SOLUTIONS - 3 · FORMULAE

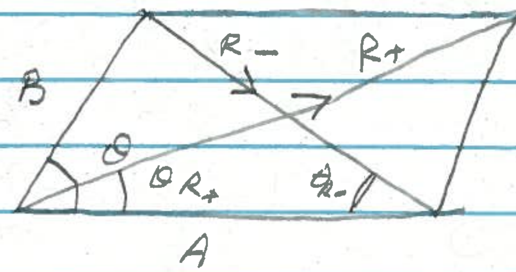
VECTOR ALGEBRA

If $\vec{V}' = S\vec{V}$, $\vec{V}' \parallel \vec{V}$

and magnitude of $V' = SV$

ADDITION OF VECTORS $\vec{R} = \vec{A} + \vec{B}$

Three Methods



Geometry Choose Scale, Draw parallelogram with A and B as sides,

Long diagonal gives

$R_+ = A + B$, measure angle θ_{R+}

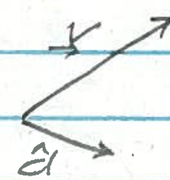
Short diagonal yields $R_- = A - B$, also θ_{R-}

Trig/Algebra $\vec{R} = \vec{A} + \vec{B}$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\theta_R = \left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

Method of components

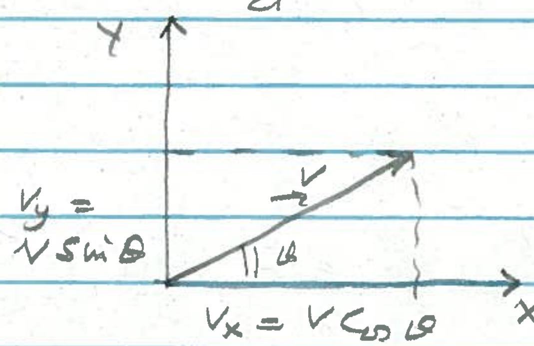


First Define Component

$$V_d = V\cos(\vec{V}, d)$$

Put $\vec{v}$ in xy-plane

$\vec{V} = V\cos\theta\,\hat{x} + V\sin\theta\,\hat{y}$



$V = \sqrt{V_x^2 + V_y^2}$

$\tan\theta = \dfrac{V_y}{V_x}$

Many Vectors $\vec{V}_1, \vec{V}_2, \ldots \vec{V}_N$

$$\vec{V}_1 = V_{1x}\hat{x} + V_{1y}\hat{y}$$
$$\vec{V}_2 = V_{2x}\hat{x} + V_{2y}\hat{y}$$
$$\vdots$$
$$\vec{V}_N = V_{Nx}\hat{x} + V_{Ny}\hat{y}$$

$$\vec{R} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \cdots + \vec{V}_N$$
$$= \left(\sum_{1\to N} V_i \cos\theta_i\right)\hat{x} + \left(\sum V_i \sin\theta_i\right)\hat{y}$$
$$= R_x\hat{x} + R_y\hat{y}$$

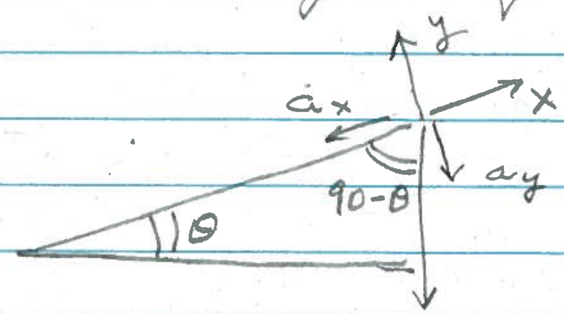
$$R = \sqrt{R_x^2 + R_y^2}, \quad \tan\theta_R = \frac{R_y}{R_x}$$

Motion ON A RAMP - TILTED AXES.

ACCELERATION DUE TO Earth always straight down:

For Earth

$$\vec{a}_x = -(9.8\sin\theta)\hat{x}$$
$$\vec{a}_y = -(9.8\cos\theta)\hat{y}$$



Motion along ramp only.

