_B^{ext} + \left(\vec{F}_{A\to B} + \vec{F}_{B\to A}\right)\right]\Delta t$$

$$\Delta\left(m_A\vec{v}_A + m_B\vec{v}_B\right) = \vec{F}_{AB}^{ext}\,\Delta t$$

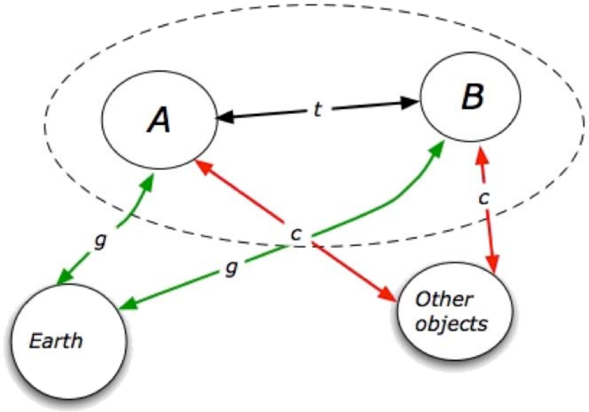


Foothold idea:
Momentum Conservation: 2

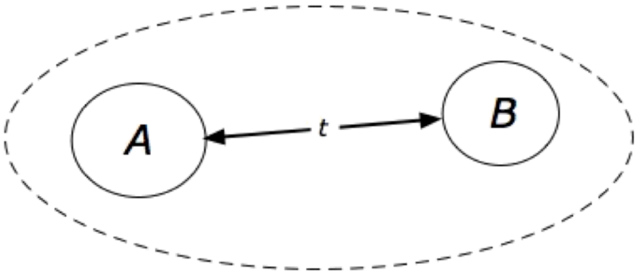
- So: If two objects interact with each other in such a way that the <u>external</u> forces on the pair cancel, then their momentum is conserved.

$$\Delta\left(m_A\vec{v}_A + m_B\vec{v}_B\right) = 0$$

$$m_A\vec{v}_A^{\,i} + m_B\vec{v}_B^{\,i} = m_A\vec{v}_A^{\,f} + m_B\vec{v}_B^{\,f}$$



$\Rightarrow$



Foothold principles: Randomness

- Matter is made of of molecules in constant motion and interaction. This motion moves stuff around.
- A large molecule, small organelle, or even a bit of dust or pollen in the air is continually bombarded from all sides by moving molecules of water or air.
- On the average, they are hit equally from all sides – but since they are not that much bigger than the bombarding molecules sometimes more hit them from one side than the other. The values fluctuate around the averages and these fluctuating imbalances produce a jiggling motion ("Brownian motion")

Foothold principles: Diffusion

- If the distribution of a chemical is non-uniform, the fluctuations in how its molecules are being struck by the molecules of the fluid it is in will tend to result in the chemical moving from more dense regions to less.
- This ***diffusion*** is **not** directed but is an emergent phenomenon arising from the combination of random motion and non-uniform concentration.

Foothold principles: Fick's first Law

- If a set of molecules is not distributed uniformly in 1D (there is a concentration gradient) there will be an effective flow of those molecules according to $J = -D\frac{dn}{dx}$ (or in 3D) $\vec{J} = -D\vec{\nabla}n$
- In a gas, the diffusion constant *D* is given by $\frac{1}{2\sqrt{3}}\lambda\bar{v}$
- In a liquid, the diffusion constant is given by $D = \frac{k_B T}{6\pi\mu R}$

Foothold principles: Fick's second Law

- The average square displacement of a random walking molecule in a thermal bath after a time *t* is given in 3D by Fick's second law:

$$\left\langle \Delta r^2 \right\rangle = \left\langle \Delta x^2 \right\rangle + \left\langle \Delta y^2 \right\rangle + \left\langle \Delta z^2 \right\rangle = 6D\Delta t$$

- The radius of a small blob of chemical in a liquid will grow at this rate.
- The displacement, $\Delta r = \sqrt{\left\langle \Delta r^2 \right\rangle}$, only grows like $\sqrt{\Delta t}$. For larger organisms, this is too slow and is the reason transport systems for air and blood have evolved.

Kinds of Matter

- Classify objects by how they deform.
  - *Solid*: don't change shape if you leave them alone or push on them (not too hard!)
  - *Gel*: look solid if you don't touch them but are "squishy" and change shape easily (jello, butter, clay,…)
  - *Liquid*: Have no shape of their own. Flow to fill a container but have constant volume.
  - *Gas*: Have neither shape nor volume but fill any container.
  - LOTS MORE!

