October 29, 2013 Physics 131 Prof. E. F. Redish

- **Theme Music:** **Elvis Presley**

  All Shook Up

- **Cartoon:** **Scott & Borgman**

  Zits

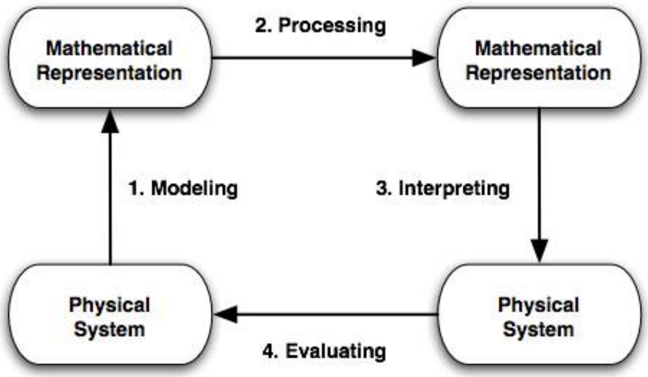
10/29/13 Physics 131 1

Foothold principles: Randomness

- Matter is made of of molecules in constant motion and interaction. This motion moves stuff around.
- If the distribution of a chemical is non-uniform, the randomness of molecular motion will tend to result in molecules moving from more dense regions to less.
- This is **not** directed but is an emergent phenomenon arising from the combination of random motion and non-uniform concentration.

10/29/13 Physics 131 4

Making a mathematical model



10/29/13 Physics 131 **5**

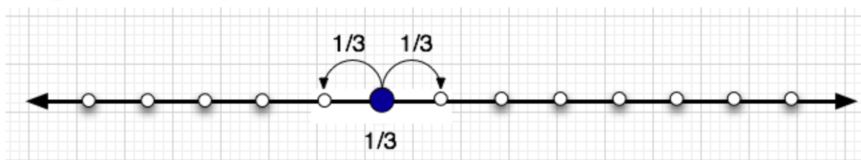
A new start

- Our mathematical model based on identifying position, velocity, and all the forces on an object and then calculating the motion using Newton's second law is too hard for a small particle being hit by many molecules.
- An alternative starting point is to describe the result of all the forces acting on a small object as random motion.
- Average phenomena that emerge from the randomness can still be reliable even though the motion at any given instant can't be predicted.

10/29/13 Physics 131 **6**

A mathematical model without equations*

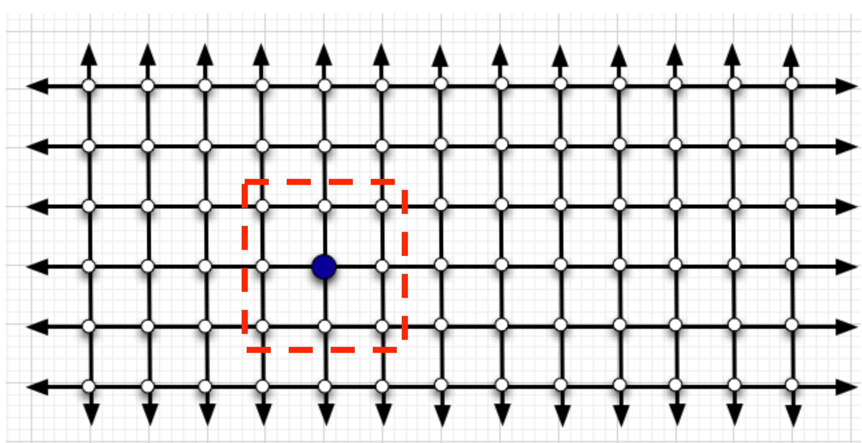
- Describe the position of our object as being on a grid. Run the clock in small time steps, letting the object move to each neighboring point (or stay where it is) with equal probability.



*Although the model is defined without equations, lots of equations can be derived about averages and distributions. See, e.g., H.C. Berg, "Random walks in biology".