Simplifies to case of ONE DIMENSION WITH CONST. acceleration

$$\vec{a}_x = -(9.8\sin\theta)\hat{x}$$

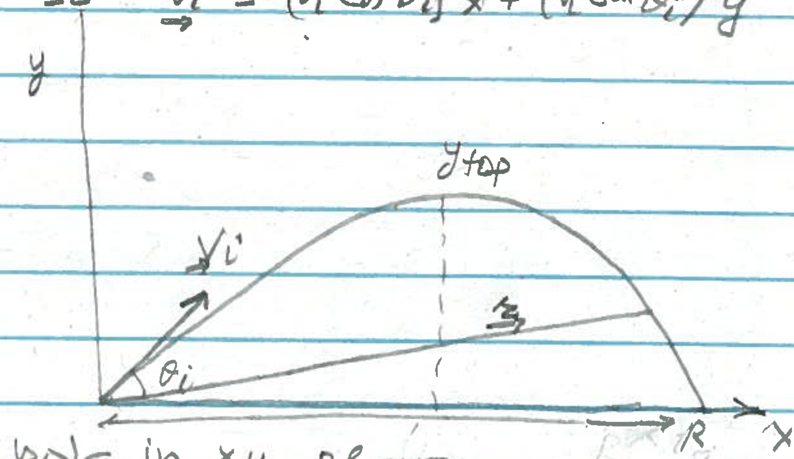
Hence

$$\vec{v} = [v_i - (9.8\sin\theta)t]\hat{x}$$
$$\vec{x} = [x_i + v_i t - (4.9\sin\theta)t^2]\hat{x}$$
$$v^2 = v_i^2 - (19.6\sin\theta)(x - x_i)$$

Projectile motion when $x_i = 0$, $y_i = 0$ and $\underset{\rightarrow}{V}$ is V_i m/s at an angle of θ above the x-axis at $t=0$. so $\underset{\rightarrow}{V_i} = (V_i \cos\theta_i)\hat{x} + (V_i \sin\theta_i)\hat{y}$

Near Earth the acceleration is

$\underset{\rightarrow}{a} = 0\hat{x} - 9.8 m/s^2 \hat{y}$



Consequently,

① Path is a parabola in xy-plane

$$y = x \tan\theta_i - 4.9\left(\frac{x}{V_i \cos\theta_i}\right)^2$$

and position vector $\underset{\rightarrow}{r} = (x\hat{x} + y\hat{y})$

② highest point

$$y_{top} = \frac{(V_i \sin\theta_i)^2}{19.6}$$

③ Time to reach y_{top}

$$t_{top} = \frac{V_i \sin\theta_i}{9.8}$$

④ At the top $\underset{\rightarrow}{a} = 0\hat{x} - 9.8 m/s^2 \hat{y}$

velocity $\underset{\rightarrow}{V} = (V_i \cos\theta_i)\hat{x} + 0\hat{y}$

⑤ Time to return to ground

$$t_{gr} = \frac{V_i \sin\theta_i}{4.9} = 2t_{top}$$

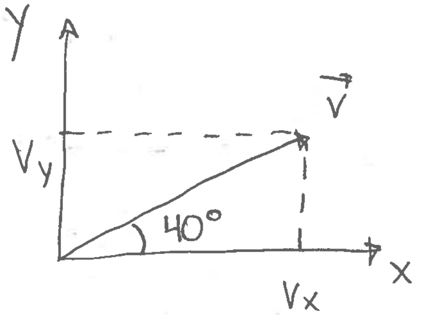
⑥ Velocity just before hitting ground

$$\underset{\rightarrow}{V} = (V_i \cos\theta_i)\hat{x} - (V_i \sin\theta_i)\hat{y}$$

⑦ Range R is distance travelled along x before returning to ground $R = \left(\frac{V_i^2 \sin 2\theta_i}{9.8}\right)$

Problem Set Chapter 3 Solutions:

3.7



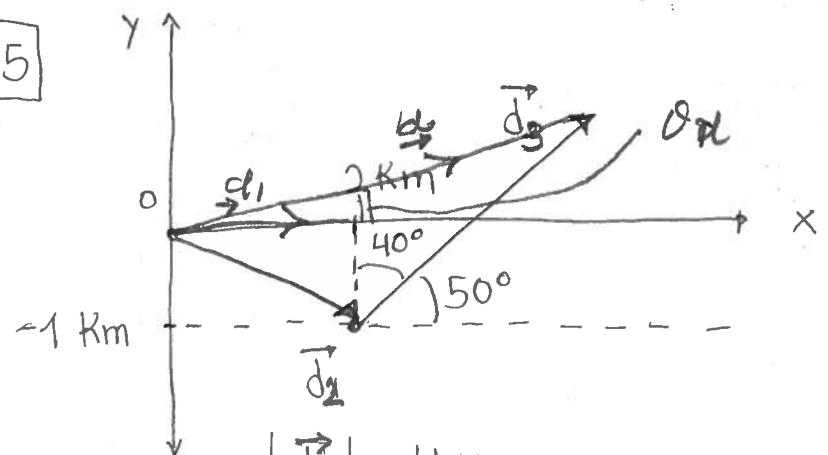
$V_y = 10\ m/s$

$\tan 40° = \frac{V_y}{V_x}$

$V_x = \frac{10\ m/s}{\tan(40°)} = \boxed{12\ m/s}$

$|\vec{V}| = \sqrt{12^2 + 10^2} = 15.56\ m/s$

3.15



$\vec{d}$ = Total displacement

$\vec{d} = \vec{d_1} + \vec{d_2} + \vec{d_3}$

$\vec{d_1} = (2\hat{x} + 0\hat{y})$ km

$\vec{d_2} = (0\hat{x} - 1\hat{y})$ km

$|\vec{d_3}| = 4$ Km

$d_{3x} = |\vec{d_3}| \cos 50°$

$d_{3y} = |\vec{d_3}| \sin 50°$

$\vec{d_3} = 4Km \cos 50°\ \hat{x} + 4Km \sin 50°\ \hat{y}$

$\vec{d_2} + \vec{d_1} = 2km\ \hat{x} - 1km\ \hat{y}$

$\tan \theta_D = \frac{2.1}{4.6} = 0.46$

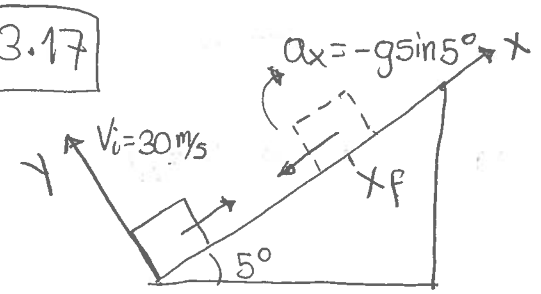
$\theta_D = 24.5°$

$\vec{d} = (4Km \cos 50° + 2Km)\hat{x} + (4Km \sin 50° - 1Km)\hat{y}$

$\boxed{\vec{d} = (4.6\ Km)\hat{x} + (2.1\ Km)\hat{y}}$

$\boxed{|\vec{d}| = \sqrt{d_x^2 + d_y^2} = 5.1Km}$

3.17



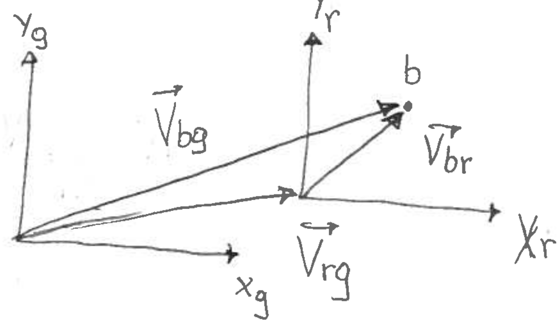
$\cancel{V_f^2}^{\,0} = V_i^2 + 2a_x(X_f - \cancel{X_i}^{\,0})$

