Physics 131- Fundamentals of Physics for Biologists I

Work
Kinetic Energy
Math: Dot Product
Arpita Upadhaya research
In many of your science classes you talk about “energy.”

What is it?

Write down:
one word or sentence or picture
one equation

Hold up your whiteboards
Kinetic Energy and Work

Consider an object moving along a line feeling a single force $F$.

When it moves a distance $\Delta x$, how much does its speed change?

\[
a = \frac{F^{net}}{m}
\]

\[
\frac{\Delta v}{\Delta t} = \frac{F^{net}}{m}
\]

\[
\frac{\Delta v}{\Delta t} \Delta x = \frac{F^{net}}{m} \Delta x
\]

\[
\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net} \Delta x}{m}
\]
\[ \frac{\Delta x}{\Delta t} \Delta v = \frac{F^{\text{net}} \Delta x}{m} \]

\[ \langle v \rangle \Delta v = \frac{F^{\text{net}} \Delta x}{m} \]

\[ \frac{v_i + v_f}{2} (v_f - v_i) = \frac{F^{\text{net}} \Delta x}{m} \]

\[ \frac{1}{2} (v_f^2 - v_i^2) = \frac{F^{\text{net}} \Delta x}{m} \]

\[ \frac{1}{2} m (v_f^2 - v_i^2) = F^{\text{net}} \Delta x \]

Definitions:

Kinetic energy = \[ \frac{1}{2} m v^2 \]

Work done by a force \( F = F \Delta x \)

Result

\[ \left( \frac{1}{2} m v^2 \right) = F^{\text{net}} x \]
Dimensions and Units of Energy and Work

- $[\frac{1}{2} \, mv^2] = M \cdot (\frac{L}{T})^2 = ML^2/T^2$
- $1 \, \text{kg} \cdot \text{m}^2/\text{s}^2 = 1 \, \text{N} \cdot \text{m} = 1 \, \text{Joule}$
- Other units of energy are common (and will be discussed later)
  - Calorie
  - eV (electron Volt)
The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.

Which puck will have the greater kinetic energy upon reaching the finish line?

1. Puck I
2. Puck II
3. Both will have the same.
4. There is not enough information to decide.
The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.

Which puck will have the greater velocity upon reaching the finish line?

1. Puck I
2. Puck II
3. Both will have the same.
4. There is not enough information to decide.
Work in another direction: The dot product

- Suppose we are moving along a line, but the force we are interested in is pointed in another direction? (How can this happen?)
- Only the part of the force in the direction of the motion counts to change the speed (energy).

\[
\text{Work } = F_{\parallel} \Delta r = F \cos \theta \Delta r \equiv \vec{F} \cdot \Delta \vec{r}
\]
Dot products in general

\[ F \parallel \Delta r \equiv \vec{F} \cdot \Delta \vec{r} \quad \vec{F} \cdot \Delta \vec{r} = F \cos \theta \Delta r \]

In general, for any two vectors that have an angle \( \theta \) between them, the dot product is defined to be

\[ \vec{a} \cdot \vec{b} = ab \cos \]

\[ \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y \]

The dot product is a scalar. Its value does not depend on the coordinate system we select.
Each row in the following table pairs a force vector with a corresponding displacement resulting in work $W$ being done.

**In which of these rows is the work done zero?**

<table>
<thead>
<tr>
<th>$\vec{F}$</th>
<th>$\vec{r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
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<tr>
<td>3.</td>
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<tr>
<td>4.</td>
<td></td>
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<tr>
<td>5.</td>
<td></td>
</tr>
<tr>
<td>6. None of the above</td>
<td></td>
</tr>
</tbody>
</table>