Foothold ideas
Properties of solids

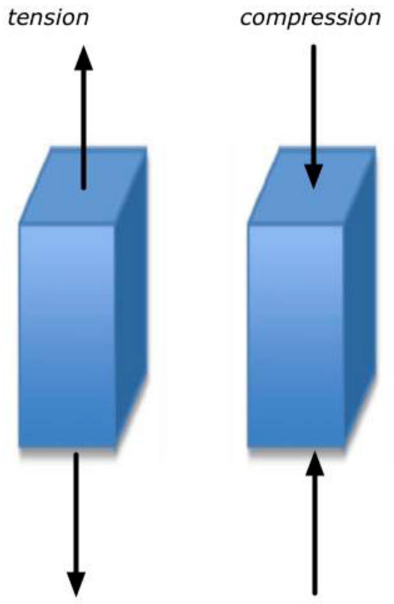
- Density $\rho = \dfrac{M}{V}$
- Stretch and squeeze:

$$F = k\Delta L$$

$$\sigma = F/A \text{ (stress)} \qquad \varepsilon = \Delta L / L_0 \text{ (strain)}$$

$$E = \sigma / \varepsilon \text{ (Young's modulus)}$$

$$k = E\frac{A}{L_0}$$



- Breaking stress

Foothold ideas: Gases – Kinetic Theory I

- We model the gas as lots of tiny little hard spheres far apart (compared to their size) and moving very fast.
- The motions are in all directions and change directions very rapidly. A model saying that on the average the total momentum is 0 (and stays 0 by momentum conservation) is a good one.
- Because there are so many particles and the collisions so sensitive to initial conditions, we can't predict the motion of individual particles for long – but emergent macroscopic averages are very stable.
- Dilute gases satisfy the Ideal Gas Law, $pV = n_{moles}RT$

The Ideal Gas Law

Chemist's form

$$pV = n_{moles}RT$$

$$n_{moles} = \frac{N}{N_A}$$

$$R = k_B N_A$$

Physicist's form

$$pV = Nk_BT$$

$$p = nmv_x^2$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

Foothold ideas: Pressure 1

- In a gas the molecules are moving very fast in all directions. On the average the momentum cancels out.
- If you put in a wall keeping the gas on only one side, only the momentum in one direction acts on the wall (N1, N2, N3), creating a force.
- In a non-flowing gas, the force/area is a constant, the pressure. It is proportional to the number of molecules and their mv^2..

Foothold ideas:
Gases – Kinetic Theory II

- Newton's laws tell us that motion continues forever unless something unbalanced tries to stop it, yet we observe motion always dies away.
- Our model of matter as lots of little particles in continual motion lets us "hide" the energy of motion that has "died away" at the macro level in the internal incoherent motion.
- The model unifies the idea of heat and temperature with our ideas of motion of macroscopic objects.

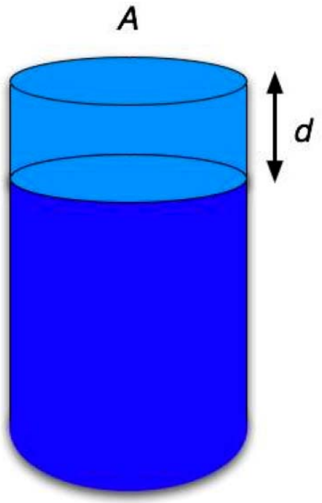
Foothold ideas: Liquids

- In a liquid the molecules are close enoug… that their mutual (short ranged) attractions hold them together (e.g. H-bonding in H_2O).
- A liquid maintains its volume but changes its shape easily in response to small forces.
- The relation of *p*, *V*, and *T* in a liquid is <u>WAY</u> more complicated than in a gas.

Foothold ideas: Pressure 2

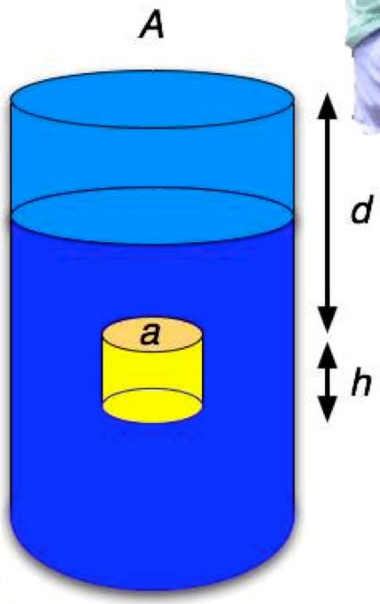
- A constrained fluid has an internal pressure –like an internal force at every point in all directions. (Pressure has no direction.)
- At a boundary or wall, the pressure creates a force perpendicular to the wall. $\vec{F} = p\vec{A}$
- The pressure in a fluid increases with depth.

$$p = p_0 + \rho g d$$



- The pressure in a fluid is the same on any horizontal plane no matter what the shape or openings of the container.