$V_i^2 = 2g \sin 5°\ X_f$

$\boxed{X_f = \frac{V_i^2}{2g \sin 5°} = 530\ m}$

$\boxed{3.23}$

b = boat
r = river
g = ground



ALL VELOCITIES ARE ALONG x

$$\vec{V}_{bg} = \vec{V}_{rg} + \vec{V}_{br}$$

Down the river: $\vec{V}_{bg1} = \vec{V}_{rg} + \vec{V}_{br1} = \dfrac{30\ \text{Km}}{3.0\ \text{h}}\hat{x} = 10\ \text{Km/h}\ \hat{x}$

During the return trip, the river is flowing against the boat:

$$\vec{V}_{bg2} = \vec{V}_{rg} + \vec{V}_{br2} = -\frac{30\ \text{Km}}{5.0\ \text{h}}\hat{x} = -6.0\ \text{Km/h}\ \hat{x}$$

So $\vec{V}_{br2} = -\vec{V}_{br1}$, thus.

$$\left.\begin{cases} \vec{V}_{rg} + \vec{V}_{br} = 10\ \text{Km/h} \\ \vec{V}_{rg} - \vec{V}_{br} = -6.0\ \text{km/h} \end{cases}\right\} 2V_{rg} = 4.0\ \text{km/h}$$

$$\boxed{\vec{V}_{rg} = 2.0\ \text{Km/h}}\ \hat{x}$$

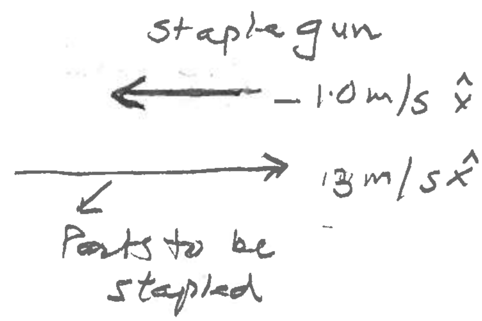
$\boxed{3.25}$

s = staple gun
p = part
g = ground

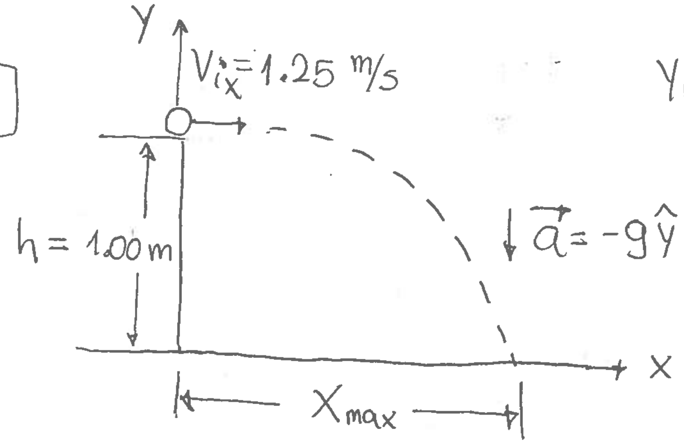
$$\vec{V}_{ps} = \vec{V}_{pg} + \vec{V}_{sg} = [3 - (-1)]\ \text{m/s}\ \hat{x}$$

$$\vec{V}_{ps} = 4.0\ \text{m/s}\ \hat{x}$$

$$\boxed{4.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \left|\ \frac{1\ \cancel{\text{s}}}{10\ \text{staples}}\right| = 0.40\ \frac{\text{m}}{\text{staple}}}$$



$\boxed{3.28}$



$$Y_f - Y_i = V_{iy}t + \frac{1}{2}a_y t^2$$
$$V_{fy} = V_{iy} + a_y t$$
$$Y_i = 1.00\,m \;,\; Y_f = 0$$
$$a_y = -g \;,\; V_{iy} = 0$$

