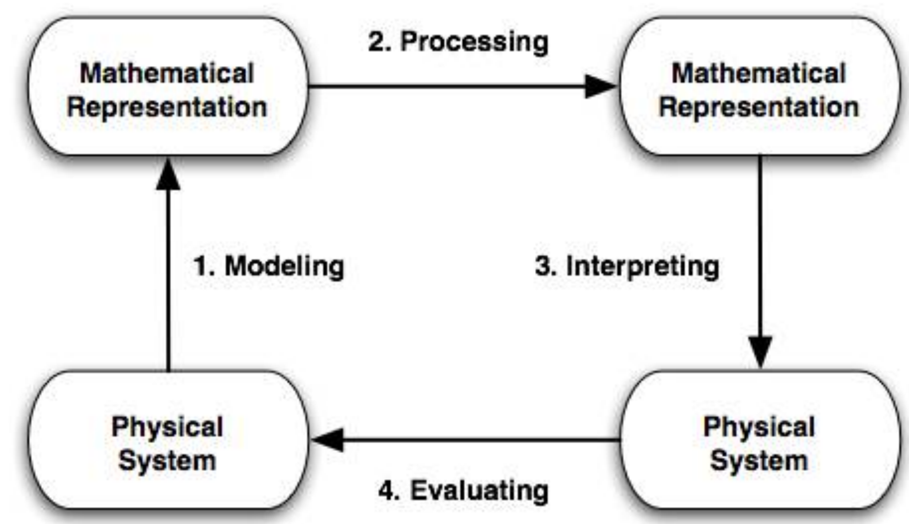
- **<u>Theme Music:</u> Take the A Train**
  Duke Ellington
- **<u>Cartoon:</u> Blondie**
  Chick Young

Foothold ideas: Modeling the world with math

- We use math to model relationships and properties.
- From the math we inherit ways to process and solve for results we couldn't necessarily see right away.
- Sometimes, mathematical models are amazingly good representations of the world. Sometimes, they are only fair. It is very important to develop a sense of when the math works and how good it is.
- Mostly, the math we use differs in important ways from the math taught in math classes.



Foothold ideas: Dimensional and unit analysis

- We label the kinds of measurement that go into assigning a number to a quantity like this:

  [x] = L means "x is a length"

  [t] = T means "t is a time"

  [m] = M means "m is a mass"

  [v] = L/T means "you get v by dividing a length by a time

- Units specify which particular arbitrary measurement we have chosen.
  - Units should be manipulated like algebraic quantities.
  - Units can be changed by multiplying by appropriate forms of "1" e.g. 1 = (1 inch)/(2.54 cm)

Foothold ideas: Dimensional analysis

- In physics we have different kinds of quantities depending on how measurements were combined to get them. These quantities may change in different ways when you change your measuring units.
- Only quantities of the same type may be equated (or added) otherwise an equality for one person would not hold for another. Equating quantities of different dimensions yields nonsense.
- Dimensional analysis tells us *how* something changes when we either
  - Change our arbitrary scale (passive change)
  - Change the scale of the object itself (active change)

Foothold Ideas:
Estimation – Quantifying experience

- **Measure your body parts**
- **Don't** look up data online or get it from friends!
- **Don't** use your calculator! Use 1-digit arithmetic
- **Do** figure out your estimations by starting with something you can plausibly know and scale up or down
- **Do** check your answer to see if it's reasonable
- **Do** learn a small number of Useful numbers

Useful numbers (people)

Numbers	
Number of people on the earth	~7 billion (7×10^9)
Number of people in the USA	~ 300 million (3×10^8)
Number of people in the state of Maryland	~ 5 million (5×10^6)
Number of students in a large state university	~30-40 thousand (3×10^4)

Useful numbers (distances)

Macro Distances	
Circumference of the earth	~24,000 miles (1000 miles/ time zone at the equator)
Radius of the earth*	$2/\pi \times 10^7$ m
Distance across the USA	~3000 miles
Distance across DC	~10 miles

