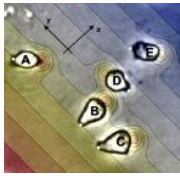
Physics 131-Physics for Biologists I



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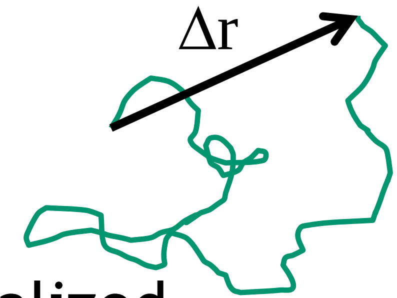
Midterm 2: November 8

Office Hours before Midterm 2:
Course Center: Monday Nov 4, 11am-12.30pm
3341 AV Williams: Wednesday Nov 6, 11.30am-1pm

Random walk in 2D



- As a result of random motion, an initially localized distribution will spread out, getting wider and wider. This phenomenon is called *diffusion*
- The square of the average distance traveled during random motion will grow with time.

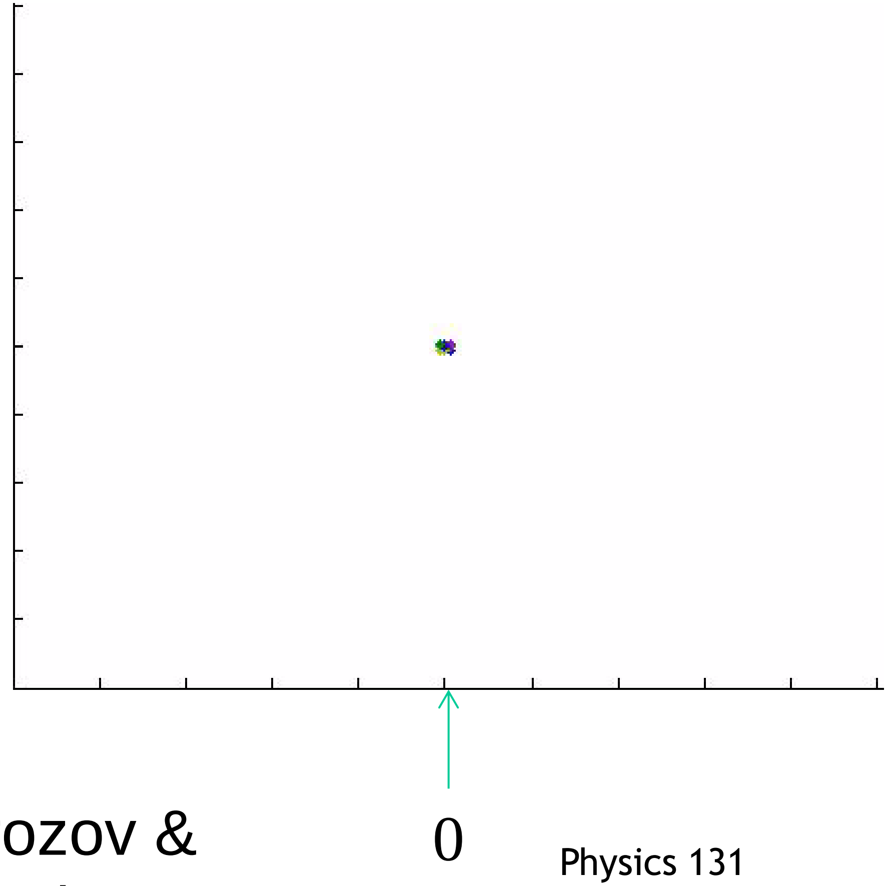
- 1 Dimension $$\left\langle (\Delta x)^2 \right\rangle = 2D\Delta t$$
- 2 Dimensions $$\left\langle (\Delta r)^2 \right\rangle = \left\langle (\Delta x)^2 + (\Delta y)^2 \right\rangle = 4D\Delta t$$
- 3 Dimensions $$\left\langle (\Delta r)^2 \right\rangle = \left\langle (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \right\rangle = 6D\Delta t$$

