D/8

PHYS121 EXAM II April 1, 2009

Prof. S. M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

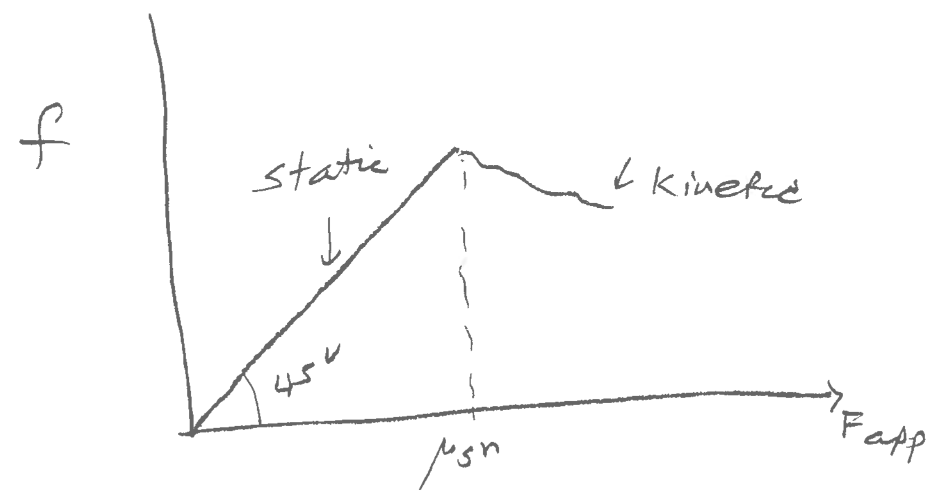
Notes

1. There are four (4) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In starting vectors give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

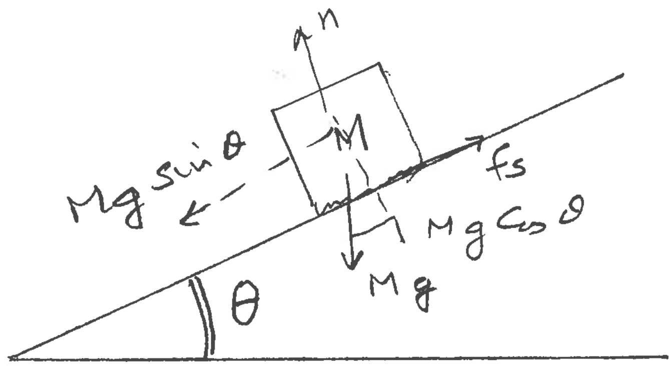
God Bless You!

Problem 1a What is the difference between static friction and kinetic friction? (7)

The force of friction arises when a force is applied to move a solid surface past another. If the applied force is less than $\mu_s n$ no motion occurs (static friction). When $F_{app} > \mu_s n$ motion begins (kinetic)



Problem 1b An object of mass M is lying on a rough inclined plane where the coefficient of static friction is 0.5. If you start increasing the angle Θ for what value of Θ will the object start to slip? Why? (18)



FORCES ON M are drawn

Here ΣF perpendicular to incline so

$n - Mg\cos\theta = 0$

hence $f_s \le \mu_s Mg \cos\theta$.

and Fapp is $Mg\sin\theta$ down the incline.

Slip will begin when

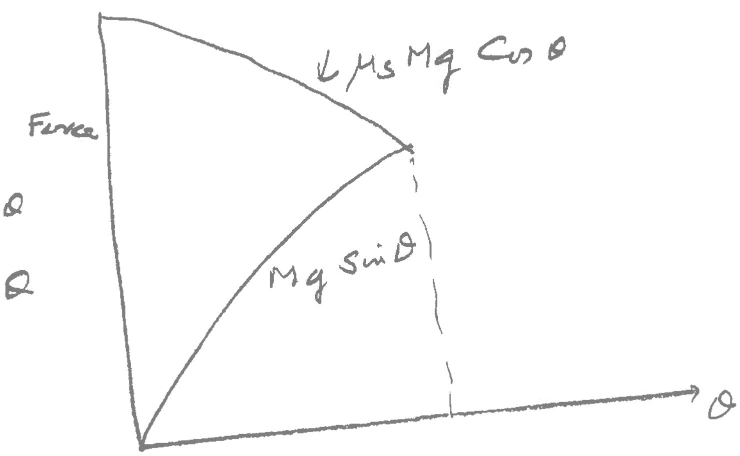
$Mg\sin\theta$ becomes equal to $\mu_s Mg\cos\theta$