Useful numbers (bio)

Bio Scales	
Size of a typical animal cell	~10-20 microns (10^{-5} m)
Size of a bacterium, chloroplast, or mitochondrion	~1 micron (10^{-6} m)
Size of a medium-sized virus	~0.1 micron (10^{-7} m)
Thickness of a cell membrane	~5-10 nm (10^{-8} m)

Foothold ideas: Measuring "where"

- In order to specify where something is we need a coordinate system. This includes:
  - Picking an origin
  - Picking perpendicular directions
  - Choosing a measurement scale
- Each point in space is specified by three numbers: (x, y, z), and a position vector– an arrow showing the displacement from the origin to that position.
- Vectors add like successive displacements or algebraically by $\vec{A} = A_x\hat{i} + A_y\hat{j}$ $\quad$ $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

Foothold ideas: Measuring "when"

- Time is a coordinate just like position
  - We need an origin (when we choose $t = 0$)
  - a direction (usually times later than 0 are +)
  - a scale (seconds, years, millennia)
- Note the difference between
  - clock reading, t
  - a time interval, Δt

This is like the difference between position and length!

Foothold ideas: 1D Velocity

- Velocity is the rate of change of position
- Average velocity

  = (how far did you go?)/(how long did it take you?)

$$\langle v \rangle = \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity = same (but for short Δt)

$$v = \frac{dx}{dt}$$

Can this velocity be negative as well as positive?

Foothold ideas: Vector velocity and speed

- A displacement – a change in position – has a direction. This means

  velocity = *displacement/time interval*

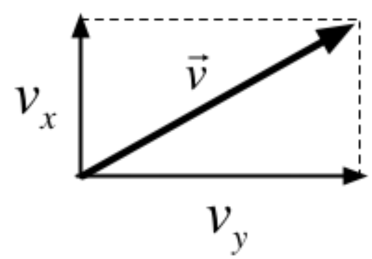
  has one too.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$v_x\hat{i} + v_y\hat{j} = \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$$

- We define speed as the magnitude of velocity. (No vector on this. Why?)

$$v = \sqrt{v_x^2 + v_y^2}$$



Foothold ideas:
3D Velocity

- Average velocity is defined by

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} = \frac{\text{vector displacement}}{\text{time it took to do it}}$$

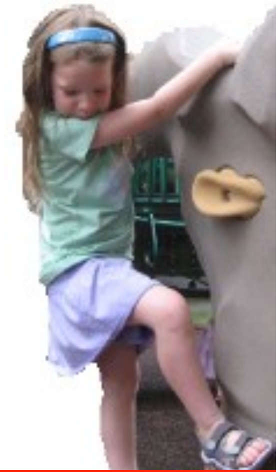
Note: an average velocity goes with a <u>time interval</u>.

- Instantaneous velocity is what we get when we consider a very small time interval (compared to times we care about)

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Note: an instantaneous velocity goes with a <u>specific time</u>.

Foothold ideas: Acceleration



- Average acceleration is defined by

$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time it took to do it}}$$

Note: an average acceleration goes with a <u>time interval</u>.

- Instantaneous acceleration is what we get when we consider a very small time interval (compared to times we care about)

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Note: an instantaneous acceleration goes with a <u>specific time</u>.

Conceptual ideas underlying Newton's Laws 1-3

1. Objects respond only to influences acting upon them at the instant that those influences act. (Object egotism) [Newton 0]
2. All outside effects on an object being equal, the object maintains its velocity (including direction). The velocity could be zero, which would mean the object is at rest. (Inertia) [Newton 1]
3. Every change in velocity an object experiences is caused by the object interacting with some other object – **forces**. (Interactions)

Conceptual ideas underlying Newton's Laws 4-6

4. If there are a lot of different objects that are interacting with the object we are considering, the overall result is the same as if we add up all the forces as vectors and produce a single effective force -- the **net force**. (Superposition)
5. When one object exerts a force on another, that force is <u>shared</u> over all parts of the structure of the object. (Mass)
6. The acceleration felt by an object at a given instant is the net force on the object at that instant divided by the object's mass. [Newton 2]
7. Whenever two objects interact, they exert forces on each other. (Reciprocity) [Newton 3]

