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PHYS121 EXAM II April 3, 2009

Prof. S. M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1. There are four (4) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In starting vectors give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

God Bless You!

Problem1a We have introduced several forces: weight, normal force, tension, spring force and friction. What is the fundamental difference between the force of friction and the other forces? (5)

ALL OF THE OTHER FORCES INITIATE MOTION. FRICTION CANNOT INITIATE MOTION, IT CAN ONLY OPPOSE MOTION INITIATED BY AN APPLIED FORCE. INDEED, IF THE LATTER IS LESS THAN $\mu_s n$ THERE IS NO MOTION AT ALL!

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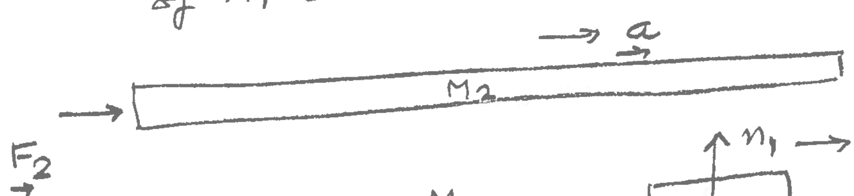
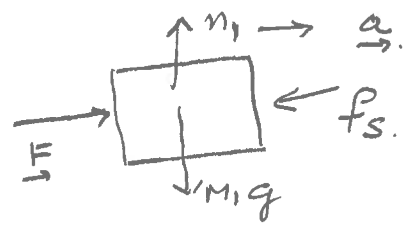
<u>Problem 1b</u> The figure shows two masses $M_1 = 2kg$, $M_2 = 5kg$. M_2 is lying on a smooth frictionless surface while the coefficient of static friction between M_1 and M_2 is 0.3. A force $\underset{\rightarrow}{F}$ is applied to M_1 as shown. (i) What is the force that moves M_2? (ii) What is the total force on M_1? (iii) What is the largest value of $\underset{\rightarrow}{F}$ so that M_1 and M_2 move together? Why? (5, 5, 10)

(i) The force that moves M_2 is the force of static friction between the surface of M_1 and M_2, $f_s \le \mu_s n_1$

$n_1 = M_1 g$



(ii) FORCES ON M_1



(iii) Newton's Law $M\vec{a} = \sum \vec{F}_i$ at that pt. at that time.

First M_2: $M_2 a = f_s$.

So $M_2 a \le \mu_s M_1 g$

$a \le \mu_s \frac{M_1}{M_2} g$.

Next M_1: $M_1 a = F - f_s$

Max. $\vec{F}$ when $f_s = \mu_s n_1$, $a = \frac{\mu_s M_1}{M_2} g$

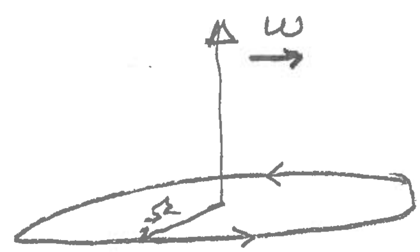
Max $\vec{F} = \left\{M_1\left[\frac{\mu_s M_1}{M_2} g\right] + \mu_s M_1 g\right\}\hat{x}$

$= \mu_s M_1 g\left[1 + \frac{M_1}{M_2}\right]\hat{x} = 0.3 \times 2 \times 9.8\left[1 + \frac{2}{5}\right]\hat{x}$

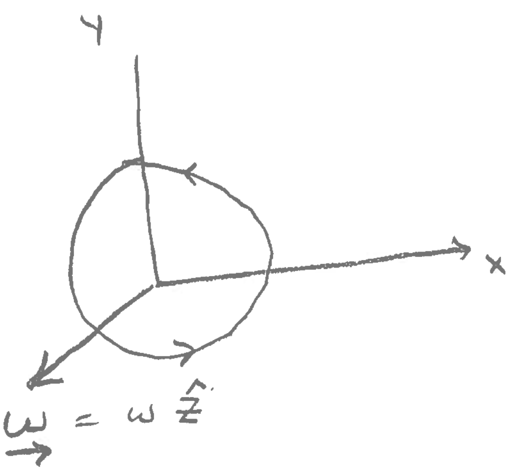
$\simeq 8.4 N \hat{x}$

<u>Problem 2a</u> In uniform circular motion an object travels on a circle of radius R at a constant speed. To describe the motion precisely, we need four vectors: position ($\underline{r}$), velocity ($\underline{v}$), centripetal acceleration ($\vec{a}_c$) and angular velocity ($\underline{w}$). Which of these four vectors does not change its direction with time? Why? (5)