Start 200 random walkers in two dimension near 0

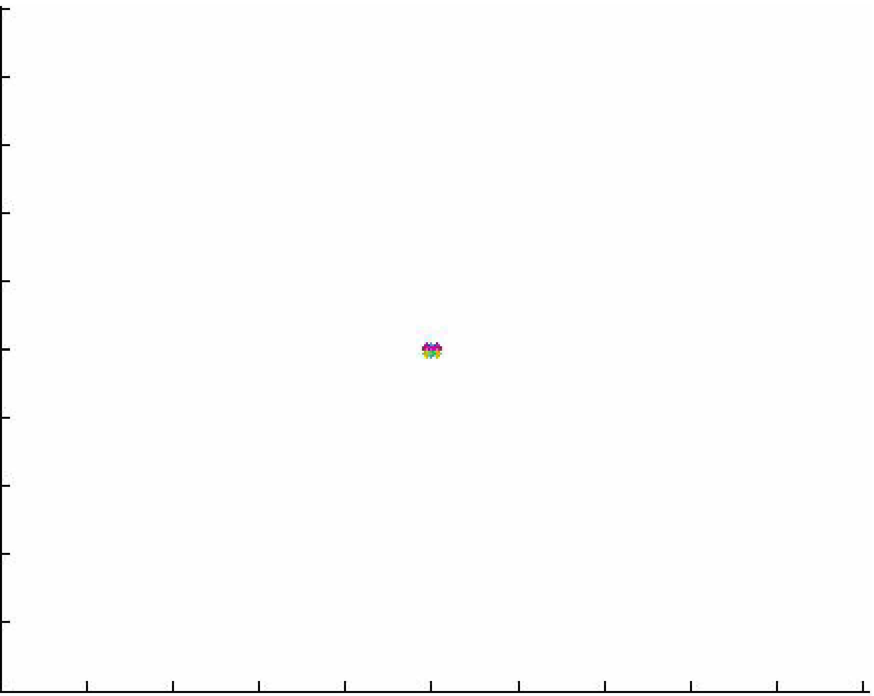
Sketch where you expect to see the 200 points after they walk some time

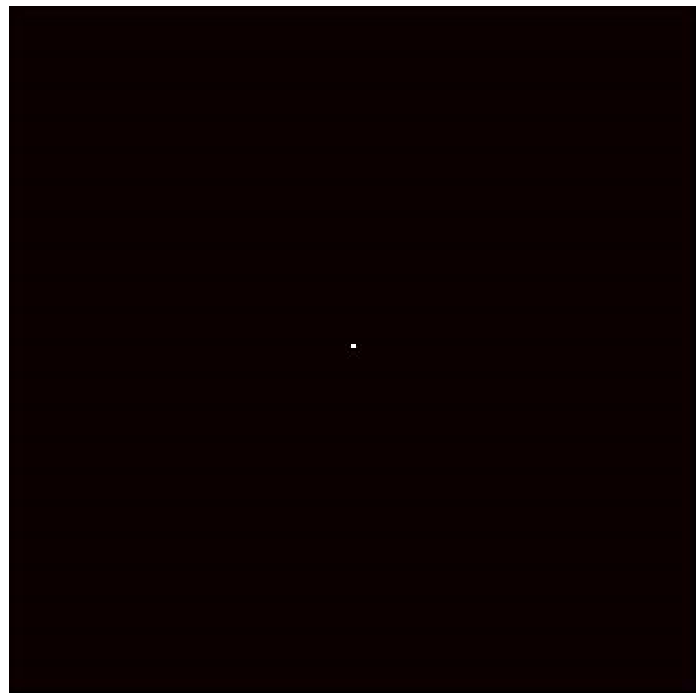
Whiteboard, TA & LA

1. Particles will form a “wave” – a ragged ring of particles moving outward.
2. They will be mostly stay near 0 no matter how long you wait.
3. **Many particles will remain near zero, but they will gradually spread out.**