Newton's Laws

- Newton 0:
  - An object responds to the forces it feels when it feels them.
- Newton 1:
  - An object that feels a net force of 0 keeps moving with the same velocity (which may = 0).
- Newton 2:
  - An object that is acted upon by other objects changes its velocity according to the rule

$$\vec{a}_A = \vec{F}_A^{net} / m_A$$

- Newton 3:
  - When two objects interact the forces they exert on each other are equal and opposite.

$$\vec{F}_{A \to B}^{type} = -\vec{F}_{B \to A}^{type}$$

Kinds of pForces

- pForces are what objects do to each other when they touch.
- If a pForce is a
  - Normal pForce N
  - Tension pForce T
  - Resistive pForces f
  - Weight pForce W
  - Electric pForce F^E
  - Magnetic pForce F^M
- Notation convention.

$$\vec{F}^{\text{type of force}}_{\text{(object causing force)}\rightarrow\text{(object feeling force)}}$$

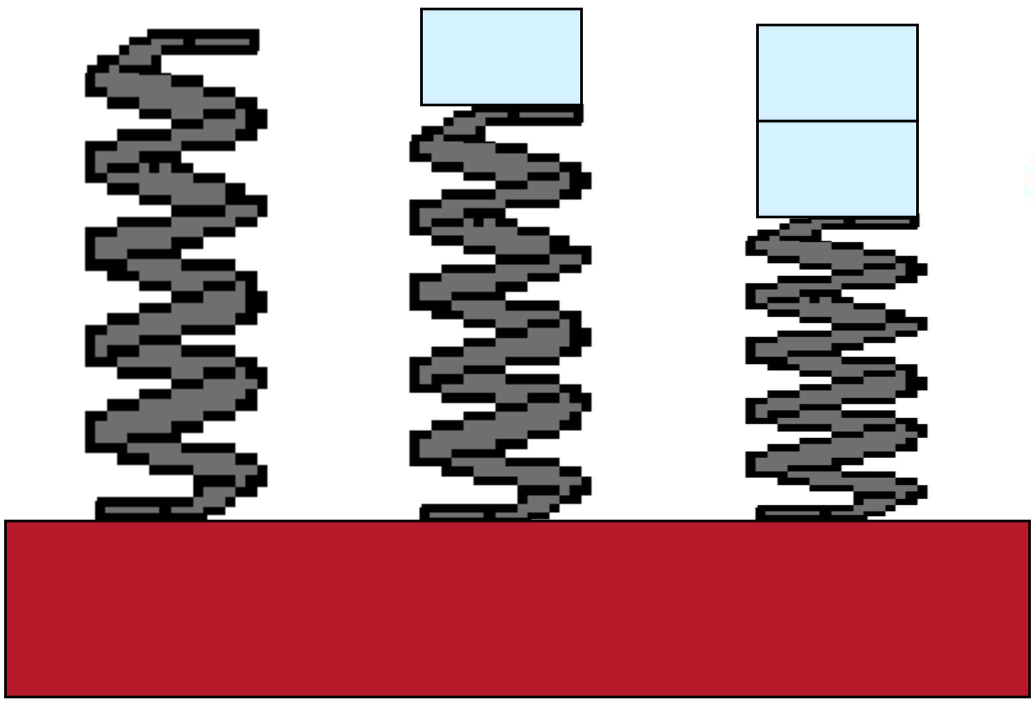
Tension:
The Ideal (Hooke's Law) Spring

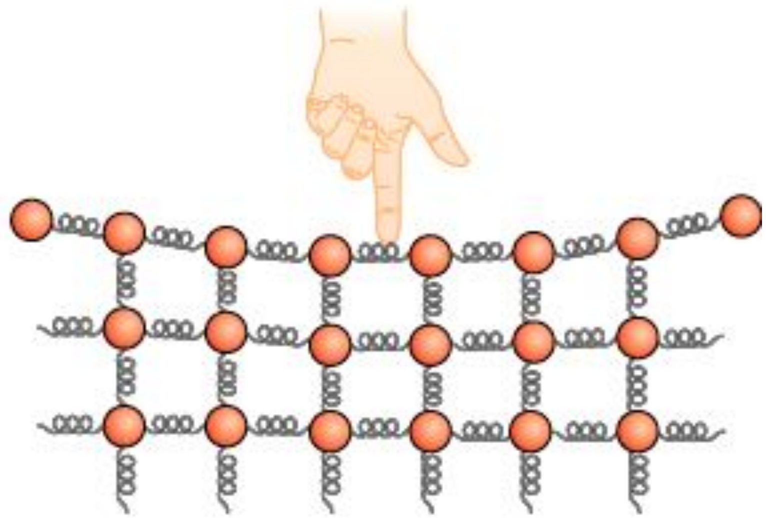
- An ideal spring changes its length in response to pulls (or pushes) from opposite directions.

$$T = k\,\Delta l$$

Δl = change in length (stretch or squeeze)

Normal Force works like a very stiff spring





Tension: Scalar vs. Vector

- Note we are using the word "tension" in two distinct ways!
- The "tension" for an internal bit of a spring, chain, or string has no direction (or rather, both directions at once). It is a <u>tension scalar</u>.
- When tension appears at the end of a spring, chain, or string, the choice of end gives us a direction and lets us create a <u>tension force</u>.

Foothold ideas: Resistive forces

- Resistive forces are contact forces acting between two touching surfaces that are parallel to the surface and tend to oppose the surfaces from sliding over each other.
- There are three types:
  - Friction (independent of velocity)
  - Viscosity (proportion to velocity)
  - Drag (proportional to the square of velocity)

Foothold Ideas: Friction