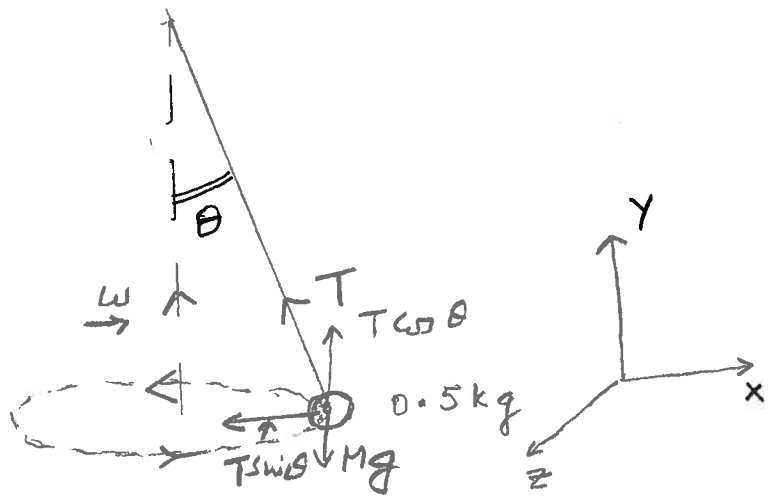
The angular velocity vector is fixed because it is perpendicular to the circle



or



<u>Problem 2b</u> Shown is a conical pendulum. A mass of 0.5 kg is hanging from the ceiling with a light string. It is moving counterclockwise on a horizontal circle of radius 0.05 m at 120 revolutions per minute. What is the angular velocity? Why? What is the value of the angle Θ and the tension T in the string? Why? (6, 7, 7)



(i) 120 revs/min $= \frac{120}{60}$ revs/sec.

$\vec{\omega} = 2\pi \times 2$ rad/s $\hat{y}$ $= 12.6$ rad/s $\hat{y}$

(ii) Mass is moving on circle so it needs a Centripetal force $\vec{F}_c = -MR\omega^2\hat{x}$

The forces acting on it are T & Mg.

Along y there is $\equiv m$ so $T\cos\theta - Mg = 0 \rightarrow (1)$

Force available $\vec{F}_{A} = -T\sin\theta\hat{x}$ which provides F_c hence

$T\sin\theta = MR\omega^2$

From (1) $T\cos\theta = Mg$

so $\tan\theta = \frac{R\omega^2}{g} = \frac{0.05 \times (4\pi)^2}{9.8} = 0.80$

$\theta = \underline{39°}$

$T = \frac{Mg}{\cos\theta} = \frac{0.5 \times 9.8}{\cos 39} = \frac{0.5 \times 9.8}{0.78} = \underline{6.3N}$

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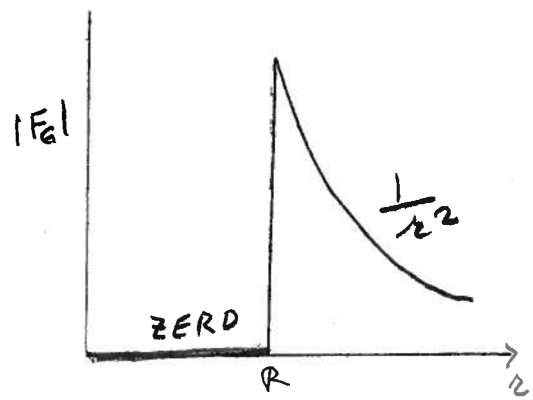
Problem 3a Newton's law of Gravitation is written as

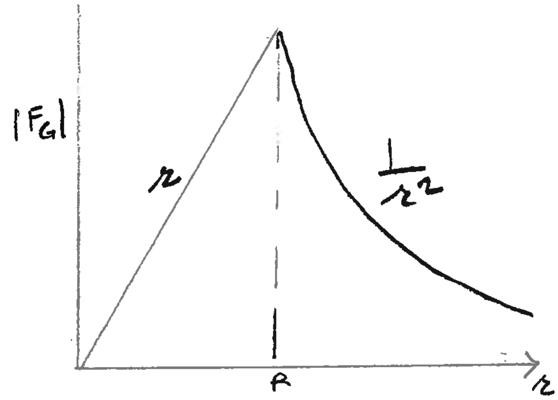
$$\underset{\rightarrow}{F_G} = -\frac{GM_1M_2}{r^2}\hat{r}$$

Why is there a "minus" sign on the right side of this equation? (5)

The minus sign tells us that the forces between M_1 & M_2 are attractive

Problem 3b The pictures show $|F_G|$, the force on a point mass m, located a distance r from the center of a sphere of radius R. Which of these spheres is hollow (shell like)? Why? (10)

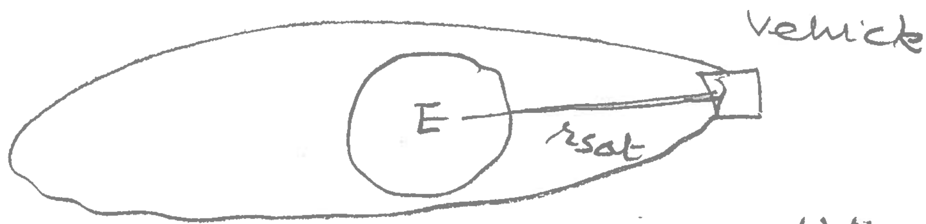




The sphere on the left is hollow as calculated by Newton.

