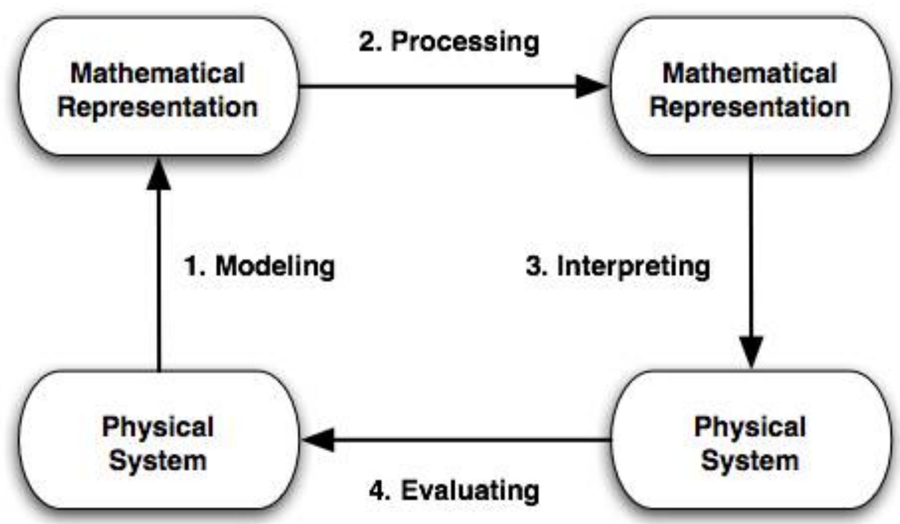
- **Theme Music: Take the A Train**
  Duke Ellington
- **Cartoon: Blondie**
  Chick Young

Foothold ideas: Modeling the world with math

- We use math to model relationships and properties.
- From the math we inherit ways to process and solve for results we couldn't necessarily see right away.
- Sometimes, mathematical models are amazingly good representations of the world. Sometimes, they are only fair. It is very important to develop a sense of when the math works and how good it is.
- Mostly, the math we use differs in important ways from the math taught in math classes.



Foothold ideas: Dimensional and unit analysis

- We label the kinds of measurement that go into assigning a number to a quantity like this:
  - [x] = L — means “x is a length”
  - [t] = T — means “t is a time”
  - [m] = M — means “m is a mass”
  - [v] = L/T — means “you get v by dividing a length by a time
- Units specify which particular arbitrary measurement we have chosen.
  - Units should be manipulated like algebraic quantities.
  - Units can be changed by multiplying by appropriate forms of “1” e.g. 1 = (1 inch)/(2.54 cm)

Foothold ideas: Dimensional analysis

- In physics we have different kinds of quantities depending on how measurements were combined to get them. These quantities may change in different ways when you change your measuring units.
- Only quantities of the same type may be equated (or added) otherwise an equality for one person would not hold for another. Equating quantities of different dimensions yields nonsense.
- Dimensional analysis tells us *how* something changes when we either
  - Change our arbitrary scale (passive change)
  - Change the scale of the object itself (active change)

Foothold Ideas:
Estimation – Quantifying experience

- **Measure your body parts**
- **Don't** look up data online or get it from friends!
- **Don't** use your calculator! Use 1-digit arithmetic
- **Do** figure out your estimations by starting with something you can plausibly know and scale up or down
- **Do** check your answer to see if it's reasonable
- **Do** learn a small number of Useful numbers

Useful numbers (people)

Numbers	
Number of people on the earth	~7 billion (7×10^9)
Number of people in the USA	~ 300 million (3×10^8)
Number of people in the state of Maryland	~ 5 million (5×10^6)
Number of students in a large state university	~30-40 thousand (3×10^4)

Useful numbers (distances)

Macro Distances	
Circumference of the earth	~24,000 miles (1000 miles/time zone at the equator)
Radius of the earth*	$2/\pi \times 10^7$ m
Distance across the USA	~3000 miles
Distance across DC	~10 miles

Useful numbers (bio)

Bio Scales	
Size of a typical animal cell	~10-20 microns (10^{-5} m)
Size of a bacterium, chloroplast, or mitochondrion	~1 micron (10^{-6} m)
Size of a medium-sized virus	~0.1 micron (10^{-7} m)
Thickness of a cell membrane	~5-10 nm (10^{-8} m)

Foothold ideas: Measuring "where"

- In order to specify where something is we need a coordinate system. This includes:
  - Picking an origin
  - Picking perpendicular directions
  - Choosing a measurement scale
- Each point in space is specified by three numbers: (*x*, *y*, *z*), and a <u>position vector</u>– an arrow showing the displacement from the origin to that position.
- Vectors add like successive displacements or algebraically by $\vec{A} = A_x\hat{i} + A_y\hat{j}$ $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

Foothold ideas: Measuring "when"

- Time is a coordinate just like position
  - We need an origin (when we choose $t = 0$)
  - a direction (usually times later than 0 are +)
  - a scale (seconds, years, millennia)
- Note the difference between
  - clock reading, t
  - a time interval, Δt

This is like the difference between position and length!