- Friction is our name for the interaction between two touching surfaces that is parallel to the surface.
- It acts to oppose the <u>relative motion</u> of the surfaces. It acts as if the two surfaces stick together a bit.
- Normal forces adjust themselves in response to external forces. So does friction – up to a point.

Static *Sliding*

$$f_{A\to B} \leq f_{A\to B}^{\max} = \mu_{AB}^{static} N_{A\to B} \qquad f_{A\to B} = \mu_{AB}^{kinetic} N_{A\to B} \qquad \mu_{AB}^{kinetic} \leq \mu_{AB}^{static}$$

- Friction can oppose motion or cause it.

Foothold ideas: Viscosity

- Viscosity is a resistive force that an object feels when it moves through a fluid as a result of the fluid sticking to the object's surface. This layer of fluid tries to slide over the next layer of fluid and the friction between the speeds that layer up and so on.
- The result is a force proportional to the velocity of the object.

$$\vec{F}^{viscous}_{fluid \to object} = -6\pi\mu R_{object}\vec{v}$$

Foothold ideas: Drag force

- The drag ("Newtonian drag") is a resistive force felt by an object moving through a fluid. It arises because the object is pushing fluid in front of it, bringing it up to the same speed it's going.
- The result is a force proportional to the density of the fluid, the area of the object, and the square of the object's velocity.

$$F^{drag}_{fluid \to object} = C d_{fluid} A_{object} v^2$$

Foothold Ideas: Gravity

- Every object (near the surface of the earth) feels a downward pull proportional to its mass:

$$\vec{W}_{E\to m} = m\vec{g}$$

where $\vec{g}$ is referred to as *the gravitational field.*

What object causes *W*?

- This is a pForce even though nothing touching the object is responsible for it.
- The gravitational field has the same magnitude for all objects irrespective of their motion and at all points.
- The gravitational field always points down.
- It is measured to be $g \approx 9.8\ \mathrm{N/kg}$

Why N/kg instead of m/s^2?