The sphere on the right is a solid of uniform density because as r increases more & more of the mass of the sphere exerts a force on the point mass.

<u>Problem 3c</u> Why are astronauts inside a vehicle in stable orbit around the Earth said to be "weightless"? (Please do not write that they are in "free fall".) (10)



For the vehicle to have stable orbit it needs a Centripetal Force

$$\vec{F}_C = - M_{sat}\, r_{sat}\, \omega_{sat}^2\, \hat{r}$$
$$= - M_{sat}\, a_c\, \hat{r} \qquad a_c = \text{centripetal acc.}$$

The Earth provides $\vec{F}_G = - \dfrac{G M_E M_{sat}}{r_{sat}^2}\hat{r} = - M_{sat}\, g(r_{sat})\,\hat{r}$

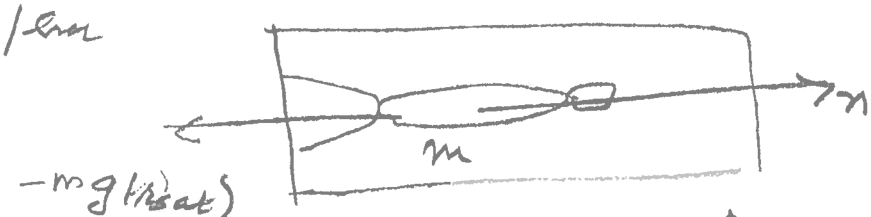
For stable orbit $\vec{F}_G = \vec{F}_C$ hence $a_c = g(r_{sat})$

Now put astronaut in vehicle

Forces on him/her are

$n\hat{r}$

$-mg(r_{sat})\hat{r}$



and he/she also has an $\vec{a}_c = - a_c\,\hat{r}$

So $\left[n - mg(r_{sat})\right] = - m a_c$

$n = m\left[g(r_{sat}) - a_c\right] = 0!$

That makes for weightlessness

Problem 4a What is a rigid body? (5)

A rigid body consists of many mass points m_i, m_j located at $\underset{\to}{r}_i$, $\underset{\to}{r}_j$, respectively, such that $(\underset{\to}{r}_i - \underset{\to}{r}_j)$ is a constant.
It neither changes shape nor size when it moves.

Problem 4b What is the difference between force and torque? (10)

FORCE CAUSES LINEAR ACCELERATION (TRANSLATION)

$M \underset{\to}{a} = \sum \underset{\to}{F}_i$ at that pt. at that time

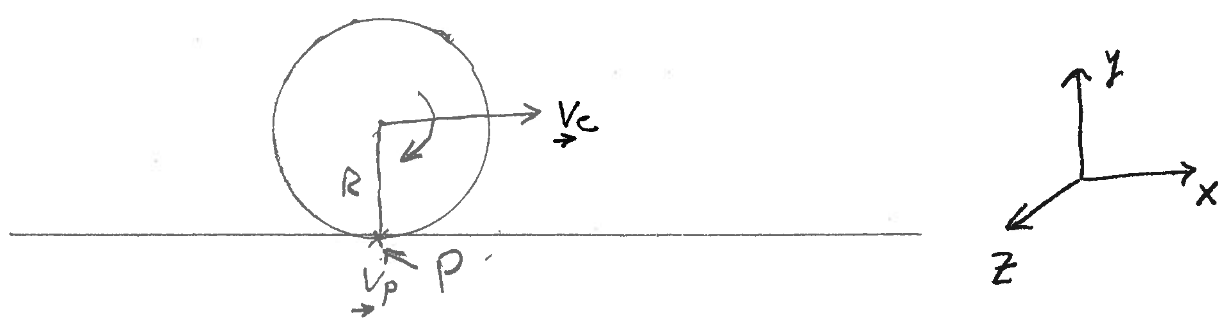
TORQUE: CAUSES ANGULAR ACCELERATION (ROTATION)

$I \underset{\to}{\alpha} = \sum \underset{\to}{\tau}_i$ about given axis

To have a torque one must apply a force $\underset{\to}{F}$ at some distance $(\underset{\to}{r})$ away from the axis of rotation so that

$\underset{\to}{\tau} = [\underset{\to}{r} \times \underset{\to}{F}]$

Problem 4c A disk of diameter 20cm is rolling without slipping on a horizontal surface. If the velocity at the center is $\underset{\rightarrow}{v_c} = 10 m/s\hat{x}$. What must be the angular velocity of the disk? Why? (10)



IN ORDER TO ROLL W/O SLIP THE VELOCITY AT THE POINT OF CONTACT (P) MUST BE ZERO AT ALL TIMES. If the disk has an angular velocity $\vec{\omega} = -\omega\hat{z}$ it will acquire a tangential velocity at P

$$\vec{V}_t = -R\omega\hat{x}$$

So $\vec{V}_p = V_c\hat{x} - R\omega\hat{x} = 0$

requires $\omega = \dfrac{V_c}{R} = \dfrac{10}{0.1}$ rad/s

$$\vec{\omega} = -100 \text{ rad/s } \hat{z}$$