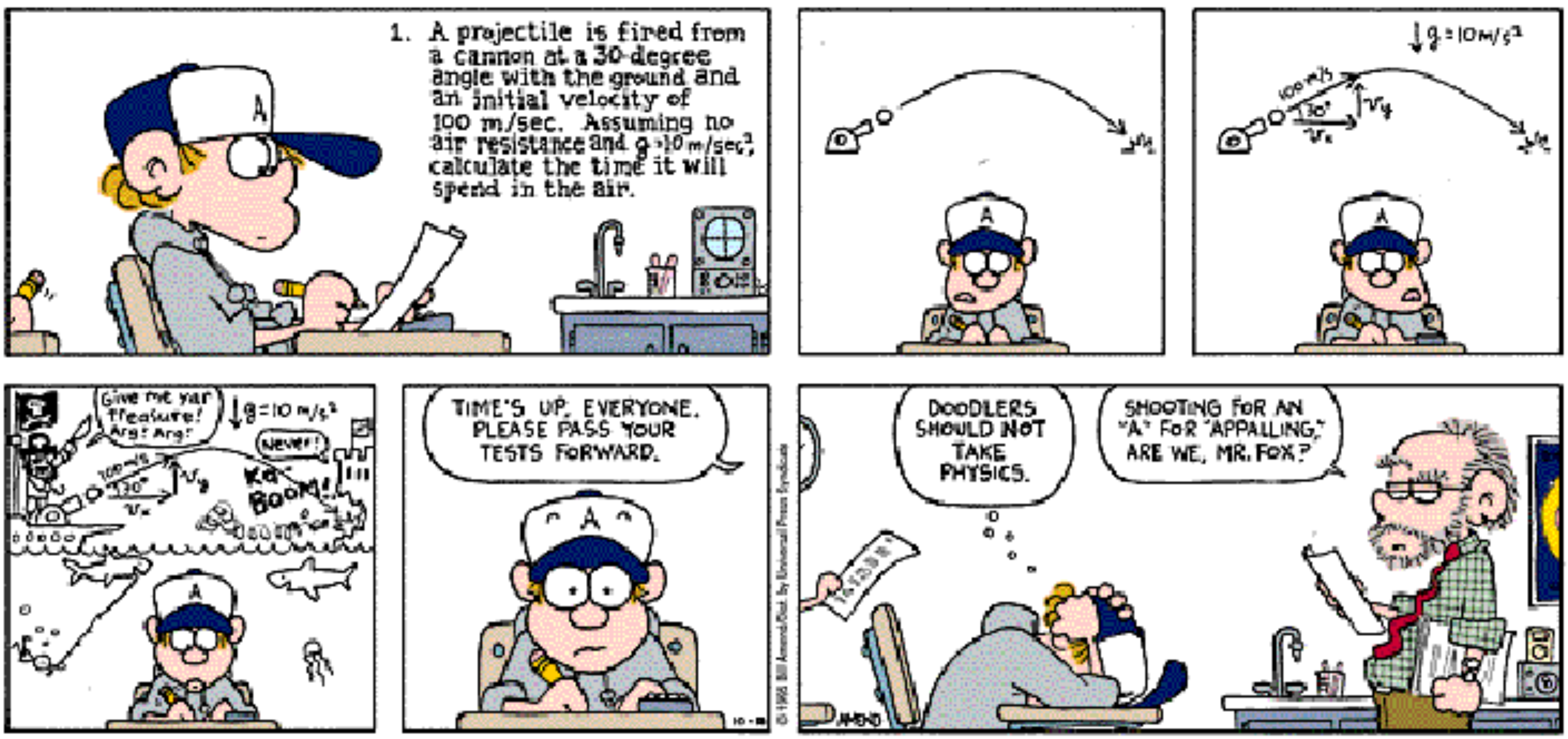
■ Theme Music: Duke Ellington

Take the A Train

■ Cartoon: Bill Amend *FoxTrot*



Foothold ideas: Viscosity

- Viscosity is a resistive force that an object feels when it moves through a fluid as a result of the fluid sticking to the object's surface. This layer of fluid tries to slide over the next layer of fluid and the friction between the speeds that layer up and so on.
- The result is a force proportional to the velocity of the object.

