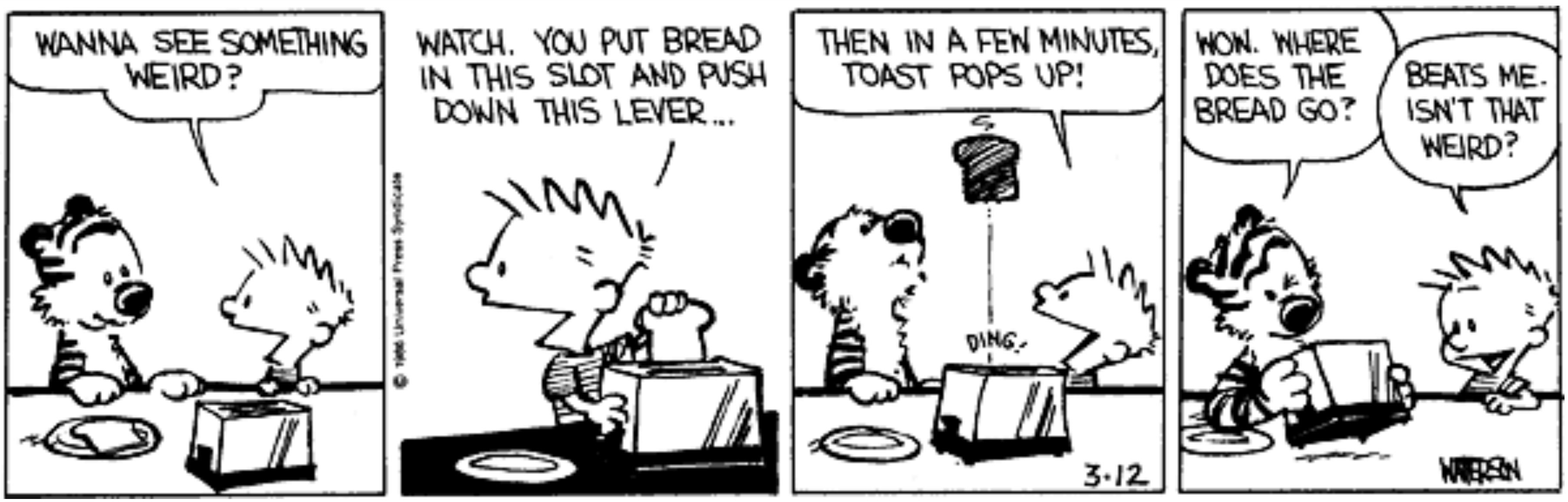
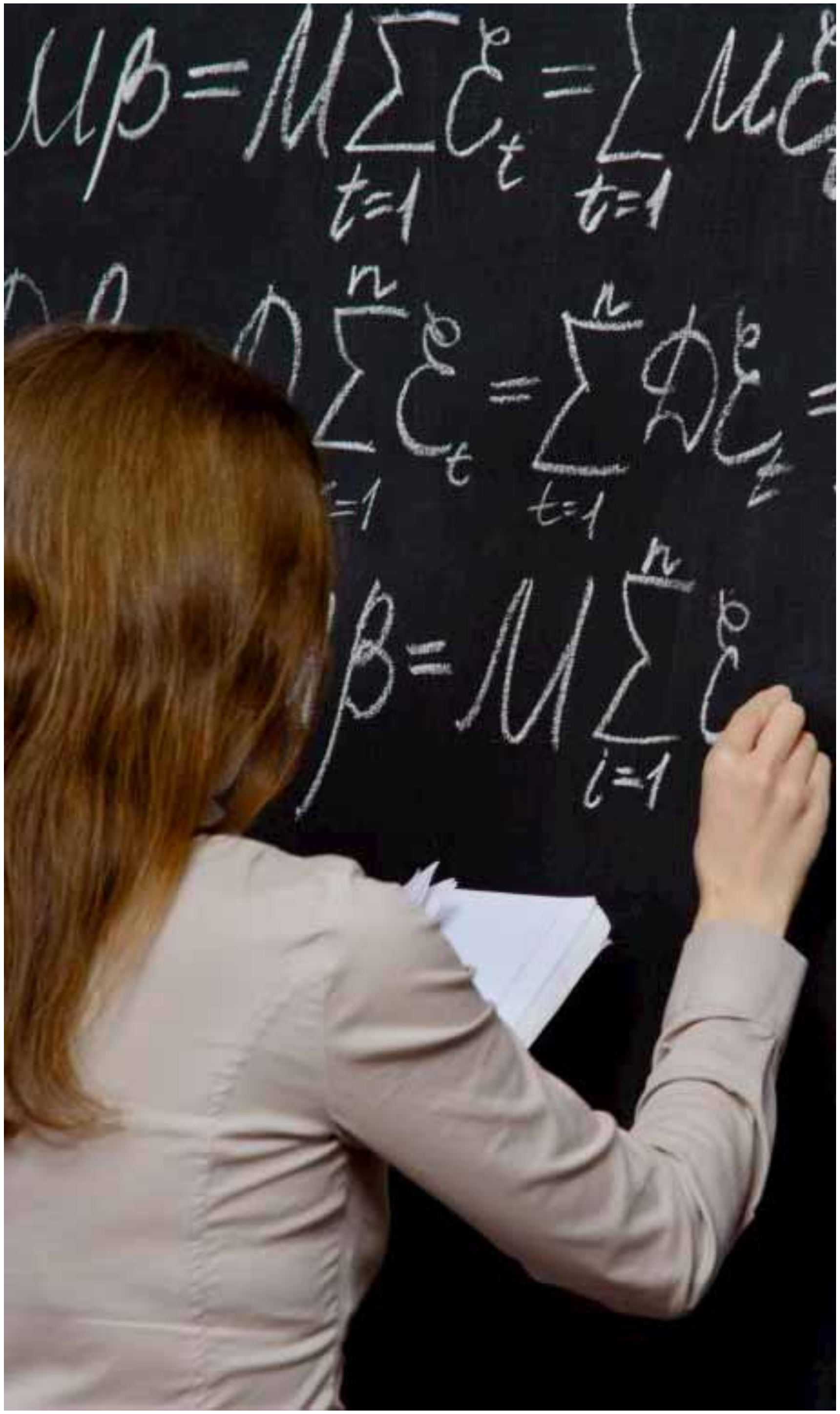
November 9, 2015 Physics 131 Prof. E. F. Redish