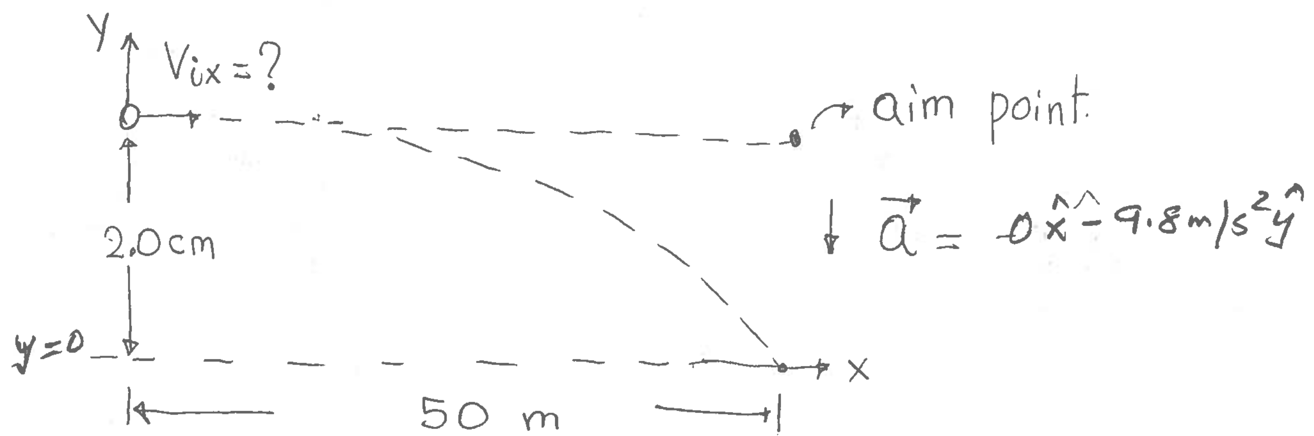
$$X_f - X_i = V_{ix}t + \frac{1}{2}a_x t^2$$
$$V_{fx} = V_{ix} + a_x t$$

$$X_i = 0 \;,\; X_f = X_{max}$$
$$V_{ix} = 1.25\,m/s \;,\; a_x = 0$$

$$X_{max} = 1.25\,m/s\ t$$
$$\boxed{X_{max} = 0.565\ m}$$

$$-1.00\,m = -\frac{1}{2}g t^2$$
$$t = \sqrt{\frac{2.00\,\cancel{m}}{9.8\,\cancel{m}/s^2}} \Rightarrow \boxed{t = 0.452}$$

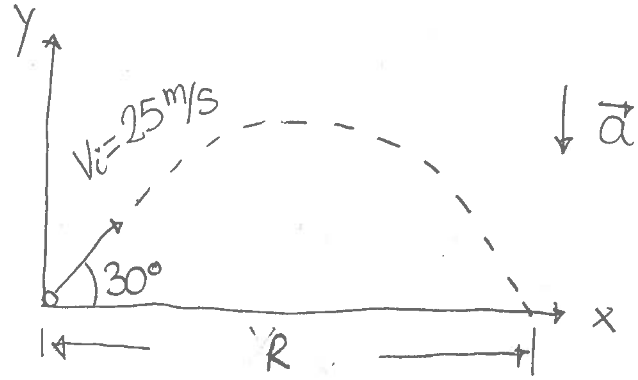
$\boxed{3.31}$



$$X_f - X_i = V_{ix}t + \frac{1}{2}a_x t^2$$
$$V_{fx} = V_{ix} + a_x t$$
But : $X_i = 0$, $X_f = 50\,m$
$$a_x = 0$$
$$X_f = V_{ix}t$$
$$50\,m = V_{ix}t$$
$$\boxed{V_{ix} = 782\ m/s}$$

$$Y_f - Y_i = V_{iy}t + \frac{1}{2}a_y t^2$$
$$V_{fy} = V_{iy} + a_y t$$
But $Y_i = 0.020\,m \;,\; Y_f = 0$
$$V_{iy} = 0 \;,\; a_y = -g$$
$$0.020\,m = \frac{1}{2}g t^2$$
$$\boxed{t = 0.064\ s}$$