$$\vec{F}^{viscous}_{fluid \to object} = -6\pi\mu R_{object}\vec{v}$$

Foothold ideas: Drag force

- The drag ("Newtonian drag") is a resistive force felt by an object moving through a fluid. It arises because the object is pushing fluid in front of it, bringing it up to the same speed it's going.
- The result is a force proportional to the density of the fluid, the area of the object, and the square of the object's velocity.

$$F^{drag}_{fluid \to object} = C d_{fluid} A_{object} v^2$$

Reynolds' Number

- Generally, for an object moving in a fluid both drag and viscosity are present. However, often, one is much more important.
- The ratio of the two forces (drag / viscosity) is called the Reynolds' Number (leaving out a few dimensionless constants)

$$\text{Re} = \frac{dvR}{\mu}$$

- Small objects in water (v, R small) are generally dominated by viscosity; larger objects in air (v, R large) tend to be dominated by drag.

Foothold Ideas: Gravity

- Every object (near the surface of the earth) feels a downward pull proportional to its mass:

$$\vec{W}_{E\to m} = m\vec{g}$$

What object causes *W*?

where $\vec{g}$ is referred to as *the gravitational field.*

- This is a pForce even though nothing touching the object is responsible for it.
- The gravitational field has the same magnitude for all objects irrespective of their motion and at all points.
- The gravitational field always points down.
- It is measured to be $g \approx 9.8$ N/kg

Why N/kg instead of m/s^2?

Model: Charge
A hidden property of matter

- Matter is made up of two kinds of electric matter (positive and negative) that have equal magnitude and that cancel when they are together and hide matter's electrical nature.
- Matter with an equal balance is called <u>neutral</u>.
- Like charges repel, unlike charges attract.
- The algebraic sum of postive and negative charges is a constant (i.e, $N_+ - N_- = \text{const.}$)

Conductors and Insulators

- Insulators
  - In some matter, the charges they contain are bound and cannot move around freely.
  - Excess charge put onto this kind of matter tends to just sit there.
- Conductors
  - In some matter, charges in it can move around throughout the object.
  - Excess charge put onto this kind of matter redistributes itself or flows off (if there is a conducting path to ground).
- Unbalanced charges attract neutral matter (polarization)

Foothold idea: Coulomb's Law

- All objects attract each other with a force whose magnitude is given by

$$\vec{F}_{q\to Q} = -\vec{F}_{Q\to q} = \frac{k_C qQ}{r_{qQ}^2}\hat{r}_{q\to Q}$$

- k_C is put in to make the units come out right.

$$k_C = 9\times 10^9\ \text{N-m}^2/\text{C}^2$$

Foothold ideas: Fields

- A *field* is a concept we use to describe anything that varies in space. It is a set of values assigned to each point in space (e.g., temperature or wind speed).
- A *force field* is an idea we use for non-touching forces. It puts a force vector at each point in space, summarizing the effect of all objects that would exert a force on a particular object placed at that point.
- A *gravitational*, *electric, or magnetic field* is a force field with something (a "coupling strength") divided out so the field no longer depends on what test object is used.

$$\vec{g} = \frac{\vec{F}_{\text{acting on } m}}{m} \qquad \vec{E} = \frac{\vec{F}_{\text{acting on } q}}{q}$$

Field is the value at a position in space " *r* " assuming that the force is measured by placing the object at *r* .

