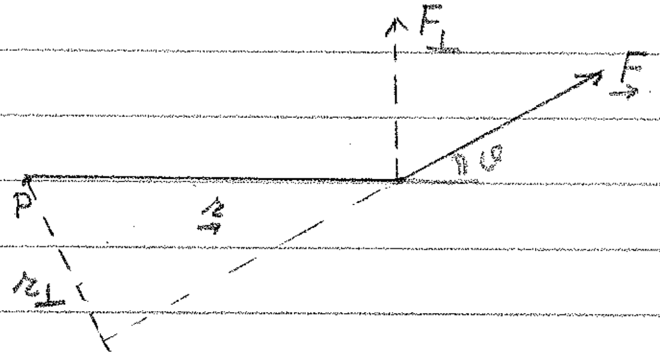
WEEK- 8 FORMULAE

TORQUE

$\underset{\rightarrow}{\tau} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{F}]$

Magnitude $\tau = rF\sin(\underset{\rightarrow}{r}, \underset{\rightarrow}{F}) = rF_{\perp} = r_{\perp}F$



For $\equiv m$ of a Rigid Body $\underset{\rightarrow}{a} \equiv 0$, $\underset{\rightarrow}{\alpha} \equiv 0$

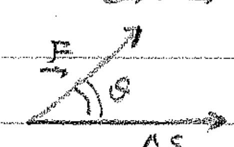
Hence $\sum \underset{\rightarrow}{F_i} = 0$

$\sum \underset{\rightarrow}{\tau_i} = 0$

Spring Force

$\underset{\rightarrow}{F_{sp}} = -k\,\Delta x\,\hat{x}$

Mechanical work $\Delta W = \underset{\rightarrow}{F} \cdot \underset{\rightarrow}{\Delta S} = F\Delta S\cos(\underset{\rightarrow}{F}, \underset{\rightarrow}{\Delta S}) = F_{\parallel}\Delta S$



Kinetic Energy $K = \frac{1}{2}MV^2$ (TRANSLATION)

$K = \frac{1}{2}I\omega^2$ (Rotation)

CHANGE OF POTENTIAL ENERGY $\Delta P = -(\underset{\rightarrow}{F_{cons}} \cdot \underset{\rightarrow}{\Delta S})$

where $\underset{\rightarrow}{F_{cons}}$ is a CONSERVATIVE FORCE

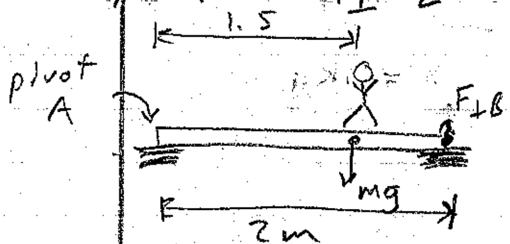
$P_g(y) = Mgy$ $\quad P_{sp} = \frac{1}{2}k(\Delta x)^2$

Solutions for problems: 8-1, 6, 9, 14, 17, 40, 45, 49
10-4, 5, 10, 13

+ direction of F

$\downarrow(-\hat{y})$ • (pivot point) $\uparrow(\hat{y})$

8.1) $\vec{\tau} = r F_{\perp} \hat{z}$



Board is in $\equiv$ m

Using A $F_{\perp w} = -mg$ (look at box above)

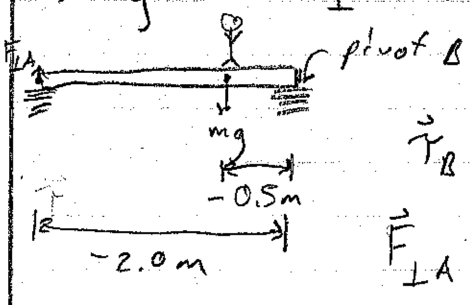
torque about A $\vec{\tau}_A = (1.5m)(-mg) + (2.0m) F_{\perp B} \hat{z} = 0$ ← the board doesn't actually move

$m = 64\,kg$

$F_{\perp B} = \frac{(1.5m)(64kg)(9.8m/s^2)}{(2.0m)} \hat{y}$ where $F_{\perp B}$ is the normal force exerted on board

$F_{\perp B} = 470.4\,N\hat{y} \rightarrow \boxed{470\,N\,\hat{y}}$

Using B $F_{\perp} = mg$ (see box above)



$\vec{\tau}_B = (+0.5m)(mg) + (+2.0m)(\vec{F}_{\perp A})\hat{z} = 0$

$\vec{F}_{\perp A} = \frac{-(+0.5m)\,64kg\,9.8m/s^2}{2.0m}(-\hat{y}) = 156.8\,N\hat{y}$

