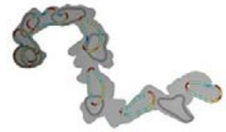
Physics 131- Fundamentals of Physics for Biologists I



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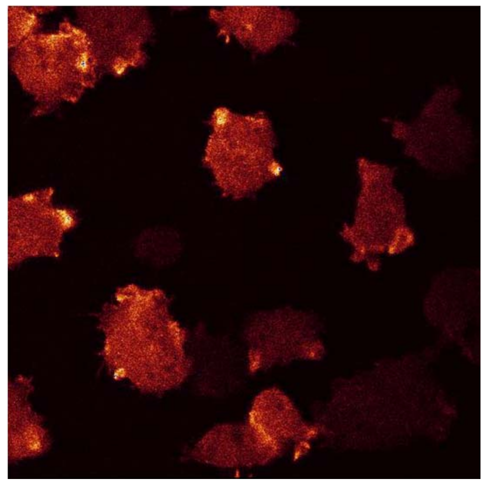


<u>Waves inside cells</u>
Waves are **biochemical and mechanical**

Outline

<u>Simple examples of emergent behavior of collections of molecules</u>

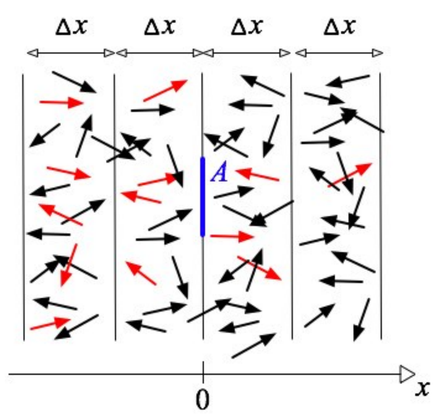
- Diffusion
  - Fick's Law
- Kinetic theory
  - Ideal Gas law

Foothold ideas: Kinetic Theory of a Gas

- We model the gas as lots of tiny little hard spheres far apart (compared to their size) and moving very fast.
- The motions are in all directions and change directions very rapidly. A model saying that on the average the total momentum is 0 (and stays 0 by momentum conservation) is a good one.
- Because there are some many particles and the collisions so sensitive to initial conditions, we can't predict the motion of individual particles for long.

Diffusion: Fick's law (1D analysis)

- Uniform fluid (black) containing (red) molecules with a varying concentration.
- Fluid molecules jiggle the (red) molecules around.

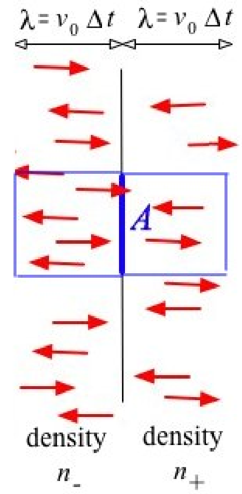


How many cross *A* in a time Δ*t*?

- Number hitting A from left
  $\frac{1}{2} n_{-} \left(A v_0 \Delta t \right)$
- Number hitting A from right
  $\frac{1}{2} n_{+} \left(A v_0 \Delta t \right)$
- Net flow across A
  $\frac{1}{2} \left(n_{-} - n_{+} \right) \left(A v_0 \Delta t \right)$
- Define flux (per unit area per unit time) as J therefore:

$$J A \Delta t = \frac{1}{2} \left(n_{-} - n_{+} \right) \left(A v_0 \Delta t \right)$$

$$J = \frac{1}{2} \Delta n \left(v_0 \right)$$



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Fick's law

- 1D result

$$J = -D \frac{dn}{dx} \quad D = \frac{1}{2} \lambda v_0$$

Does not yield the trajectory of molecules, but tells us, how a collection of molecules is distributed

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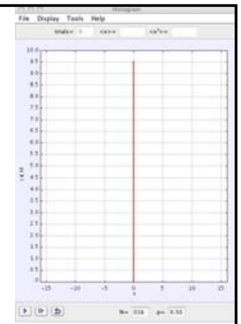
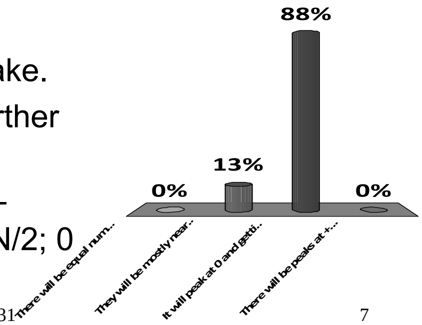
In this simulation, a "walker" starts at 0 and steps left and right with equal probability. We will let it take N steps. If we release a lot of walkers from the origin at once, on the average, what will our distribution of particles look like?