- **Theme Music:** **Bruce Springsteen**
  Working on a Dream
- **Cartoon:** **Bill Watterson**
  Calvin & Hobbes





The Equation of the Day

The Work-Energy Theorem

$$\vec{F}^{net} \cdot \Delta\vec{r} = \Delta\left(\tfrac{1}{2}mv^2\right)$$

Physics 131

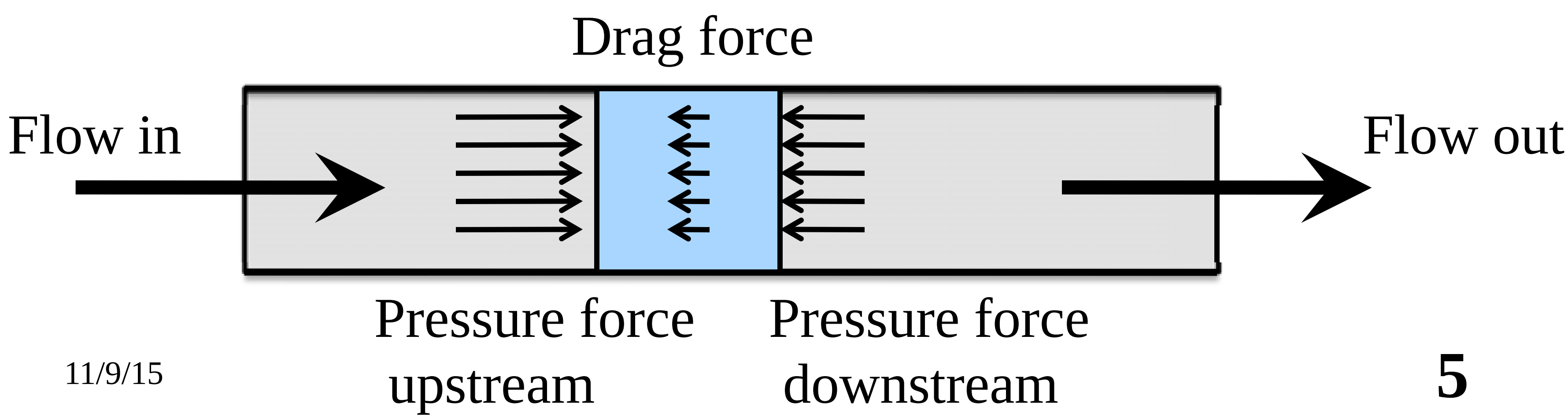
Remember: Viscous Drag

- A fluid flowing in a pipe doesn't slip through the pipe frictionlessly.
- The fluid sticks to the walls moves faster at the middle of the pipe than at the edges.
  As a result, it has to "slide over itself" (shear).
- There is friction between layers of fluid moving at different speeds that creates a viscous drag force, trying to reduce the sliding.
- The drag is proportional to the speed and the length of pipe.

$$F_{drag} = 8\pi\mu L v$$

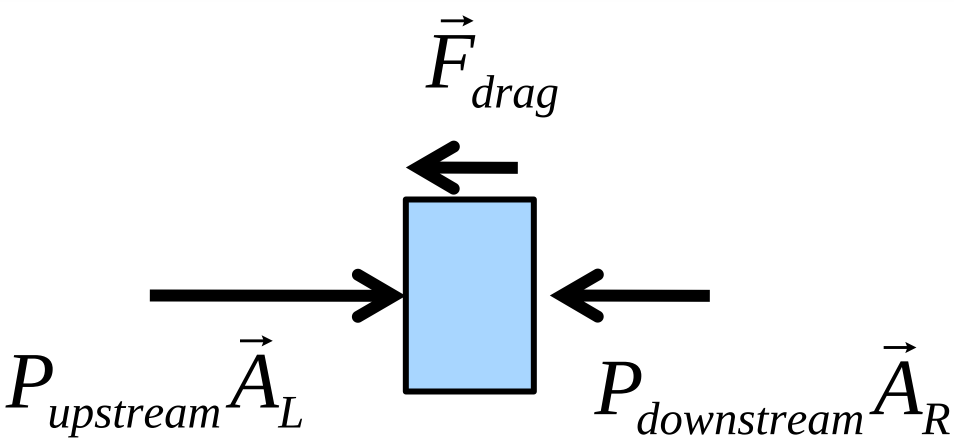
Implication: Pressure drop

- If we have a fluid moving at a constant rate and there is drag, N2 tells us there must be another force to balance the drag.
- The internal pressure in the fluid must drop in the direction of the flow to balance drag.



The Hagen-Poiseuille Law

- If the pressure drop balances the drag (and thereby maintains a constant flow) N2 tells us



$$\Delta P\, A = 8\pi\mu L v$$

$$\Delta P\, A = 8\pi\mu L\left(\frac{Q}{A}\right)$$

$$\Delta P = \left(\frac{8\pi\mu L}{A^2}\right)Q = \left(\frac{8\mu L}{\pi R^4}\right)Q$$

$$\Delta P = ZQ$$

Ramble on “functional dependence”

- One of the most important things an equation can tell you is how one thing depends on another.
- Simply saying “more this means more that” or “less this means less that” doesn’t tell you much.
- But saying “y changes like x to the n^{th} power” gives you “much more powerful information”. (Sorry.)
- Examples
  - – Diffusion vs. directed vs. caged motion
  - – HP equation
  - – Worm

Energy

- N2 tells us that a force can change an object's velocity in one of two ways:
  - It can change the speed
  - It can change the direction
- Analyzing changes in speed leads us to study energy.
- Analyzing changes in direction leads us to study rotations.