Foothold ideas: Electric Forces and Fields

- When we focus our attention on the electric force on a particular charge (a test charge) we see the force it feels factors into the magnitude of its charge times a factor that depends on position (and the other charges).

$$\vec{F}_{q_0}^{E\,net} = \frac{k_C q_0 q_1}{r_{01}^2}\hat{r}_{1\to 0} + \frac{k_C q_0 q_2}{r_{02}^2}\hat{r}_{2\to 0} + \frac{k_C q_0 q_3}{r_{03}^2}\hat{r}_{3\to 0} + \ldots \frac{k_C q_0 q_N}{r_{0N}^2}\hat{r}_{N\to 0}$$

$$\vec{F}_{q_0}^{E\,net} = q_0 \vec{E}(\vec{r}_0)$$

$$\vec{E}(\vec{r}_0) = \frac{k_C q_1}{r_{01}^2}\hat{r}_{1\to 0} + \frac{k_C q_2}{r_{02}^2}\hat{r}_{2\to 0} + \frac{k_C q_3}{r_{03}^2}\hat{r}_{3\to 0} + \ldots \frac{k_C q_N}{r_{0N}^2}\hat{r}_{N\to 0}$$

Momentum: Definition

- We define <u>momentum</u>:

$$\vec{p} = m\vec{v}$$

- This is a way of defining "the amount of motion" an object has.
- Our "delta" form of N2 becomes

$$\vec{F}^{net} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

which we can rewrite as

$$\vec{F}^{net} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

The Impulse-Momentum Theorem

- Newton 2 $$\vec{a} = \vec{F}^{net} / m$$
- Put in definition of *a* $$\frac{d\vec{v}}{dt} = \frac{\vec{F}^{net}}{m}$$
- Multiply up by Δt $$m\Delta\vec{v} = \vec{F}^{net}\Delta t$$
- Define Impulse $$\vec{\mathrm{I}}^{net} = \vec{F}^{net}\Delta t$$
- Combine to get Impulse-Momentum Theorem $$\Delta\vec{p} = \vec{\mathrm{I}}^{net}$$

Momentum Conservation: 1

- If two objects, A and B, interact with each other and with other ("external") objects, By the IMT

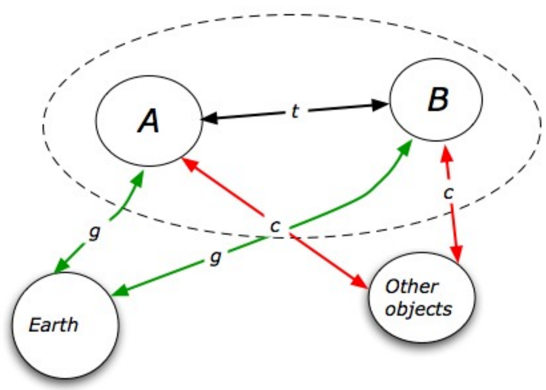
$$m_A\,\Delta\vec{v}_A = (\vec{F}_A^{ext} + \vec{F}_{B\to A})\Delta t$$

$$m_B\,\Delta\vec{v}_B = (\vec{F}_B^{ext} + \vec{F}_{A\to B})\Delta t$$

- Adding:

$$m_A\,\Delta\vec{v}_A + m_B\,\Delta\vec{v}_B = \left[\vec{F}_A^{ext} + \vec{F}_B^{ext} + \left(\vec{F}_{A\to B} + \vec{F}_{B\to A}\right)\right]\Delta t$$