A simulation of many random walkers





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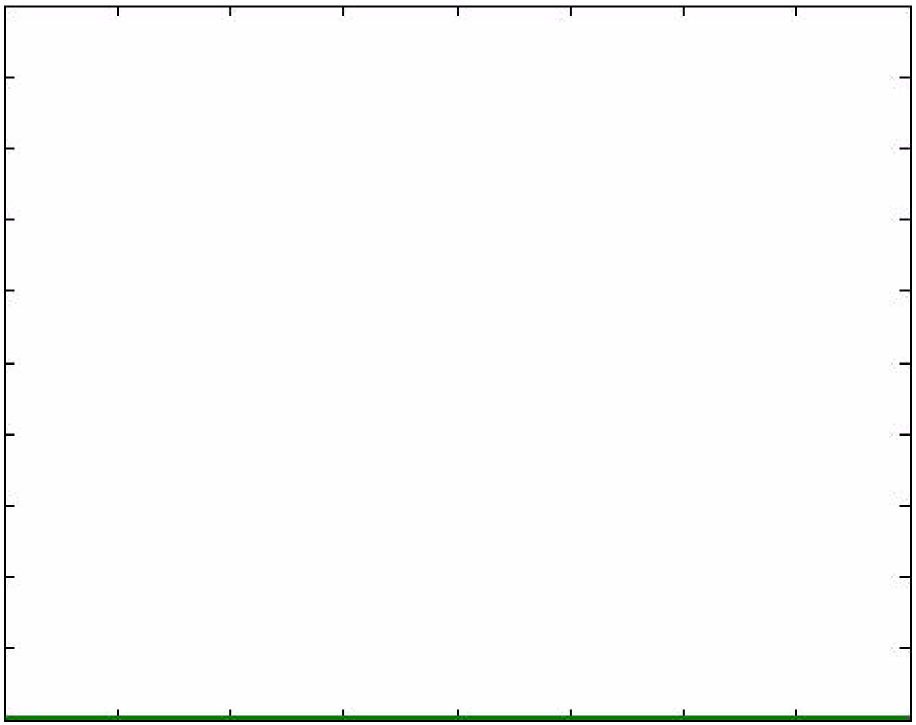
Density as a function of position

Gaussian Distribution



Measure width at half the peak height. Measure the full width.

Since we only have a fixed number of walkers, the line is choppy!
The variability is equal to $\sqrt{N}$
e.g. if on average we have 100 particles in a bin, variability will be 10

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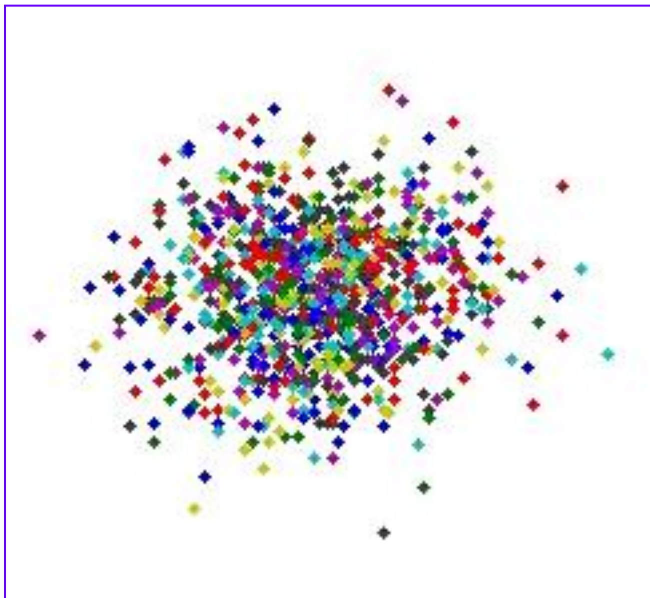
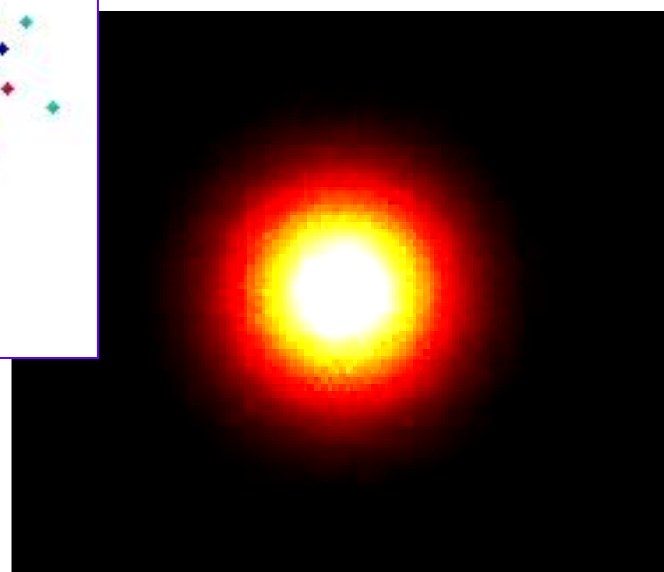
- If I wait four times longer, the width of the distribution increases by a factor?