Kinetic Energy and Work

- Consider an object moving along a line feeling a constant net force, F^{net}. When it moves a distance Δx, how much does its speed change?

$$a = F^{net} / m$$

$$\frac{\Delta v}{\Delta t} = \frac{F^{net}}{m}$$

$$\frac{\Delta v}{\Delta t}\Delta x = \frac{F^{net}}{m}\Delta x$$

$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net}\Delta x}{m}$$

$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net} \Delta x}{m}$$

$$\langle v \rangle \Delta v = \frac{F^{net} \Delta x}{m}$$

$$\frac{v_i + v_f}{2} (v_f - v_i) = \frac{F^{net} \Delta x}{m}$$

$$\tfrac{1}{2} (v_f^2 - v_i^2) = \frac{F^{net} \Delta x}{m}$$

$$\tfrac{1}{2} m (v_f^2 - v_i^2) = F^{net} \Delta x$$

<u>Definitions:</u>

Kinetic energy

$$= \tfrac{1}{2} m v^2$$

Work done
by a force F

$$= F \; \Delta x$$

Result:

$$\Delta(\tfrac{1}{2} m v^2) = F^{net} \; \Delta x$$

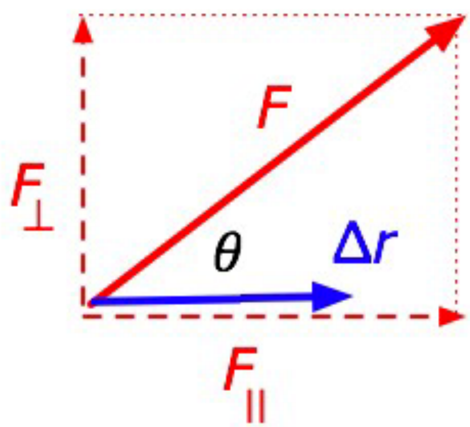
The Work-Energy Theorem

Foothold ideas: Kinetic Energy and Work

- Newton's laws tell us how velocity changes. The Work-Energy theorem tells us how speed (independent of direction) changes.
- Kinetic energy = $\frac{1}{2}mv^2$
- Work done by a force = $F_x \Delta x$ or $F_{||} \Delta r$ (part of force || to displacement)
- Work-energy theorem: $\Delta(\frac{1}{2}mv^2) = F_{||}^{net} \Delta r$

Work in another direction: The dot product

- Suppose we are moving along a line, but the force we are interested in in pointed in another direction? (How can this happen?)
- Only the part of the force in the direction of the motion counts to change the speed (energy).



$$\text{Work} = F_{||}\,\Delta r = F\cos\theta\,\Delta r \equiv \vec{F}\cdot\Delta\vec{r}$$

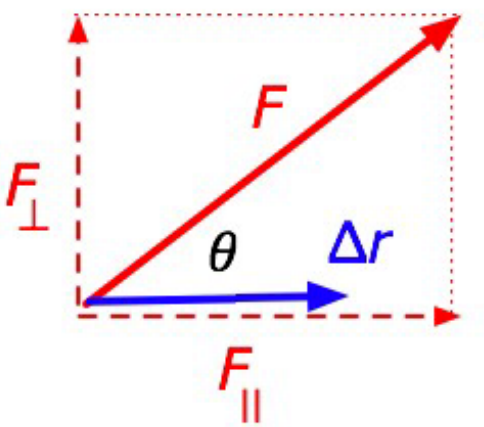
Dot products in general

$$F_{\parallel}\,\Delta r \equiv \vec{F}\cdot\Delta\vec{r} \qquad\qquad \vec{F}\cdot\Delta\vec{r} = F\cos\theta\,\Delta r$$

In general, for any two vectors
that have an angle θ between them,
the dot product is defined to be

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$$



The dot product is a scalar.
Its value does not depend on the coordinate system we select.