$(160\,N\hat{y})$

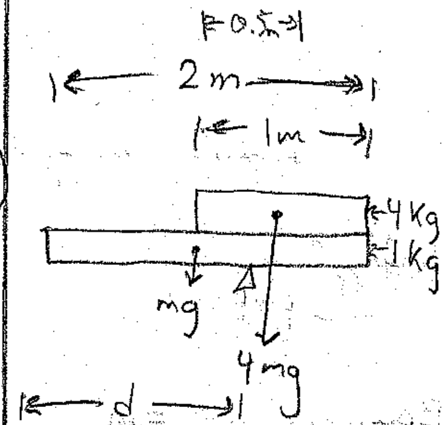
↑ see box above

Having found $F_{\perp B}$ you could also use $(F_{\perp A} + F_{\perp B} - Mg)y = 0$ to get

$F_{\perp A} = 64 \times 9.8 - 470.4$

$= 156.8\,N$ $\vec{F}_{\perp A} = 156.8\,N\hat{y}$

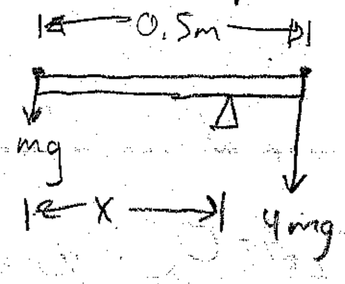
8.6)



$$\boxed{\downarrow(-\hat{y}) \;\text{pivot}\; \uparrow(\hat{y})}$$

$m = 1\,kg$

draw simple diagram



Torques on both sides of pivot must add to zero

$\vec{\tau}_m + \vec{\tau}_{4m} = 0$

$x\,mg + (0.5_m - x)(-4mg)\hat{z} = 0$

see box

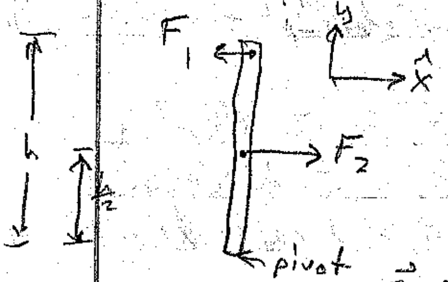
$x + (0.5_m - x)(-4)\hat{z} = 0 \rightarrow 4x - 2.0m - x\hat{z} = 0$

$5x\,\hat{z} = 2.0m\,\hat{z} \qquad x = \frac{2}{5}m$

$$\boxed{d = 1m + x = 1.4\ m}$$

from diagram

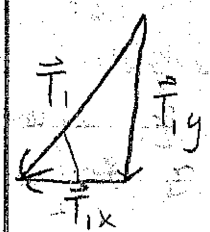
8.9) The angles of the strings matter in this problem we will return to θ later.



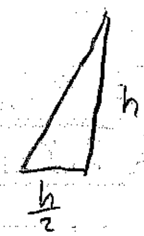
By inspection F_2 must be double F_1 and in opposite direction b/c it is exerted half the distance from the pivot

$$\vec{\tau} = F_1 h + (F_2)\frac{h}{2}\hat{z} = 0 \rightarrow F_1\hat{x} = \frac{-F_2}{2}\hat{x} \quad F_2\hat{x} = -2F_1\hat{x}$$

The rest is trig

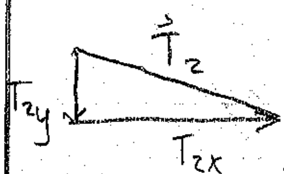


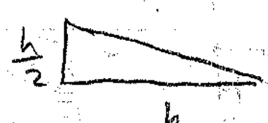
T_{1x} & T_{1y} have same ratio in triangle as length of sides



$$\left[\frac{T_{1y}}{T_{1x}} = \frac{opp}{adj} = 2\right]$$

we know $\vec{T}_{1x} = -F_1\hat{x}$ so $\vec{T}_{1y} = -2F_1\hat{y}$





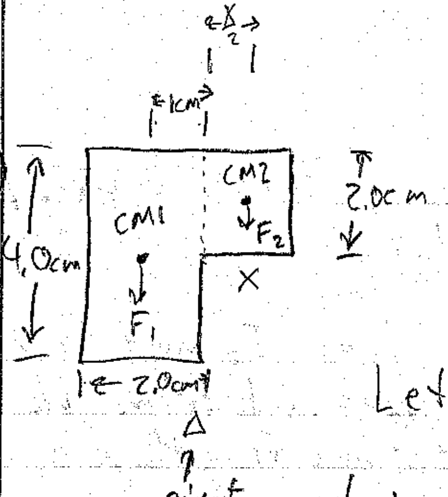
$$\frac{T_{2y}}{T_{2x}} = \frac{opp}{adj} = \frac{1}{2}$$