1. There will be equal numbers near +N/2 and –N/2
2. They will be mostly near 0 no matter how many steps you take.
3. It will peak at 0 and getting farther will decrease in probability.
4. There will be peaks at + and – values but not at +N/2 and –N/2; 0 will be less likely.



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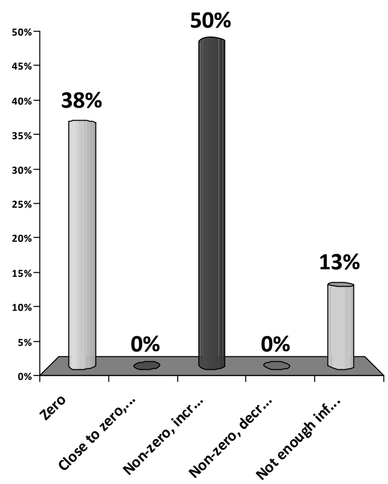
The average distance travelled is

1. Zero
2. Close to zero, does not depend on time
3. Non-zero, increases with time
4. Non-zero, decreases with time
5. Not enough information



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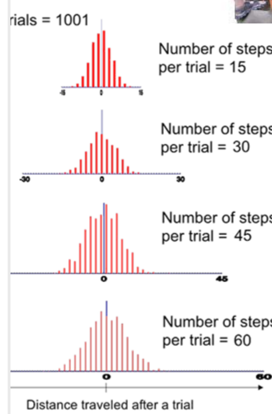
Foothold ideas: Random walk in 1D

- As a result of random motion, an initially localized distribution will spread out, getting wider and wider. This phenomenon is called *diffusion*
- The square of the average distance traveled during random motion will grow with time:
- $$\left\langle (\Delta x)^2 \right\rangle = 2D\Delta t$$
- *D* is called *the diffusion constant* and has dimensionality [*D*] = L^2/T



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The gradient

- If we want to take the derivative of a function of one variable, *y* = *df/dx*, it's straightforward.
- If we have a function of three variables – *f(x,y,z)* – what do we do?
- The gradient is the **vector derivative.** To get it at a point (*x*,*y*,*z*)
  - Find the direction in which *f* is changing the fastest.
  - Take the derivative by looking at the rate of change in that direction.
  - Put a vector in that direction with its magnitude equal to the <u>maximum</u> rate of change.
  - The result is the vector called $\vec{\nabla} f$

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Fick's law

- 1D result

$$J = -D\frac{dn}{dx} \quad D = \tfrac{1}{2}\lambda v_0$$

- For all directions (not just 1D) Fick's law becomes

$$\vec{J} = -D\vec{\nabla}n$$

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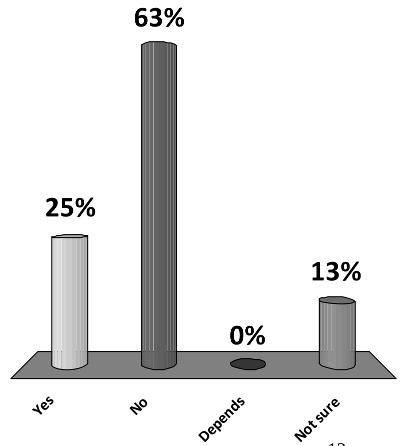
Does n have the same dimension in both equations?

$$J = -D\frac{dn}{dx} \qquad \vec{J} = -D\vec{\nabla}n$$

1. Yes
2. No
3. Depends
4. Not sure



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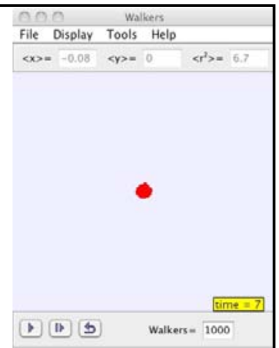
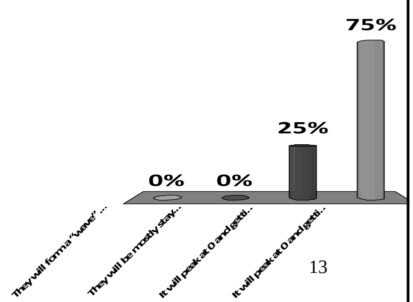
In this simulation, a lot of “walkers” starts in 2D near 0 and step in a random directions with equal probability. As time grows, what will happen to the distribution of walkers – number as a function of distance??