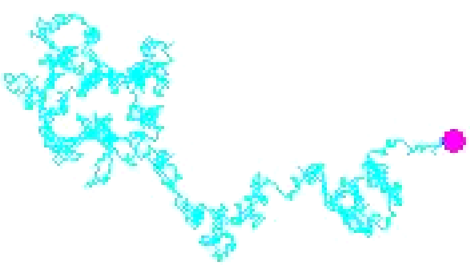
Width increases by factor of 2

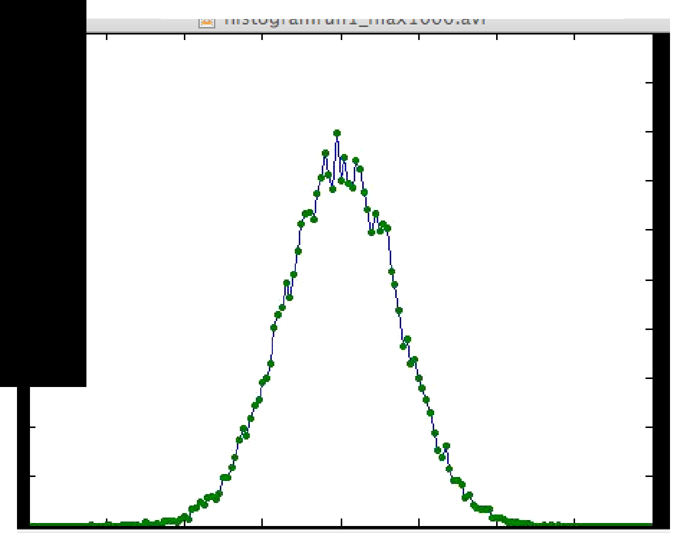
Whiteboard, TA & LA

2D Simulations: Multiple representations









How many cross *A* in a time Δt?

- Number hitting A from left

$$\tfrac{1}{2} n_{-} \left(A v_0 \Delta t \right)$$

- Number hitting A from right

$$\tfrac{1}{2} n_{+} \left(A v_0 \Delta t \right)$$

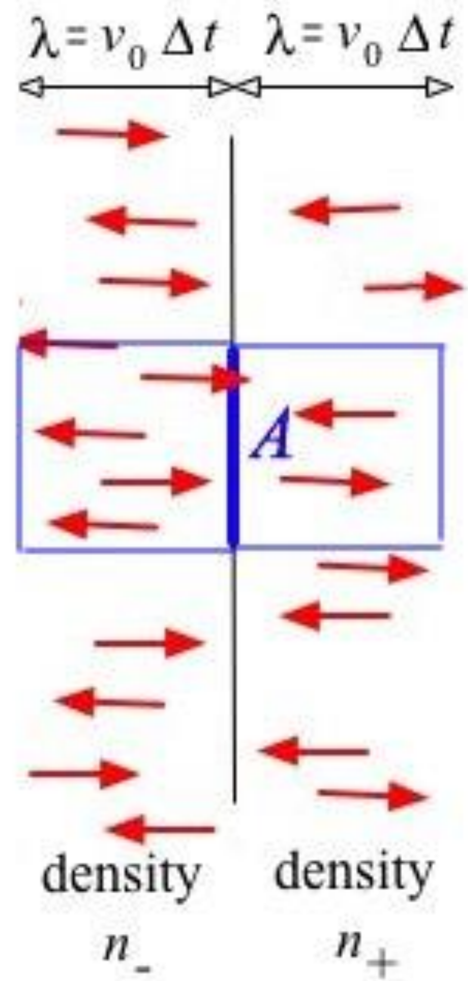
- Net flow across A

$$\tfrac{1}{2} \left(n_{-} - n_{+} \right) \left(A v_0 \Delta t \right)$$

- Define flux (per unit area per unit time) as J therefore:

$$J A \Delta t = \tfrac{1}{2} \left(n_{-} - n_{+} \right) \left(A v_0 \Delta t \right)$$

$$J = \tfrac{1}{2} \Delta n \left(v_0 \right)$$

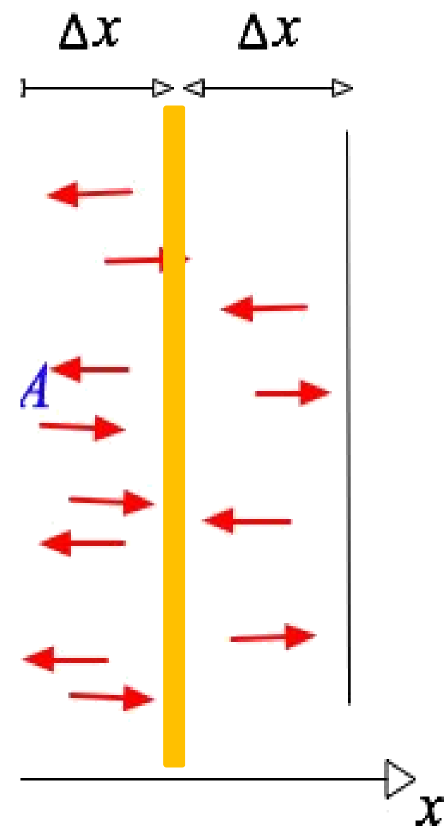


Fick's law

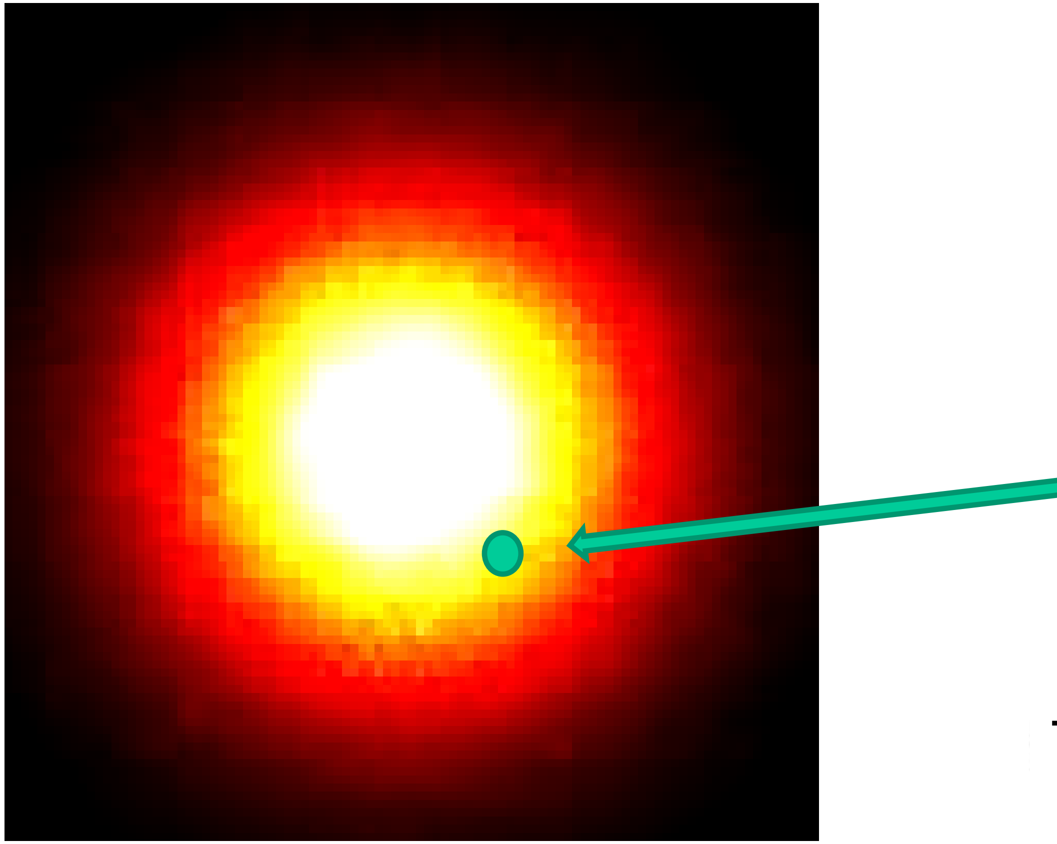
- 1D result

$$J = -D\frac{dn}{dx} \qquad D = \tfrac{1}{2}\lambda v_0$$

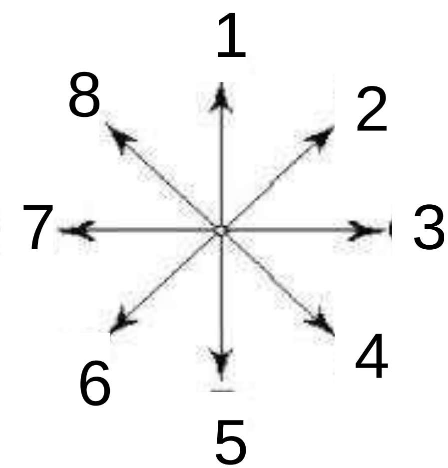
Atoms move randomly in two containers. More atoms are on the left than on the right of a yellow gate. When the gate is suddenly lifted, some of the randomly moving atoms travel across the gate.



1. **More go to the right**
2. More go to the left
3. Equal amount goes left and right in random motion
4. Not enough information



In which Direction does the flux J point?



The gradient

- If we want to take the derivative of a function of one variable, $y = df/dx$, it's straightforward.
- If we have a function of three variables – $f(x,y,z)$ – what do we do?
- The gradient is the **vector derivative.**
  To get it at a point (x,y,z)
