

1. An object of mass m is attached to a horizontal spring. The mass is stretched to a displacement A from equilibrium and released from rest undergoing simple harmonic oscillations on a frictionless surface with period T . (3+2 points)

- a) By how much does the period increase or decrease when the mass is doubled? Give the physical reason for the change.

$T = 2\pi\sqrt{m/k}$ When the mass is doubled the period increases by a factor of $\sqrt{2}$. Physically, this is because a more massive object has more inertia, so it "takes more effort" and more time to change the motion.

- b) By how much does the period increase or decrease when the initial displacement (A) is doubled? Explain.

The period does not depend on the amplitude since amplitude is just an initial condition. This means that objects that have a higher amplitude (mass m and spring constant k being the same) will move faster to be able to maintain the same period.

2. What happens to the time period of a pendulum of length 1 m when it is taken to the moon? (3 points) A qualitative explanation will do.

The period for a pendulum is given by $T = 2\pi\sqrt{L/g}$. When you take it to the moon, where gravity is lower, the period increases. This means that the pendulum takes longer to complete each oscillations and runs slower.

3. A mass oscillates when attached to a spring on a horizontal frictionless surface with amplitude of 3m and maximum velocity of 0.75 m/s. Consider a single cycle of motion for the following questions. How many times during a cycle is

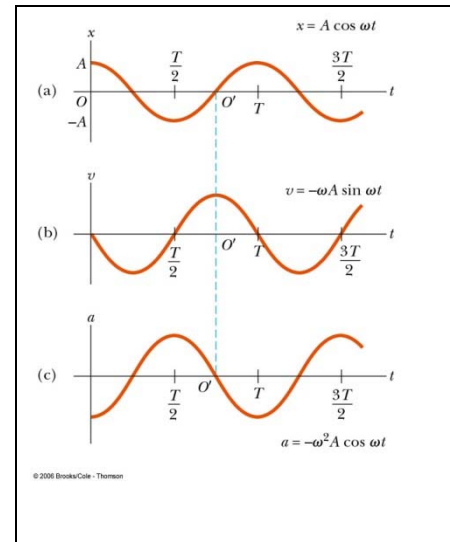
- the velocity zero?
- the velocity -0.5 m/s?
- the "speed" 0.5 m/s?
- the potential energy zero? (6 points) (Hint: X vs. t and V vs. t graphs will help.) (6)

In the pictures shown, the cycle starts at $t=0$ and ends at $t=T$ (which is not included). The velocity is -0.5 m/s twice. See the second graph. Since 0.5 is less than 0.75 , the maximum, the velocity is 0.5 twice in the interval 0 to T .

The velocity is zero twice: at the start and at $t=T/2$. The zero velocity shows up as zero slope in X vs t graph.

The speed 0.5 is less than the maximum, it occurs 4 times in a cycle. (2 times for positive velocity and 2 times for negative velocity)

The potential energy is zero when the displacement is zero, which happens twice in the interval.



4. The motion of an object of mass 2 kg attached to a horizontal spring on a frictionless surface is described by the equation $X(t) = (0.30 \text{ m}) \cos(\pi t/6)$. What are the amplitude, period, maximum velocity, maximum acceleration, and the mechanical energy of the mass? (6 points)

Compare the given equation to $X = A \cos(\omega t)$. The amplitude is 0.30 m.

ω is the coefficient of t , which in this case is $\pi/6$ rad/s. Period is $T = 2\pi/\omega = 12$ seconds.

Maximum velocity is $\omega A = (\pi/6 \text{ rad/s}) * (0.30 \text{ m}) = 0.157 \text{ m/s}$.

Maximum acceleration is $\omega^2 A = (\pi/6 \text{ rad/s})^2 * (0.30 \text{ m}) = 0.0822 \text{ m/s}^2$.

The mechanical energy is $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(2 \text{ kg})(.157 \text{ m/s})^2 = 0.0247 \text{ J}$.