Foothold ideas: Buoyancy



- ***Archimedes' principle***:
  When an object is immersed in a fluid (in gravity), the result of the fluid's pressure variation with depth is an upward force on the object equal to the weight of the water that would have been there if the object were not.
- As a result, an object less dense than the fluid will float, one denser than the fluid will sink.
- An object less dense than the fluid floats with a fraction of its volume under the fluid equal to $\dfrac{\rho_{object}}{\rho_{fluid}}$

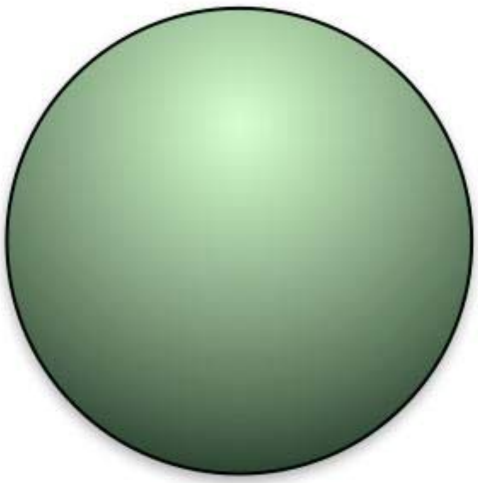
Foothold ideas: Surface tension

- Due to the intermolecular interactions holding a liquid together, the surface of a liquid experiences a tension.
- The pull across any line in the surface of the liquid is proportional to the length of the line.

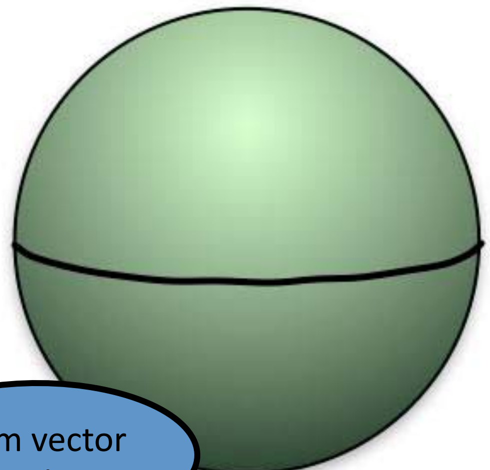
$$F_{\text{surface tension}} = \gamma L$$

Laplace Bubble Law

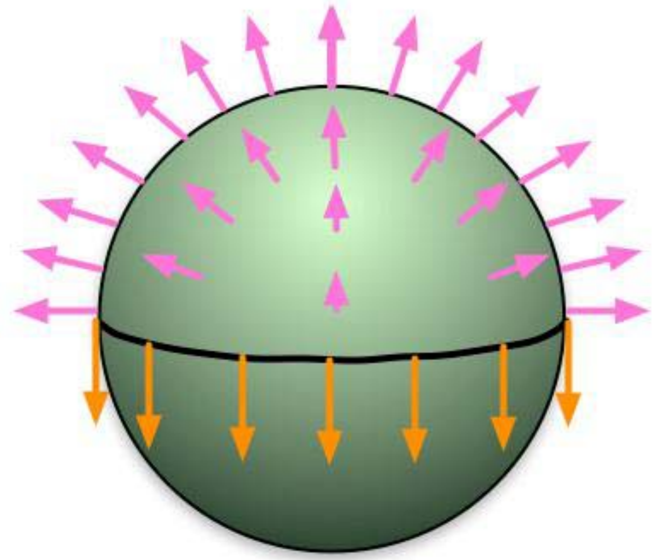
Consider a bubble



Now consider its top half



What forces act on it?



From vector averaging

$$F^{\uparrow}_{\text{air pressure inside}\rightarrow\text{top half}} = \tfrac{1}{2}pA = \tfrac{1}{2}p(2\pi r^2) = \pi p r^2$$

$$F^{\downarrow}_{\text{s.t. of bot half}\rightarrow\text{top half}} = \gamma L = \gamma(2\pi r) = 2\pi\gamma r$$

$$p = \frac{2\gamma}{r}$$

SMALLER bubble has bigger pressure!

Force from pressure inside (up) must cancel pull of surface tension from the bottom half (down)