  - Find the direction in which f is changing the fastest.
  - Take the derivative by looking at the rate of change in that direction.
  - Put a vector in that direction with its magnitude equal to the <u>maximum</u> rate of change.
  - The result is the vector called $\vec{\nabla} f$

Fick's law

- 1D result

$$J = -D\frac{dn}{dx} \qquad D = \tfrac{1}{2}\lambda v_0$$

- For all directions (not just 1D) Fick's law becomes

$$\vec{J} = -D\vec{\nabla}n$$

Diffusion is a simple example of emergent behavior.

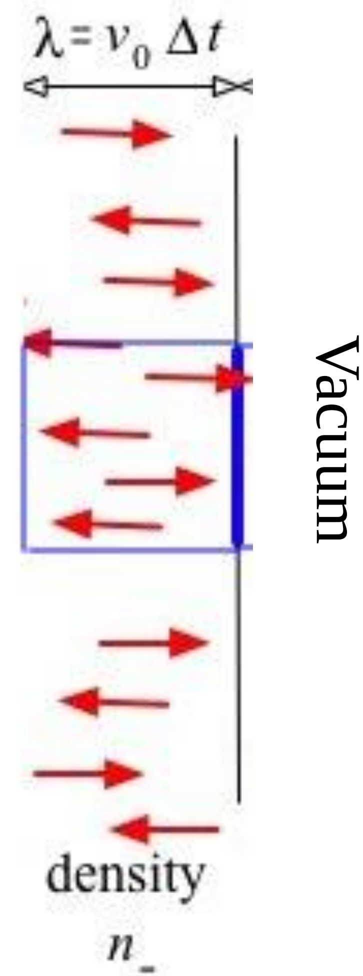
- Mean Squared Displacement $\left\langle \left(\Delta r\right)^2 \right\rangle = 6D\Delta t$

- Fick's Law $\vec{J} = -D\vec{\nabla} n$

Connect DYNAMICS (Motion) to FORCES

- Pressure
- Ideal Gas law

How much force does air exert on a sheet of size 1 m^2 with vacuum on the other side ? Equivalent to the weight of:



1. 1N
2. Glass of Milk
3. Gallon of Milk
4. Person
5. Car
6. **10Ton truck**
7. 100 foot boat
8. Building