$Mg\sin\theta = \mu_s Mg\cos\theta$

$\tan\theta = \mu_s$

$= 0.5$

$\underline{\theta = 26.6^\circ}$



Problem 2 An object of mass 0.5 kg is moving clockwise, in the *xy*-plane, at constant speed. It makes 30 revolutions per minute and the radius of the orbit is 2.5m.

a) What is the period? (2)

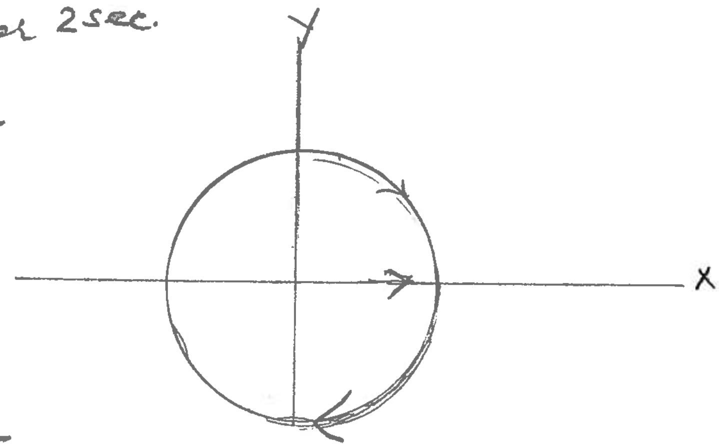
b) What is the angular velocity vector? (5)

At $t = 0$, the position vector is $\underset{\sim}{r}(0) = 2.5m\hat{x}$. Calculate and draw the i) position ii) velocity and iii) acceleration vector at $t = 1.5$secs (6, 6, 6)

a) 30 revs per min. (60sec).

1 rev per 2sec.

T = 2sec



b) $\omega = \frac{2\pi}{T}$

$\underset{\rightarrow}{\omega} = -\frac{2 \times 3.14}{2}$ rad/s $\hat{z}$. (right hand rule)

$= -3.14$ rad/s $\hat{z}$.

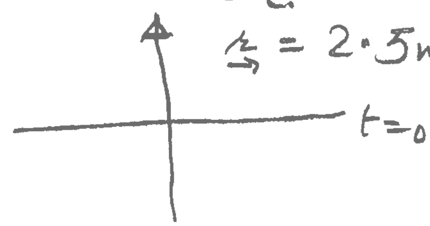
t=1.5sec.

$\underset{\rightarrow}{r} = 2.5m\,\hat{y}$.

At $t = 1.5$sec

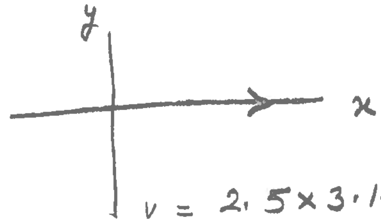
$\underset{\rightarrow}{r} = R\hat{r}$

Position vector



$\underset{\rightarrow}{v} = R\omega\hat{\tau}$.

Vel. vector

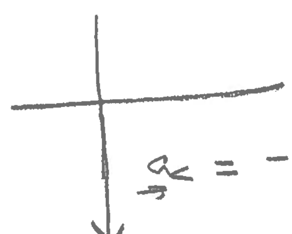


$\underset{\rightarrow}{v} = 2.5 \times 3.14$ m/s $\hat{x}$.

