SOLUTIONS — 9

FORMULAE. CONSERVATION OF ENERGY (MECHANICAL).

MECHANICAL WORK

$$\Delta W = \vec{F} \cdot \Delta\vec{S} = F \Delta S \cos(\vec{F}, \Delta\vec{S}) = F_{\parallel} \Delta S$$

$$= F_x \Delta x + F_y \Delta y + F_z \Delta z$$

b/c

$$\vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}, \quad \Delta\vec{S} = \Delta x \hat{x} + \Delta y \hat{y} + \Delta z \hat{z}$$

KINETIC ENERGY POINT MASS $K = \frac{1}{2} M V^2$

RIGID BODY TRANSLATION $K_{Tr} = \frac{1}{2} M V^2$

ROTATION $K_{Rot} = \frac{1}{2} I \omega^2$

POTENTIAL ENERGY: EARTH MASS $U_g = Mgh$, spring $U_{sp} = \frac{1}{2} k x^2$.

CONSERVATION EQUATIONS:

ISOLATED SYSTEM (NO EXTERNAL FORCE)

$$\underbrace{K_f + U_f}_{\text{Final}} = \underbrace{K_i + U_i}_{\text{Initial}}$$

[Total Energy is Constant]

IF EXTERNAL (NON-CONSERVATIVE) FORCE PRESENT

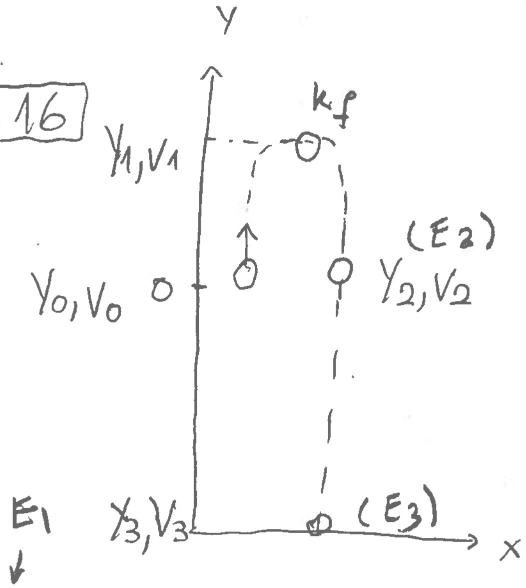
$$K_f + U_f = K_i + U_i + W_{NCF}$$

WORK DONE by NON-CONS. FORCE

Friction — NCF work done always negative

$$\Delta W_f = \vec{f}_k \cdot \Delta\vec{S} = -f_k \Delta S \cos 180^\circ$$

10.16



$y_0 = y_2 = 0$ ← Note $y=0$ not at ground.

$y_3 = -20\,m.$

$v_0 = 10\,m/s$

$v_1 = 0\,m/s$

We want to find y_1, v_2 and v_3

$E_f = E_i$

$K_i + \cancel{U_i}^{0} = \cancel{K_f}^{0} + U_f$

$\frac{1}{2} m v_0^2 = m g y_1$

$$\boxed{y_1 = \frac{v_0^2}{2g} = 5.1\,m}$$

or 25 m above the ground

$E_2 = E_i$

$K_0 = K_2 + \cancel{U_2}^{0}$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_2^2$

$$\boxed{v_2 = v_0 = 10\,m/s}$$

However, $\vec{v_2} = -10\,m/s\,\hat{y}$ ✓

while $\vec{v_0} = 10\,m/s\,\hat{y}$

$E_3 = E_i$

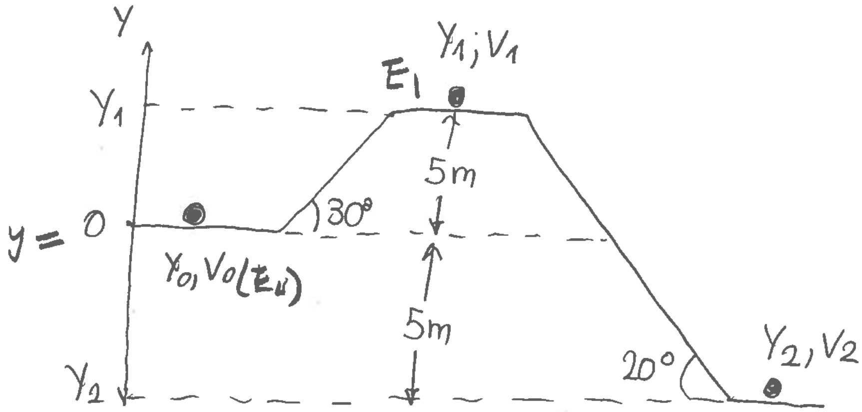
$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_3^2 + m g y_3$