1. They will form a “wave” – a ragged ring of particles moving outward.
2. They will be mostly stay near 0 no matter how long you wait.
3. It will peak at 0 and getting farther will decrease in probability, the distribution remaining mostly the same.
4. It will peak at 0 and getting farther will decrease in probability, the distribution getting wider with time.



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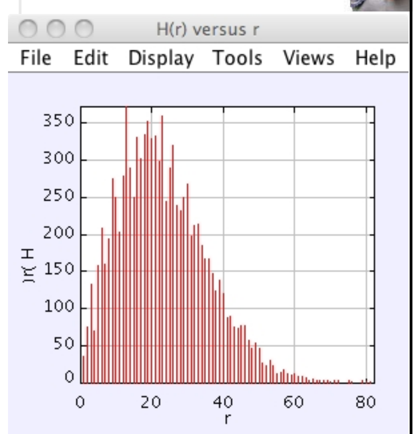
Foothold ideas: Random walk in 2D

- The density of walkers decreases uniformly as you get farther from the source.
- The total number within a given radius peaks – since the area within a radius r decreases to 0 as r gets small.
- The average squared displacement grows with time :

$$\left\langle (\Delta r)^2 \right\rangle = 4D\Delta t$$



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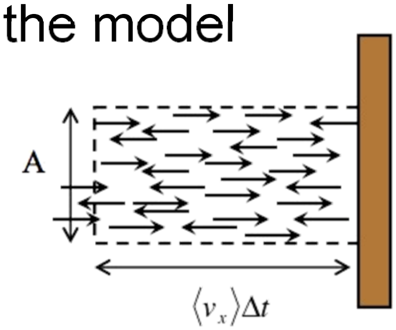
Can we understand the ideal gas law from the motion of molecules?

- Dilute gases satisfy the Ideal Gas Law

$$pV = n_{moles}RT$$

Summarizing the model

- In between collisions each molecule moves in a straight line – ignoring gravity. (We've used N1!)
- Ignore up and down motions.
- Momentum change of a molecule that bounces off the wall exerts a force on the wall.
- The force on the wall will be the change in momentum of all the molecules that bounce off the wall in a time Δ*t* divided by Δ*t.*
- Calculate this using density.



$$F = \left(\frac{2mv_x}{\Delta t}\right)\left(\tfrac{1}{2} nAv_x\Delta t\right) = nmv_x^2 A$$

$$p = \frac{F}{A} = nmv_x^2 = \frac{N}{V} mv_x^2 = \frac{N}{V}\tfrac{1}{3} m\langle v^2\rangle$$

$$pV = N\frac{2}{3}\left(\frac{1}{2}m\langle v^2\rangle\right)$$

The Ideal Gas Law

Chemist's form

$$pV = n_{moles}RT$$

$$n_{moles} = \frac{N}{N_A}$$

$$R = k_B N_A$$

Physicist's form

$$pV = Nk_B T$$

$$p = nmv_x^2$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

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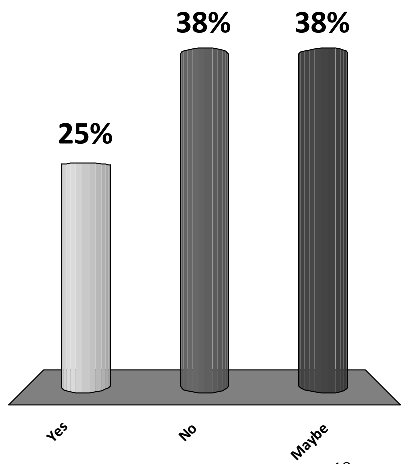
Could you still use the ideal gas law to analyze the inside of a cell

1. Yes
2. No
3. Maybe



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