$$\Delta\left(m_A\vec{v}_A + m_B\vec{v}_B\right) = \vec{F}_{AB}^{ext}\Delta t$$

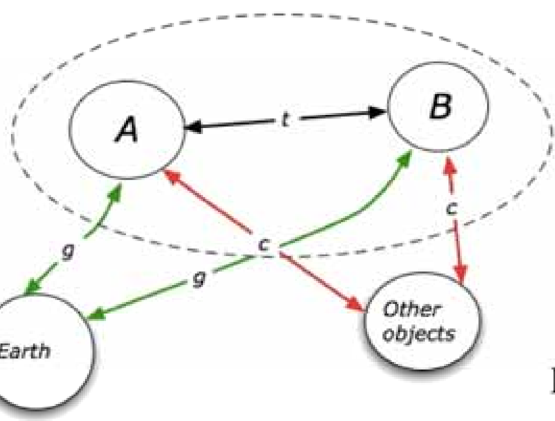


Momentum Conservation: 2

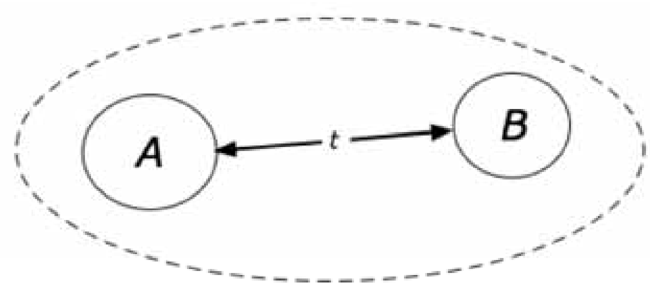
- So: If two objects interact with each other in such a way that the <u>external</u> forces on the pair cancel, then momentum is conserved.

$$\Delta\left(m_A\vec{v}_A + m_B\vec{v}_B\right) = 0$$

$$m_A\vec{v}_A^{\,i} + m_B\vec{v}_B^{\,i} = m_A\vec{v}_A^{\,f} + m_B\vec{v}_B^{\,f}$$



$\Rightarrow$



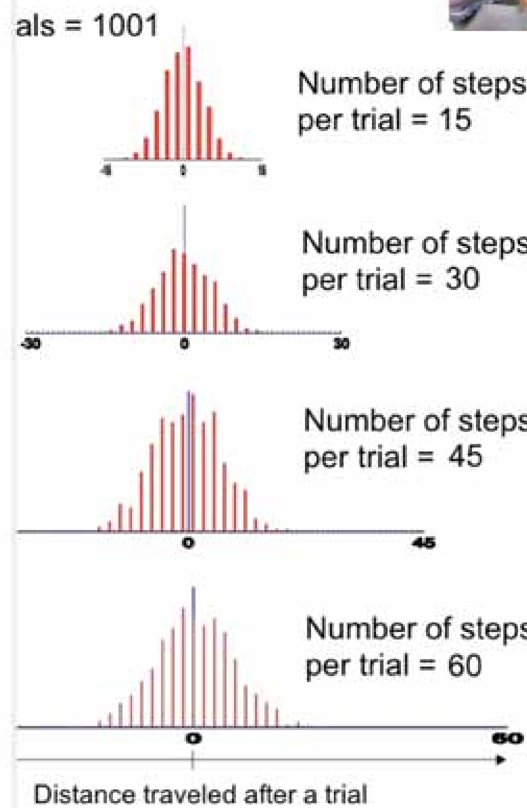
Foothold principles: Randomness

- Matter is made of of molecules in constant motion and interaction. This motion moves stuff around.
- If the distribution of a chemical is non-uniform, the randomness of molecular motion will tend to result in molecules moving from more dense regions to less.
- This is **not** directed but is an emergent phenomenon arising from the combination of random motion and non-uniform concentration.

Foothold ideas: Random walk in 1D

- As a result of random motion, an initially localized distribution will spread out, getting wider and wider. This phenomenon is called *diffusion*
- The width of the distribution will grow like

