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PHYS 121 **EXAM 3** **May 1, 2009**
Prof. S. M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1) There are four (4) problems in this exam. Please make sure that your copy has all of them.

2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.

3) Write your answers on the sheets provided.

4) Do not forget to write the units.

5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

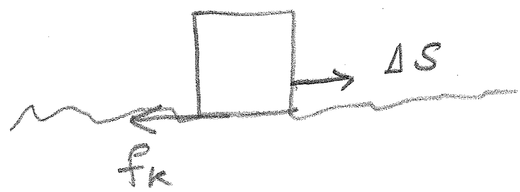
God Bless You!

$$k_B = 1.383x10^{-23}\ J/K$$
$$N_A = 6.02x10^{23}$$
$$m_p = 1.6x10^{-27}\ kg$$

Problem 1a: Show that the work done by the force of friction is always negative? (5)

WORK: $\Delta W = \vec{F} \cdot \vec{\Delta S}$

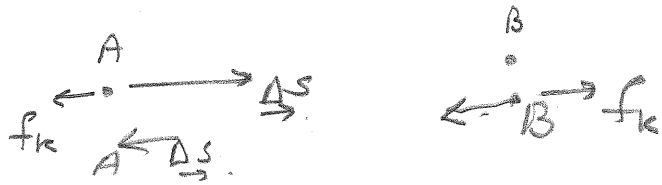
The force of (kinetic) friction $\vec{f_k}$ always acts opposite to the displacement vector $\vec{\Delta S}$



So $\Delta W = \vec{f_k} \cdot \vec{\Delta S} = f_k \Delta S \cos(180^\circ)$
$= -f_k \Delta S$.

Problem 1b: Why is it not possible to define a potential energy for the force of friction? (5)

Consider a path from A to B. As shown above



going from A to B work done $\Delta W_{AB} = -f_k \Delta S_{AB}$

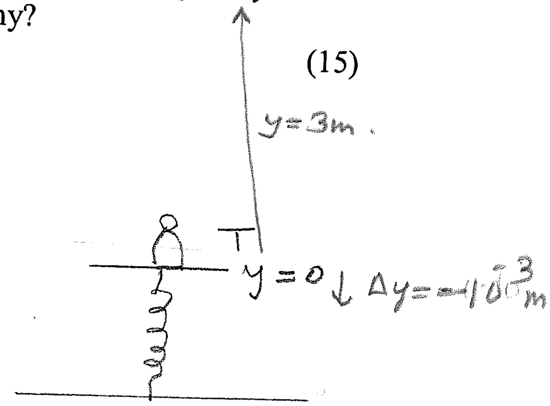
Next go from B to A. $\Delta W_{BA} = -f_k \Delta S$ also

Total work A→B→A $= -2 f_k \Delta S$

for a Conservative force total work would be zero.

Problem 1c: The Toy (T) is placed on a platform as shown. You squeeze the spring by 1mm and let go. If the toy has a mass of 10^{-2}kg and it jumps up to a height of y= 3m before falling back down, what is the spring constant? Why?
[Neglect air friction, g= 9.8 m/s^2]

(15)



Conservation of Energy.
No external force, So

$K_f + U_f = K_i + U_i$

$K_i = 0$

$U_i = \frac{1}{2} k (\Delta y)^2$ ← spring

$+ Mg\Delta y$ ← Earth-mass

$K_f = 0$

$U_f = Mg\,y$

$Mg(3) = \frac{1}{2} k (10^{-3})^2 - Mg(10^{-3})$

So $k = \frac{2 \times 10^{-2} \times 9.8 [3 + 10^{-3})}{10^{-6}} = 5.9 \times 10^5 \, N/m$

Problem 2a: Two objects of masses M and 3M have the same linear momentum (magnitude). Which one will have the larger kinetic energy and by what factor? Why?