$= 7.85$m/s $\hat{x}$

centripetal. acc. vector

$\underset{\rightarrow}{a_c} = -R\omega^2\hat{r}$

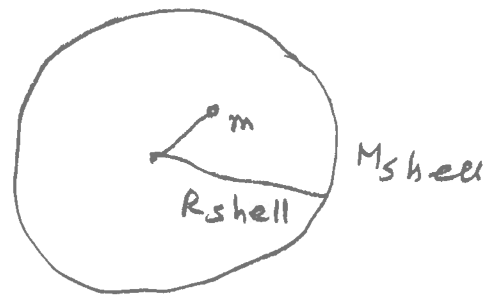


$\underset{\rightarrow}{a_c} = -2.5 \times (3.14)^2$ m/s^2 $\hat{y}$

$= -24.6$ m/s^2 $\hat{y}$.

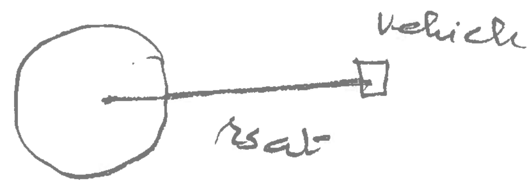
Problem 3a A point mass *m* is located inside a spherical shell of mass M_{shell} and radius R_{shell}. What is the gravitational force on the shell due to m? Why? (5)

Newton showed that
m experiences no
force due to the
shell.
By the 3rd law
shell cannot experience
any force from m



Problem 3b Why are astronauts in a vehicle in stable orbit around the Earth said to be "weightless"? (Please do not write that they are in free fall.) (10)

For vehicle in a circular orbit we must find a centripetal force



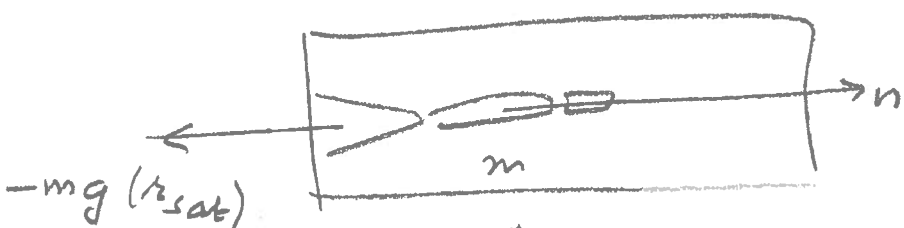
$$\vec{F}_c = -M_{sat}\, r_{sat}\, \omega_{sat}^2\, \hat{r} = -M_{sat}\, a_c\, \hat{r}$$

The Gravitational force of Earth's

$$\vec{F}_G = -\frac{G M_E M_{sat}}{r_{sat}^2}\hat{r} = -M_{sat}\, g(r_{sat})\,\hat{r}$$

For stable orbit $\vec{F}_G = \vec{F}_c$ so $a_c = g(r_{sat})$

Now put astronaut inside



Forces on him/her are $n\hat{r}$ + $-mg(r_{sat})\hat{r}$

and centripetal acc. is $\vec{a}_c = -a_c\hat{r}$

So $$n - mg(r_{sat}) = -ma_c$$

$$n = m[g(r_{sat}) - a_c] = 0$$

Weightlessness because n is zero!

<u>Problem 3c</u> The moon is a satellite of Earth and all satellites of Earth have keplerian orbits. The radius of the moon's orbit is about $4x10^5$ km and the period is about 27 days. What would be the radius of the orbit of a satellite whose period is one (1) day? Why?

(10)

For keplerian orbits around Earth

$$T_{sat}^2 = \frac{4\pi^2}{G M_E} R_{sat}^3.$$

so Moon will have

$$T_{Moon}^2 = \frac{4\pi^2}{G M_E} R_{Moon}^3$$

and

$$\left(\frac{T_{sat}}{T_{Moon}}\right)^2 = \left(\frac{R_{sat}}{R_{Moon}}\right)^3.$$

or

$$\frac{R_{sat}}{R_{Moon}} = \left(\frac{T_{sat}}{T_{Moon}}\right)^{2/3} = \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}.$$

so

$$R_{sat} = \frac{4 \times 10^5}{9} \text{ km.}$$

$$= 4.4 \times 10^4 \text{ km.}$$

7/8

<u>Problem 4a</u> What is the difference between Force and Torque? (7)

FORCE CAUSES TRANSLATION, Linear acceleration

$M\vec{a} = \sum \vec{F}_i$ at that pt. at that time.