10/29/13 7

If the probability is equal that in the next time step the object moves to any one of the sites in the red box, what is the probability for going to any one particular site?

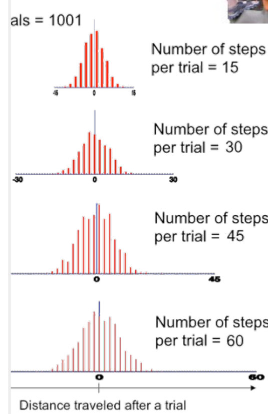


Foothold ideas: Random walk in 1D

- As a result of random motion, an initially localized distribution will spread out, getting wider and wider. This phenomenon is called *diffusion*
- The width of the distribution will grow like

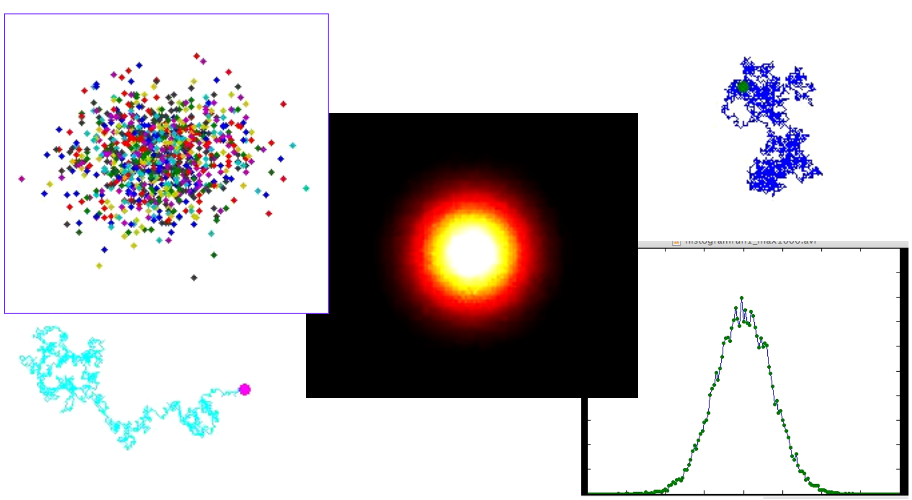
$$\langle (\Delta x)^2 \rangle = 2Dt$$

- D is called *the diffusion constant* and has dimensionality $[D] = L^2/T$



16

2D Simulations: Multiple representations

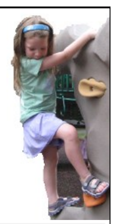


10/29/13 *Simulations by Alex Morozov & Kerstin Nordstrom* 17

2D Simulations: Multiple representations

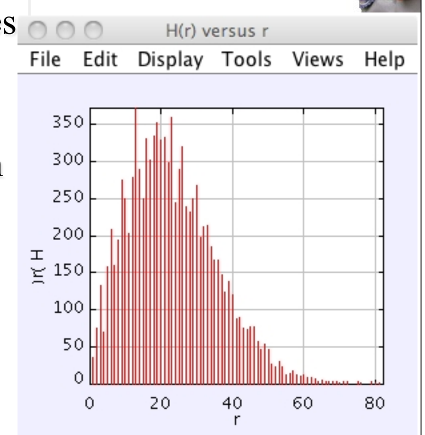
1. Watch all the particles.
2. Look at the density of the particles
   - What do the colors represent?
3. Look at a plot of the density along a slice through the middle.
   - What it will look like and what it will do.
4. Look at the motion of individual particles.

10/29/13 Physics 131 **18**



Foothold ideas: Random walk in 2D

- The density of walkers decreases uniformly as you get farther from the source.
- The total number within a given radius peaks – since the area within a radius r decreases to 0 as r gets small. ("phase space")
- The width of the peak grows with the square root of time.



10/29/13 Physics 131 **19**