(5, 5)

Kinetic Energy $K = \frac{p^2}{2M}$.

let

$M_1 = M$

$M_2 = 3M$

$K_1 = \frac{p_1^2}{2M_1}$, $K_2 = \frac{p_2^2}{2M_2}$.

$\frac{K_1}{K_2} = \frac{p_1^2}{2M_1} \cdot \frac{2M_2}{p_2^2}$ b/c $p_1 = p_2$

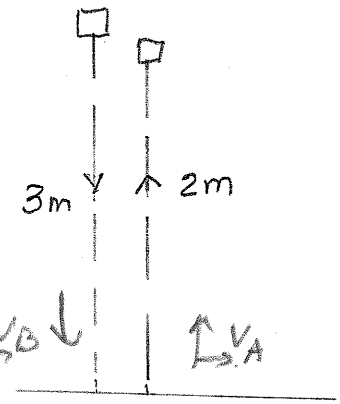
$= \frac{M_2}{M_1} = 3$

Smaller mass has 3 times the kin Energy of the bigger mass.

Problem 2b: A 1kg object drops from rest from a 3m high table. On bouncing from the Earth it rises to a height of 2m.

(i) What is the impulse received by the object?
(ii) What is the impulse received by the Earth?
(iii) Is kinetic energy conserved in this collision?
(Justify your answers)

(15)



First use conservation of energy to calculate velocities before & after bounce

$K_f + U_f = K_i + U_i$

Bef $\frac{1}{2} M V_B^2 + 0 = 0 + Mg(3)$

$\vec{V}_B = -\sqrt{6Mg}$ m/s $\hat{y} = -7.7$ m/s $\hat{y}$

Aft $0 + Mg(2) = \frac{1}{2} M V_A^2 + 0$

$\vec{V}_A = +\sqrt{4Mg}$ m/s $\hat{y} = +6.3$ m/s $\hat{y}$

(i) Impulse Received by object

$\vec{J}_{obj} = M(\vec{V}_A - \vec{V}_B) = +14$ kg-m/s $\hat{y}$

(ii) By conservation of momentum $\vec{J}_{obj} + \vec{J}_{Earth} = 0$

so $\vec{J}_{Earth} = -14$ kg-m/s $\hat{y}$

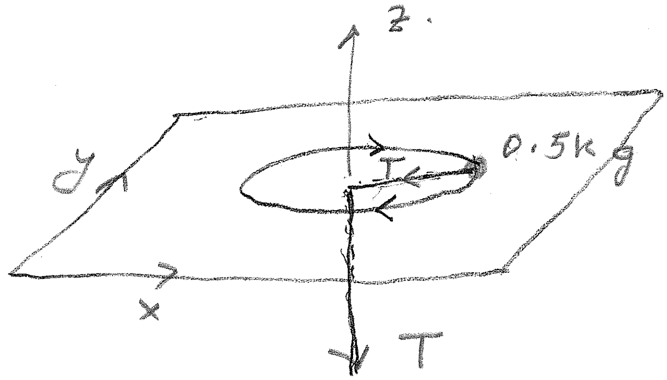
(iii) change in kin En. of Earth $\Delta K_{Earth} = \frac{J_E^2}{2M_E} \approx 0$

Since object loses K.E. and Earth gains none. K.E. NOT CONSERVED

Problem 3a: As shown an object of mass 0.5kg is in uniform circular motion on a smooth horizontal table (*xy*-pl). If the radius is 0.5m and it is going clockwise at 2m/s what is its angular momentum? If you pull on the string will the angular momentum change? Justify your answers.

(5, 5)

Angular momentum



$\vec{l} = \vec{r} \times \vec{p}$

$= -mrv\hat{z} = mr^2\vec{\omega}$

$= -0.5 \times 0.5 \times 2\,\hat{z}$

$= -0.5$ kg-m²/sec $\hat{z}$.