$\boxed{3.33}$



$\downarrow \vec{a} = -g/6\, \hat{y} = -1.63\,m/s^2\, \hat{y}$

$Y_f - Y_i = V_{iy} t + \frac{1}{2} a_y t^2$

$V_{fy} = V_{iy} + a_y t$

$\vec{V_i} = V_{ix}\,\hat{x} + V_{iy}\,\hat{y}$

$Y_f = 0,\ Y_i = 0$

$a_y = -g/6$

$\cos 30^\circ = \frac{V_{ix}}{V_i}$; $\sin 30^\circ = \frac{V_{iy}}{V_i}$

$V_{ix} = 12.5\sqrt{3}\ m/s$; $V_{iy} = 12.5\ m/s$

$0 = 12.5\,m/s\ t - \frac{1}{12} g t^2$

$0 = t\left(12.5\,m/s - \frac{1}{12} g t\right)$ $\begin{cases} t = 0 \\ \text{or} \\ t = \frac{12 \cdot 12.5\,m/s}{g} = \boxed{15.3\ s} \end{cases}$ a)

b) $X_f - X_i = V_{ix} t + \frac{1}{2} a_x t^2$ $\begin{cases} a_x = 0 \\ X_i = 0,\ X_f = X_{max} \end{cases}$

$R = 12.5\sqrt{3}\ m/s\ t \Rightarrow \boxed{R_{Max} = 331\ m}$

c) $Y_f - Y_i = V_{iy} t + \frac{1}{2} a_y t^2$ But : $\begin{cases} Y_f = 0 \\ Y_i = 0 \end{cases}$; $a_y = -g$.

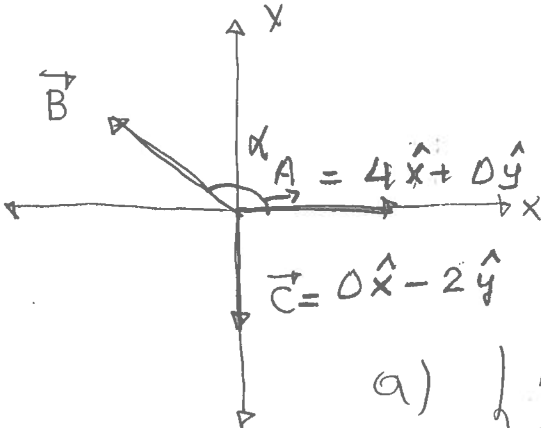
$0 = 12.5\,m/s\ t - \frac{1}{2} g t^2 \rightarrow \begin{cases} t = 0 \\ \text{or} \\ t = \frac{2 \cdot 12.5\,m/s}{g} = 2.55\ s \end{cases}$

ON Earth

$R_E = 12.5\sqrt{3}\,m/s\ t \Rightarrow \boxed{R_E = 55.2\ m}$

$R_M - R_E = 331 - 55 = 276\ m$

$\boxed{3.43}$

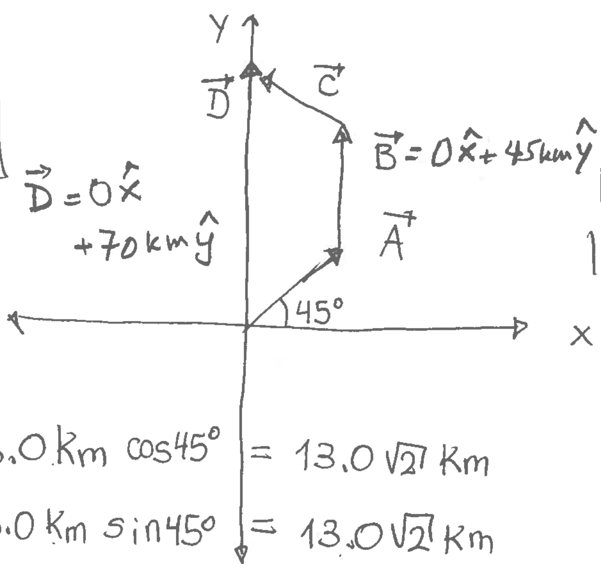


$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = 2\hat{x} + 0\hat{y}$$
$$= 4\hat{x} + B_x\hat{x} + B_y\hat{y} - 2\hat{y} = 2\hat{x}$$
$$= (4 + B_x)\hat{x} + (B_y - 2)\hat{y} = 2\hat{x}$$

a) $\begin{cases} 4 + B_x = 2 \Rightarrow \\ B_y - 2 = 0 \Rightarrow \end{cases}$ $\boxed{\begin{matrix} B_x = -2 \\ B_y = 2 \end{matrix}}$

b) $\vec{B} = -2\hat{x} + 2\hat{y}$ $\begin{cases} |\vec{B}| = B = \sqrt{4+4} = 2\sqrt{2} \\ \tan\alpha = -1 \Rightarrow \alpha = 135° \end{cases}$