$v_3 = \sqrt{v_0^2 - 2g y_3}$

$$\boxed{v_3 = 22\,m/s}$$

$\vec{v_3} = -22\,m/s\,\hat{y}$

10.21



$E_1 = E_0$

$\frac{1}{2} m V_0^2 = K_1 + mgY_1$

$K_1 = \frac{1}{2} m V_0^2 - mgY_1$

$= 7.5 \times 10^4 J - 7.4 \times 10^4 J$

$= 10^3 J$

Thus the car does make it to the top.

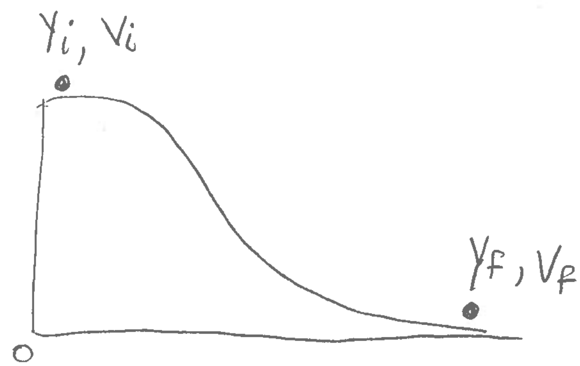
$E_2 = E_0$

$\frac{1}{2} m V_0^2 = \frac{1}{2} m V_2^2 + mgY_2$

$\sqrt{V_0^2 - 2gY_2} = V_2$

$\boxed{\vec{V_2} = 14 \ m/s \ \hat{x}}$

10.28



$V_i = 35 \ Km/h$

$Y_i = 15 m.$

$Y_f = 0 m$

Find V_f

$35 \frac{Km}{h} \left| \frac{1 \ h}{3600 s} \right| \frac{1\,000 m}{1 \ Km} \right| = 9.7 \ m/s$

$K_f + U_f = K_i + U_i$

$\frac{1}{2} m V_i^2 + mgY_i = \frac{1}{2} m V_f^2 \Rightarrow V_f = \sqrt{V_i^2 + 2gY_i} = 19.7 \ m/s = 71 \ Km/h$

$\vec{V_f} = 71 \ Km/h \ \hat{x}$

Slightly over the speed limit

$\boxed{10.35}$

Before

m_p $\quad m_c = 12\, m_p$

$v_{ip} = 2.0\times10^{7}\,m/s$ $\quad v_{ic} = 0\,m/s$

OMIT.

After

$v_{fp} = ?$ $\quad v_{fc} = ?$

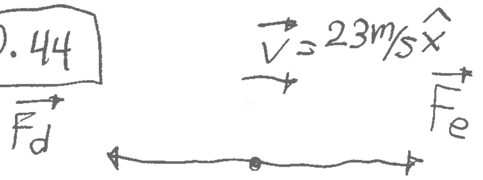
$$v_{fp} = \left(\frac{m_p - m_c}{m_p + m_c}\right) v_{ip} = -1.7\times10^{7}\,m/s \Rightarrow \vec{v}_{fp} = -1.7\times10^{7}\,m/s\ \hat{x}$$

$$v_{fc} = \left(\frac{2m_p}{m_p + m_c}\right) v_{ip} = 3.1\times10^{6}\,m/s \Rightarrow \vec{v}_{fc} = 3.1\times10^{6}\,m/s\ \hat{x}$$

$\boxed{10.42}$

$$P_w = \frac{W}{\Delta t} = \frac{U}{\Delta t} = \frac{\frac{1}{2}kx^2}{\Delta t} = \frac{4.0\,J}{0.30\,s} = 13\,W$$

$\boxed{10.44}$



$\vec{F_d} = -500\,N\ \hat{x}$

to travel at constant speed $\vec{a} = 0$, thus:

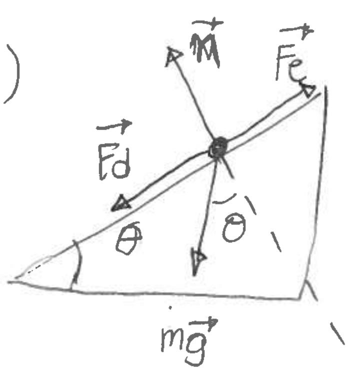
$\vec{F_e} + \vec{F_d} = 0$

$\vec{F_e} = 500\,N\ \hat{x}$

a)

$P = \vec{F_e}\cdot\vec{v} = 1.2\times10^{4}\,W$

b)



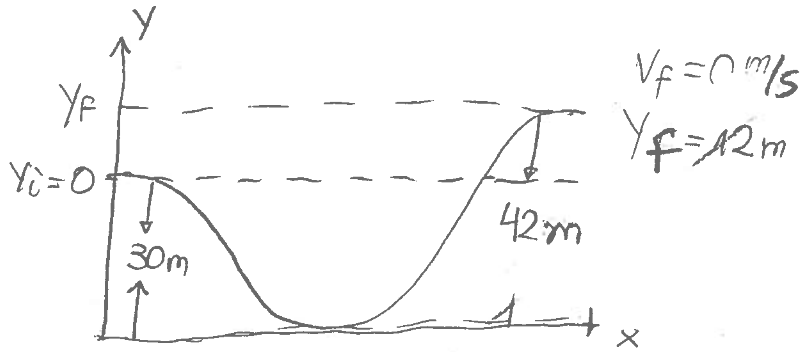
$F_e - F_d - mg\sin\theta = 0 \Rightarrow F_e = F_d + mg\sin\theta$

$n - mg\cos\theta = 0$

$\boxed{\vec{F_e} = 842\,N\ \hat{x}}$

$\boxed{P = \vec{F_e}\cdot\vec{v} = 1.9\times10^{4}\,W}$

10.53



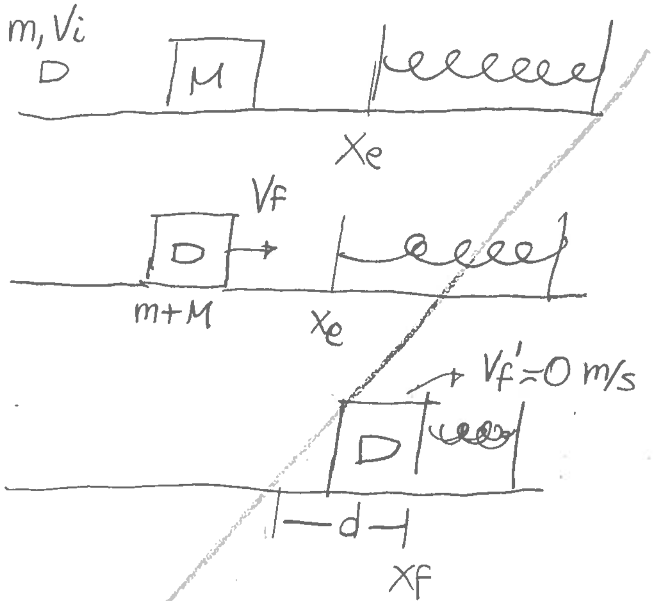
$$E_f = E_i$$

$$\frac{1}{2} m V_i^2 = m g Y_f$$

$$V_i = \sqrt{2 g Y_f} = 15 \text{ m/s}$$

$$\boxed{\vec{V}_i = 15 \text{ m/s } \hat{x}}$$

10.61 a)



omit

$$P_i = P_f$$

$$m V_i = (m+M) V_f$$

$$V_f = \left(\frac{m}{m+M}\right) V_i$$

$$E_f = E_f'$$

$$\frac{1}{2} (m+M) V_f^2 = \frac{1}{2} k d^2$$

$$\frac{m^2}{(m+M)} V_i^2 = k d^2$$

$$\boxed{V_i = \frac{\sqrt{k(m+M)}}{m} d}$$

b) $\vec{V_i} = 200\,m/s\,\hat{x}$

c). $$\frac{E_i - E_f}{E_i} = \frac{\frac{1}{2} m V_i^2 - \frac{1}{2}(m+M) V_f^2}{\frac{1}{2} m V_i^2} = \frac{m V_i^2 - \frac{m^2}{m+M} V_i^2}{m V_i^2}$$

$$= 1 - \frac{m}{m+M} = \underline{99.8\,\%}$$