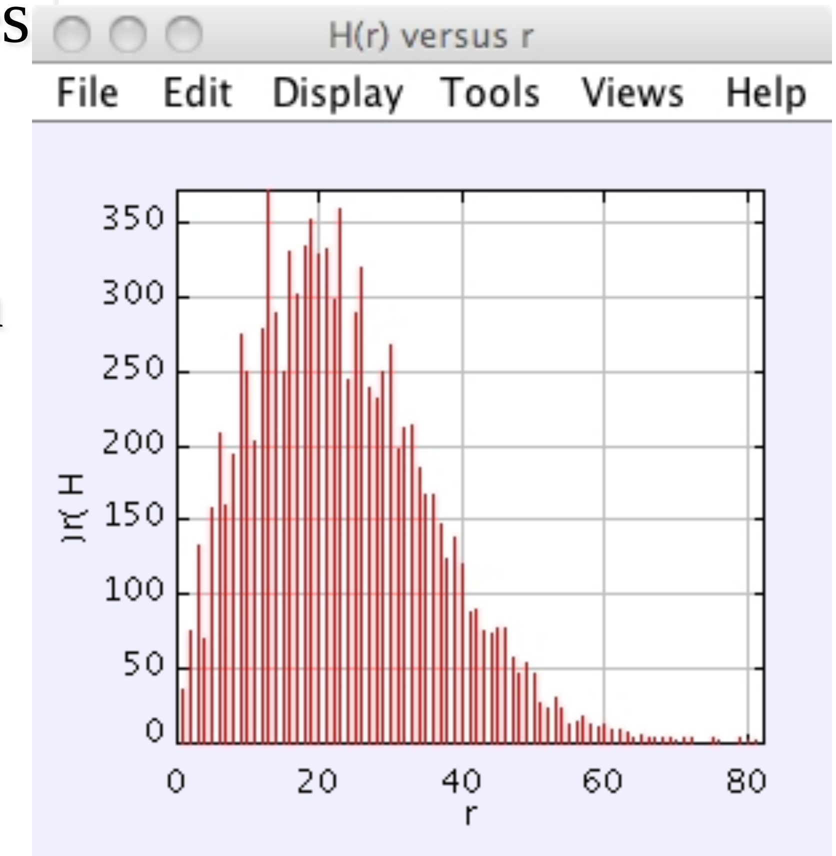
$$\left\langle (\Delta x)^2 \right\rangle = 2Dt$$



- *D* is called *the diffusion constant* and has dimensionality $[D] = L^2/T$

Foothold ideas: Random walk in 2D

- The density of walkers decreases uniformly as you get farther from the source.
- The total number within a given radius peaks – since the area within a radius r decreases to 0 as r gets small. ("phase space")
- The width of the peak grows with the square root of time.



Foothold ideas: Kinetic Theory I

- We model the gas as lots of tiny little hard spheres far apart (compared to their size) and moving very fast.
- The motions are in all directions and change directions very rapidly. A model saying that on the average the total momentum is 0 (and stays 0 by momentum conservation) is a good one.
- Because there are some many particles and the collisions so sensitive to initial conditions, we can't predict the motion of individual particles for long.
- Dilute gases satisfy the Ideal Gas Law, $pV = n_{moles}RT$

Foothold ideas: Kinetic Theory II

- Newton's laws tell us that motion continues forever unless something unbalanced tries to stop it, yet we observe motion always dies away.
- Our model of matter as lots of little particles in continual motion lets us "hide" the energy of motion that has "died away" at the macro level
  in the internal incoherent motion.
- The model unifies the idea of heat and temperature with our ideas of motion of macroscopic objects.

The Ideal Gas Law

Chemist's form

$$pV = n_{moles}RT$$

$$n_{moles} = \frac{N}{N_A}$$

$$R = k_B N_A$$

Physicist's form

$$pV = Nk_BT$$

$$p = nmv_x^2$$

$$\frac{3}{2}k_BT = \frac{1}{2}mv^2$$

Kinds of Matter

- Classify objects by how they deform.
  - *Solid*: don't change shape if you leave them alone or push on them (not too hard!)
  - *Gel*: look solid if you don't touch them but are "squishy" and change shape easily (jello, butter, clay,…)
  - *Liquid*: Have no shape of their own. Flow to fill a container but have constant volume.
  - *Gas*: Have neither shape nor volume but fill any container.
  - LOTS MORE!

Foothold ideas: Pressure

- A constrained fluid has an internal pressure –like an internal force at every point in all directions. (Pressure has no direction.)
- At a boundary or wall, the pressure creates a force perpendicular to the wall. $\vec{F} = p\vec{A}$
- The pressure in a fluid increases with depth. (Why?)

$$p = p_0 + \rho g d$$

- When immersed in a fluid, an object feels an (upward) BF equal to the weight of the displaced fluid. (Archimedes' Principle)