we know $\vec{T}_{2x} = 2F_1\hat{x}$ so $\vec{T}_{2y} = -F_1\hat{y}$

$$|\vec{T}_1| = \sqrt{F_1^2 + 4F_1^2} = \sqrt{5}F_1 \qquad \frac{|\vec{T}_1|}{|\vec{T}_2|} = 1 \quad :)$$

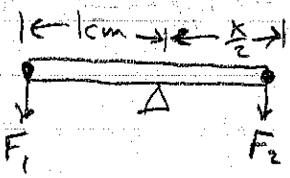
$$|\vec{T}_2| = \sqrt{4F_1^2 + F_1^2} = \sqrt{5}F_1$$

8.14)



Let's solve for case where pivot is shown. If the pivot lies beyond this pt the object will tip.

Let's define $F_1 = \sigma\, 8.0cm^2\, g$
$F_2 = \sigma\, 2.0cm(x)g$
where σ is a constant that has to do with the density.



$\vec{\tau} = gF_1(1cm) + g(-F_2)\left(\frac{x}{2}\right)\hat{z} = 0$

$\vec{\tau} = g\sigma\, 8cm^3 - \sigma g 2.0cm\, \frac{x^2}{2}\hat{z} = 0$

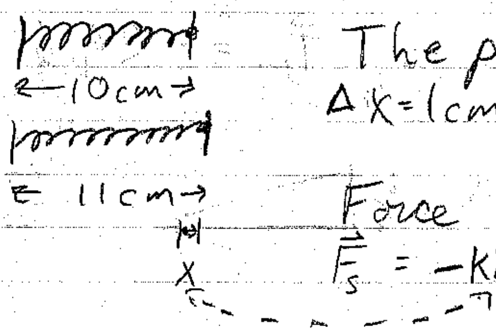
divide both sides by $(1cm)(\sigma)$

$x^2 = 8cm^2$

$\boxed{x \leq 2.8cm}$

If x is larger $\vec{\tau}$ from $\vec{F}_2$ will become larger than $\vec{\tau}$ from $\vec{F}_1$

8.17)



The problem states that for $\Delta x = 1cm$ $\vec{F} = -F\hat{x}$

Force for a spring
$\vec{F}_s = -K\Delta x\, \hat{x}$

let's solve for K $\quad -F\hat{x} = -K(1cm) \quad K = \frac{F}{cm}$

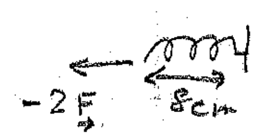
$\vec{F} = -\frac{F}{cm}\Delta x\, \hat{x} \qquad$ if $\vec{F} = -3F\hat{x}$

a $\quad -3F\hat{x} = -\frac{F}{cm}\Delta x\, \hat{x} \qquad \boxed{\Delta x = 3cm}$

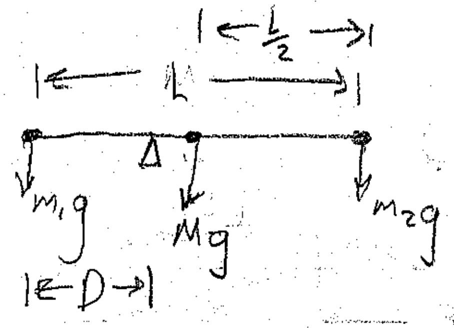
b. For force to be 2F compression must be 2cm

$2\vec{F} = -F\Delta x\, \hat{x}$

$\Delta x = -2cm.$



8.40)



For $\equiv m$ $\sum \vec{\tau}_i = 0$

$$\vec{\tau} = \left[m_1 g D - Mg\left(\tfrac{L}{2} - D\right) - m_2 g (L - D)\right]\hat{z} = 0$$

note that this changes sign if $D > \frac{L}{2}$

solve for D

$$(m_1 + m_2 + M) g D \hat{z} = \left(\tfrac{1}{2}M + m_2\right) g L \hat{z}$$

$$\boxed{D = \frac{\left(\frac{1}{2}M + m_2\right)L}{(m_1 + m_2 + M)}}$$

if $m_1 = m_2$ we know that $D = \frac{L}{2}$. Does this work?