Foothold ideas: Velocity

- Average velocity is defined by

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} = \frac{\text{vector displacement}}{\text{time it took to do it}}$$

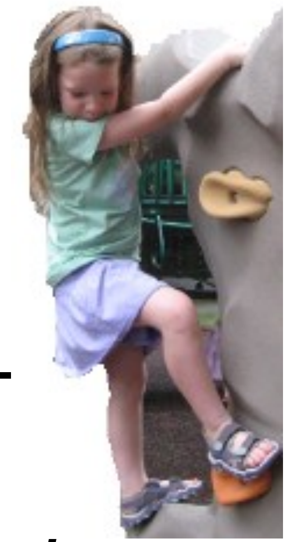
Note: an average velocity goes with a <u>time interval</u>.

- Instantaneous velocity is what we get when we consider a very small time interval (compared to times we care about)

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Note: an instantaneous velocity goes with a <u>specific time</u>.

Foothold ideas: Vector velocity and speed



- A displacement – a change in position – has a direction. This means

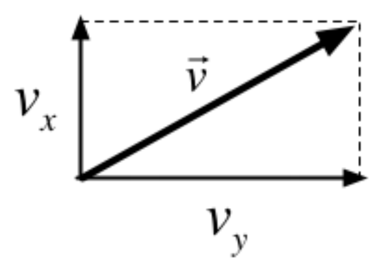
  velocity = *displacement/time interval*

  has one too. $\vec{v} = \frac{d\vec{r}}{dt}$

$$v_x\hat{i} + v_y\hat{j} = \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$$

- We define speed as the magnitude of velocity. (No vector on this. Why?)

$$v = \sqrt{v_x^2 + v_y^2}$$



Foothold ideas: Acceleration

- Average acceleration is defined by

$$\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time it took to do it}}$$

Note: an average acceleration goes with a <u>time interval</u>.

- Instantaneous acceleration is what we get when we consider a very small time interval (compared to times we care about)

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Note: an instantaneous acceleration goes with a <u>specific time</u>.

What have we learned? Representations and consistency

- Visualizing where an object is at different times → a position graph
- Visualizing how fast an object is moving at different times → a velocity graph
- Position graph → velocity graph

  slopes $\langle v \rangle = \dfrac{\Delta x}{\Delta t}$

- Velocity graph → position graph

  areas $\Delta x = \langle v \rangle \Delta t$

What have we learned? Representations and consistency

- Visualizing how fast an object is at different times → a velocity graph
- Visualizing how an object changes v at different times → a velocity graph
- Velocity graph → accel. graph
- Accel. graph → velocity graph

slopes $\langle a \rangle = \dfrac{\Delta v}{\Delta t}$

area $\Delta v = \langle a \rangle \Delta t$
s

System Schemas

- A tool that allows you to be explicit about defining what you are going to choose to talk about and with how much complexity you are going to treat it.
- Specify
  - Relevant objects (and structures if needed)
  - Interactions between objects

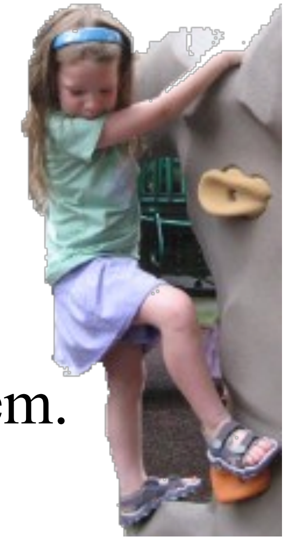
Conceptual ideas underlying Newton's Laws 1-3

1. Objects respond only to influences acting upon them at the instant that those influences act. (Object egotism) [Newton 0]
2. All outside effects on an object being equal, the object maintains its velocity (including direction). The velocity could be zero, which would mean the object is at rest. (Inertia) [Newton 1]
3. Every change in velocity an object experiences is caused by the object interacting with some other object – **forces**. (Interactions)

Conceptual ideas underlying Newton's Laws 4-6

4. If there are a lot of different objects that are interacting with the object we are considering, the overall result is the same as if we add up all the forces as vectors and produce a single effective force -- the **net force**. (Superposition)
5. When one object exerts a force on another, that force is <u>shared</u> over all parts of the structure of the object. (Mass)
6. The acceleration felt by an object at a given instant is the net force on the object at that instant divided by the object's mass. [Newton 2]
7. Whenever two objects interact, they exert forces on each other. (Reciprocity) [Newton 3]

Newton's Laws



- Newton 0:
  - An object responds to the forces it feels when it feels them.
- Newton 1:
  - An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).
- Newton 2:
  - An object that is acted upon by other objects changes its velocity according to the rule

$$\vec{a}_A = \vec{F}^{net}_A \Big/ m_A$$

- Newton 3:
  - When two objects interact the forces they exert on each other are equal and opposite.

$$\vec{F}^{type}_{A\to B} = -\vec{F}^{type}_{B\to A}$$

Kinds of Forces

- Forces how objects influence each other trying to change velocity.
- Types of forces
  - Spring, Normal, Tension Force *N, T* $$T = k\Delta L$$
  - Friction Force *f* $$f \le \mu N$$
  - Weight Force *W* $$\vec{W} = m\vec{g}$$
- Notation convention.

$$\vec{F}^{\text{type of force}}_{\text{(object causing force)}\rightarrow\text{(object feeling force)}}$$

Foothold ideas:

The Spring – Tension and Normal forces

- A spring changes its length in response to pulls (or pushes) from opposite directions.

$$T = k\,\Delta l$$

- Normal forces arise from the springiness of ordinary solids (a very stiff spring).