$\boxed{3.48}$



$|\vec{A}| = 26.0$ Km
$|\vec{B}| = 45.0$ Km
$|\vec{D}| = 70.0$ km

$A_x = 26.0\,\text{Km} \cos 45° = 13.0\sqrt{2}$ Km
$A_y = 26.0\,\text{Km} \sin 45° = 13.0\sqrt{2}$ Km

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$
$$\vec{C} = \vec{D} - \vec{A} - \vec{B}$$
$$\vec{C} = 70.\,\text{Km}\,\hat{y} - 13.0\sqrt{2}\,\text{Km}(\hat{x} + \hat{y}) - 45.0\,\hat{y}$$

$\boxed{\vec{C} = -18.4\ \text{Km}\ \hat{x} + 6.6\ \text{Km}\ \hat{y}}$ $\boxed{|\vec{C}| = 19.5\ \text{Km}}$

$\tan\alpha = -\dfrac{6.6\ \cancel{\text{Km}}}{18.4\ \cancel{\text{Km}}} \Rightarrow$ $\boxed{\alpha \simeq 160.2°}$

$\boxed{3.54}$

m = man

s = sidewalk

g = ground

d = distance from gate to baggage claim

$\vec{V}_{mg} = \vec{V}_{ms} + \vec{V}_{sg}$

(1) if the sidewalk is broken then: $V_{sg} = 0$

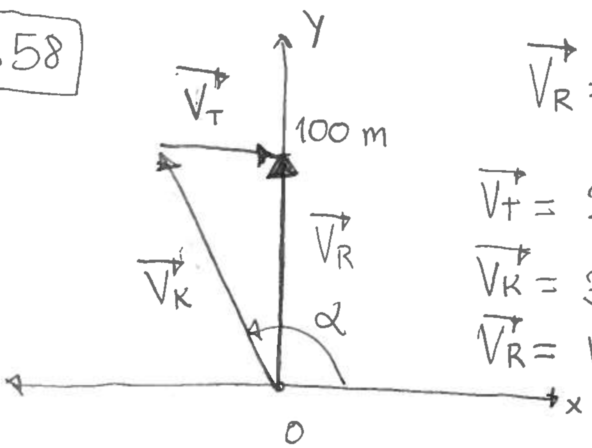
(2) if the man doesn't walk while riding the sidewalk then: $V_{ms} = 0$

$$V_{ms} = \frac{d}{50s} \quad ; \quad V_{sg} = \frac{d}{75s}$$

$$V_{mg} = \frac{d}{t} = \frac{d}{50s} + \frac{d}{75s} \Rightarrow \frac{1}{t} = \frac{1}{50s} + \frac{1}{75s}$$

$$\boxed{t = 30s}$$

$\boxed{3.58}$



$$\vec{V}_R = \vec{V}_K + \vec{V}_T$$

$$\vec{V}_T = 2.0 \, m/s \; \hat{x}$$

$$\vec{V}_K = 3.0 \, m/s \left(\cos\alpha \, \hat{x} + \sin\alpha \, \hat{y} \right)$$

$$\vec{V}_R = V_R . \hat{y}$$

$$V_R \hat{y} = \left(2.0 \, m/s + 3.0 \, m/s \cos\alpha\right) \hat{x} + 3.0 \, m/s \sin\alpha \, \hat{y}$$

$$2.0 \, m/s + 3.0 \, m/s \cos\alpha = 0$$

$$\cos\alpha = -2/3$$

a) $\boxed{\alpha = 132°}$

$$V_R = 3.0 \, m/s \cdot \sin\alpha$$

$$V_R = 2.2 \, m/s$$

b) $100 \, m = 2.2 \, m/s \cdot t \Rightarrow \boxed{t = 44.7 \, s}$