Theme Music: Queen
Under Pressure
Cartoon: Bill Watterson
Calvin & Hobbes
The Equation of the Day

Ideal Gas Law

\[ pV = nRT \]

\[ pV = Nk_B T \]
Quiz 7

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Avg. = 5.4
Summarizing the model

- In between collisions each molecule moves in a straight line – ignoring gravity. (We’ve used N1!)
- Ignore up and down motions.
- Momentum change of a molecule that bounces off the wall exerts a force on the wall.
- The force on the wall will be the change in momentum of all the molecules that bounce off the wall in a time $\Delta t$ divided by $\Delta t$.
- Calculate this using density.

$$ F = \frac{2mv_x}{\Delta t} \left( \frac{1}{2} nAv_x \Delta t \right) = nmv_x^2 A $$

$$ p = F = nmv_x^2 = \frac{N}{V} mv_x^2 = \frac{N}{V} \frac{1}{3} m \langle v^2 \rangle $$

**Average force is an emergent property!**
The Ideal Gas Law

Chemist’s form

\[ pV = n_{\text{moles}}RT \]

Physicist’s form

\[ pV = Nk_B T \]

\[ p = nmv^2 \]

\[ \frac{3}{2} k_B T = \frac{1}{2} mv^2 \]
In a gas the molecules are moving very fast in all directions. On the average the momentum cancels out.

If you put in a wall keeping the gas on only one side, only the momentum in one direction acts on the wall (N1, N2, N3), creating a force.

In a non-flowing gas, the force/area is a constant, the pressure. It is proportional to the number of molecules and their $mv^2$. 
Foothold ideas: Gases – Kinetic Theory II

- Newton’s laws tell us that motion continues forever unless something unbalanced tries to stop it, yet we observe motion always dies away.

- Our model of matter as lots of little particles in continual motion lets us “hide” the energy of motion that has “died away” at the macro level in the internal incoherent motion.

- The model unifies the idea of heat and temperature with our ideas of motion of macroscopic objects.
Interpreting

- The “physicist’s form” of the ideal gas law lets us interpret where the $p$ comes from and what $T$ means.

- $p$ arises from molecules hitting a wall and transferring momentum to it;

- $T$ corresponds to the energy of motion of one molecule (up to a constant factor).

\[
p = n m v_x^2 \quad \quad k_B T = \frac{2}{3} \left( \frac{1}{2} m v^2 \right)
\]
Their average speed is \(~400\) m/s!
(\(~1000\) mi/hr)

10/28/15