TORQUE CAUSES ROTATION, ANGULAR ACCELERATION

$I\vec{\alpha} = \sum \vec{\tau}_i$ about the chosen axis.

To have a torque the force $\vec{F}$ must be applied at some distance $\vec{r}$ away from the axis

$\vec{\tau} = [\vec{r} \times \vec{F}]$

Problem 4b Shown is a pulley of moment of inertia 0.01 kg-m^2 and radius 5cm. A light string supports the masses.

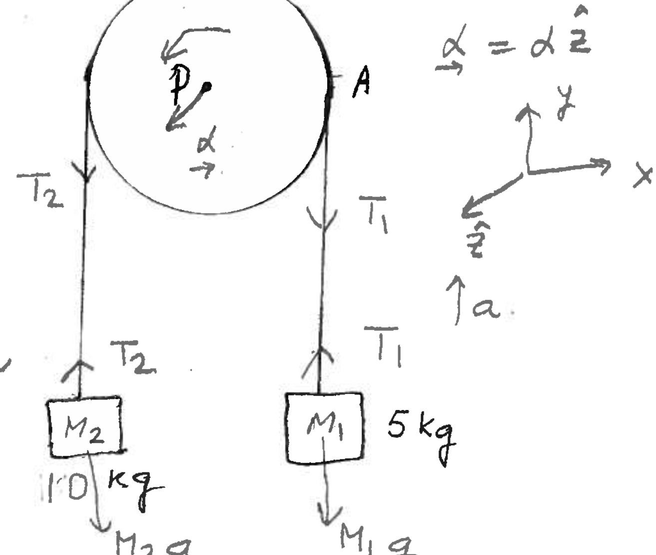
a) What is the torque about an axis through P? Why?
b) What is the angular acceleration? Why?
c) What is the tangential acceleration at A if the string does not slip? Why?

(6, 6, 6)

Use the Eqns. written in Prob. 4a.

$I = \frac{M_P R_P^2}{2}$

$\vec{\alpha} = \alpha \hat{z}$



At M_1

$M_1 a = T_1 - M_1 g$ — ①

At M_2

$-M_2 a = T_2 - M_2 g \rightarrow$ ②

For $\hat{z}$ axis Through P

$I\alpha = R_P T_2 - R_P T_1 \rightarrow$ ③

Since there is no slip $R_P \alpha = a \rightarrow$ ④

From ③ + ④ $\quad \frac{I\alpha}{R_P} = (T_2 - T_1) = \frac{I a}{R_P^2} \rightarrow$ ⑤

Next ① − ② $\quad (M_1 + M_2) a = (M_2 - M_1) g + (T_1 - T_2)$

$= (M_2 - M_1) g - \frac{I a}{R_P^2}$ ← from (5)

So $\quad a = \frac{(M_2 - M_1) g}{M_1 + M_2 + \frac{I}{R_P^2}} = \frac{(10 - 5)\, 9.8}{10 + 5 + \frac{0.01}{2} \times \frac{1}{(0.05)^2}}$

$= \frac{5 \times 9.8}{17} = 2.88\ m/s^2$

$\vec{a}_2 = -2.88\ m/s^2 \hat{y}, \quad \vec{a}_1 = +2.88\ m/s^2 \hat{y}$

at (A)

$\vec{\alpha} = \frac{a}{R_P} \hat{z} = \frac{2.88}{0.05}\ rad/s^2\ \hat{z} = 57.6\ rad/s^2\ \hat{z}$

$\vec{\tau} = I \vec{\alpha} = 0.58\ N\text{-}m\ \hat{z}$