$$D = \frac{\left(\frac{1}{2}M + m\right)L}{(2m + M)} = \frac{L}{2}$$

8.45) a) just before the mass touches the scale simply reads the weight of the mass.

$$F = +mg\hat{y} = +(5\,kg)(-10\,m/s^2)\hat{y} = 50\,N\,\hat{y}$$

b) The spring starts at equilibrium, therefore compression is the 'y' in the $\vec{F} = -ky\hat{y}$ equation.

$$\vec{F}_{scale} + \vec{F}_{spring} + 50N = 0 \text{ (mass does not move)}$$

$$-k\Delta y + 20N - 50N\hat{y} = 0$$ y was compressed $\Delta y = -2.0\,cm$

$$-k(-2.0\,cm)\hat{y} = 30N\hat{y}$$

$$\boxed{k = 15\frac{N}{cm}}$$

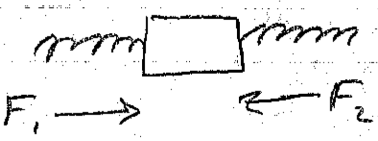
c) $-ky + 0N - 50N = 0$

$$-\frac{15N}{cm} y\hat{y} = 50N\hat{y}$$

$$\boxed{y = -\frac{10}{3}\,cm}$$

8.49) Equilibrium $\rightarrow F_{net} = 0$

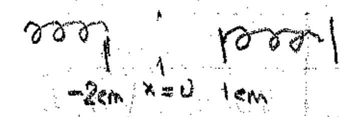
a) $\vec{F}_1 + \vec{F}_2 = 0$



$[-K_1 X_1 - K_2 X_2]\hat{x} = 0$

$-\frac{10N}{m} \cdot \frac{1m}{100cm}(-2.0cm) - \frac{20N}{m} \cdot \frac{1m}{100cm} X_2 \hat{x} = 0$

$\boxed{\Delta X_2 = 1.0cm} \rightarrow$



b) $\Delta X_1 = 15cm - 2.0cm$ $\quad \Delta X_2 = -15cm - 1.0cm$

sketched by

$= 13cm$ $\quad \Delta X_2 = -16cm$ ← compressed by 16cm

$-K\Delta X_1 - K\Delta X_2 \hat{x} = F_{net}$

$-0.1\frac{N}{cm}(13cm) - 0.2\frac{N}{cm}(16cm)\hat{x} = \boxed{-4.5N}\hat{x}$

remember where the equilibrium pt of each spring is located!

Chapter 10

10.4) $\Delta W = \underline{F} \cdot \Delta \underline{S} = F \Delta S \cos(\underline{F}, \Delta \underline{S}) = F_{\parallel} \Delta S$

$W_1 = T_1 \cos(20°)(3m)$

$= 326N \cos(20°)(3m) = \boxed{919 J}$

$W_2 = T_2 \cos(30°)(3m)$
$= 223N \cos(30°)(3m) = \boxed{579 J}$

notice: no direction! WORK IS A SCALAR.

10.5) a) So b/c we are on the moving sidewalk and moving at a constant velocity the Force on me is zero (Newton's 2nd law).

$W = Fd = 0$ b/c $F = 0$

extra

you acquire kinetic energy!

Obviously we experience a force & work is done when we get on the sidewalk.
$W = \frac{1}{2}(60kg)(0.7m/s)^2 = 14.7J$

b) It takes effort to lift something against gravity. Our F is mg in this problem. This is the normal force → that is why it is positive.

$W = F \times$ distance along force $[F_{\parallel} \Delta S]$

$W = mgd = (60kg)(9.8m/s^2)(4.5m) = 2,646 J$

c) You are giving the down escalator energy as you go down so the answer is simply negative the answer above.
$W = (60kg)(9.8m/s^2)(-4.5m) = -2,646J$

10.10) The boats have kinetic energy which we must counter.

$$KE = \frac{1}{2}(1200\,Kg)(1.2\,m/s)^2 = 864\,J$$

But we are taking this energy <u>from</u> the boats

$$\boxed{W = -864\,J}$$

10.13) $K_{rot} = \frac{1}{2} I \omega^2$

moment of inertia

$$4\times10^6\,J = \frac{1}{2} I \left(2\pi \frac{20{,}000\,rev}{\cancel{min}} \frac{\cancel{min}}{60s}\right)^2 = 2.2\times10^6\,I$$

$$\boxed{I = \frac{4\times10^6\,J}{2.2\times10^6 \frac{1}{s^2}} = 1.8\,Kg\cdot m^2}$$