[$\vec{\omega} = -\frac{v}{r}\hat{z} = -\frac{2}{0.5}$ rad/s $\hat{z}$]

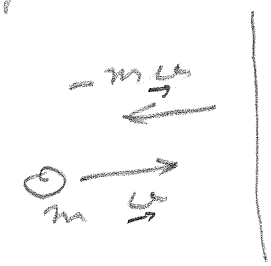
If you pull on the string the magnitude of T changes but on the object $\vec{T} \parallel \hat{r}$ so there is no torque due to T. Since $\vec{\tau} = 0$, $\vec{l}$ = Const. [conservation of angular mom^m].

Problem 3b: Why does a gas exert pressure on the walls of its container? Explain? (10)

First

At a finite temperature, atoms of the gas are in random motion

Second Each time an atom of mass m bounces off a wall, since the collision is totally elastic and the wall picks up NO kinetic energy, it picks up a momentum $\Delta \vec{p}_{wall}(\text{one collision}) = +2m\vec{u}$



Third Let us calculate the number of collisions an area A of the wall gets every second, call this N_s.

Then change of momentum of wall per sec

$$\Delta \vec{p}_{wall}(\text{per sec}) = 2m\vec{u}\, N_s$$

But $\dfrac{\Delta \vec{p}}{\Delta t} = \langle \vec{F}_s \rangle$ ← Av. force on wall of Area A

Pressure $P = \dfrac{F}{A}$!

Problem 3c: In a container there is a mixture of He (m_{He}=4mp) and Krypton (m_{kr}= 84 mp) (where mp is the proton mass) at 500K. Which particles will have the higher kinetic energy? Why?

(5)

The two gases are in equilibrium so they have the same temperature and kinetic energy

$$\frac{1}{2} m \langle u^2 \rangle = \frac{3}{2} k_B T$$

i's determined only by Temperature so they both have same kin Energy

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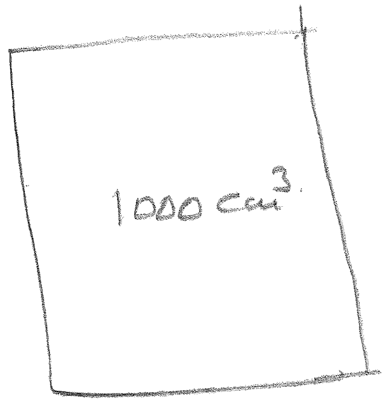
<u>Problem 4a</u>: A beaker of Aluminum is filled to the brim with $1000\,cm^3$ of water at 20C. If its temperature is raised to 80C how much water will spill out if $\alpha_{Al} = 23\times10^{-6}\,C^{-1}$ (linear expansion) and $\beta_{Water} = 210\times10^{-6}\,C^{-1}$. (volume expansion) (15)

Linear Expansion

$$\ell = \ell_0\left[1+\alpha(\theta_f-\theta_i)\right]$$

Vol. Expansion

$$V = V_0\left[1+\beta(\theta_f-\theta_i)\right]$$



For Aluminum

$$V_{80}^{Al} = V_{20}\left[1+3\times23\times10^{-6}(80-20)\right]$$

For Water

$$V_{80}^{W} = V_{20}\left[1+210\times10^{-6}(80-20)\right]$$

Spill

$$V_{spill} = V_{80}^{W} - V_{80}^{Al}$$

$$= 1000\left[210-69\right]\times10^{-6}\times60\ cm^3$$

$$= 1000\times141\times6\times10^{-3}\ cm^3$$

$$= \underline{0.85\ cm^3}$$

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Problem 4b: Write down the dimensions (not units) of (Hint Acceleration is LT^{-2})

(i) Pressure
(ii) Temperature
(iii) Angular momentum
(iv) Kinetic Energy

(10)

Pressure $F/A \rightarrow \frac{MLT^{-2}}{L^2}$ $ML^{-1}T^{-2}$

Temperature Θ Θ^1

ANGULAR MOMENT $\ell = mrv$ ML^2T^{-1}

Kinetic energy $K = \frac{1}{2}MV^2$ ML^2T^{-2}