SOLUTIONS – 13

FORMULAE

MECHANICAL EQUIVALENT OF HEAT: 4.18J of WORK MIMICS EFFECTS OF 1 Calorie of DQ (HEAT)

Pressure of a gas

$$P = \frac{1}{3} m \frac{N}{V} \langle v^2 \rangle \qquad [\langle \underline{v} \rangle = 0]$$

$$= \frac{N k_B T}{V}$$

$$\text{so} \quad \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$

1st Law CONSERVATION OF ENERGY IN THERMODYNAMIC PROCESS

$$DQ = dU + DW$$

2nd Law DEALS WITH DIRECTION OF THERMODYNAMIC PROCESSES

Engine Efficiency (Carnot Cycle)

$$e = 1 - \frac{T_c}{T_H} \qquad \left[\frac{DQ_H}{T_H} + \frac{DQ_c}{T_c} \right] = 0$$

Coefficient of Performance

$$COP = \frac{T_H}{T_H - T_c}$$

To provide direction needs a property which is "UNI-DIRECTIONAL". ENTROPY $\rightarrow dS \overset{R}{=} \frac{DQ}{T}$

$$\rightarrow \boxed{dS \geq 0} \quad \text{ADIABATIC PROCESS}$$

11-6 Daily intake

$$= 1\,\text{day} \times \frac{24\,\text{hr}}{1\,\text{day}} \times \frac{60\,\text{min}}{1\,\text{hr}} \times \frac{60\,\text{s}}{1\,\text{min}} \times \frac{100\,\text{J}}{1\,\text{s}} \times \frac{1\,\text{cal}}{4.19\,\text{J}} \times \frac{1\,\text{kcal}}{1000\,\text{cal}}$$

$= 2060$ Cal

11-13 (a) Work done in one repetition

$= (40\,\text{kg})(9.8\,\text{ms}^{-2})(0.5\,\text{m})$ $\qquad \Delta W = \underset{\rightarrow}{F} \cdot \underset{\rightarrow}{\Delta S}$

$\simeq 200\,\text{J}$

(b) Energy expended per day

$$= \frac{(200\,\text{J/rep})(20\,\text{reps/day})}{25\%}$$

$= 16000\,\text{J/day}$

(c) # donuts needed $\qquad$ *This is actually kcal

$$= \frac{16000\,\text{J/day}}{400\,\text{Cal}^{*}/\text{donut}}$$

$$= \frac{16000\,\text{J/day}}{400\,\text{Cal/donut}} \times \frac{1\,\text{Cal}}{1\,\text{kcal}} \times \frac{1\,\text{kcal}}{1000\,\text{cal}} \times \frac{1\,\text{cal}}{4.2\,\text{J}}$$

$= 0.0095$ donuts/day

11-16 The temperatures of each gas in the mixture are the same. Thus their molecules are having the same average kinetic energy.

$$\frac{1}{2} m_{Ar} V_{rms,Ar}^2 = \frac{1}{2} m_{Ne} V_{rms,Ne}^2$$

$$\Rightarrow V_{rms,Ar} = V_{rms,Ne}\sqrt{\frac{m_{Ne}}{m_{Ar}}} = 400\,\text{m s}^{-1}\sqrt{\frac{20}{40}} = 300\,\text{ms}^{-1}$$

11-18 $\quad V_{rms} = \sqrt{\dfrac{3k_B T}{m}} \rightarrow T = \dfrac{m V_{rms}^2}{3k_B}$

The original temperature $T_0 = 273\,K$

(a) If $V_{rms}' = \frac{1}{2} V_{rms,0}$, then

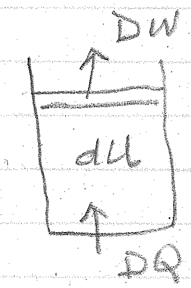
$$T' = \frac{m V_{rms}'^2}{3k_B} = \frac{1}{4} T_0 = 70\,K$$

(b) If $V_{rms}' = 2 V_{rms,0}$, then

$$T' = \frac{m V_{rms}'^2}{3k_B} = 4T_0 = 1000\,K$$

11-20 The first law of thermodynamics gives

$$DQ = dU + DW$$



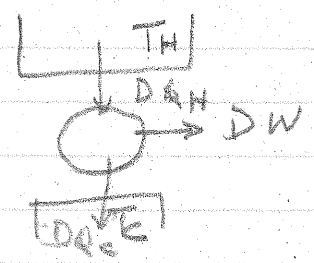
In this case, $DW = -400\,J$
$DQ = +600\,J$

Then the change in internal energy $= -400\,J + 600\,J$
$= 200\,J$

11-23 (a) efficiency

$$= \frac{55-40}{55} \times 100\%$$

$= 27\%$



(b) Work done / cycle $\quad DW = DQ_H + DQ_c$

$DQ_H + DQ_c = 55\,kJ - 40\,kJ = 15\,kJ \qquad [DQ_c \text{ is -ive}]$

11-27 For a Carnot engine, its efficiency is given by

$$e = 1 - \frac{T_C}{T_H}$$

$$\Rightarrow T_C = T_H(1-e)$$

$$T_H = 427°C = (427+273)K = 700K$$
$$T_C = 700K(1-0.6) = 280K$$

11-32 For a Carnot fridge, the coefficient of performance

$$COP_{max} = \frac{T_H}{T_H - T_C} = \frac{293K}{293K - 253K} \simeq 7.3$$

11-33(a)

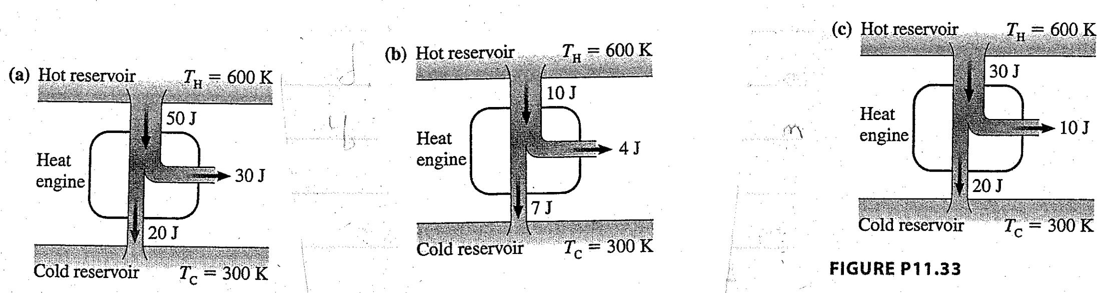


FIGURE P11.33

Engine (a) $DQ_H = 50J$, $DQ_C = -20J$, $DW_{out} = 30J$

$50 - 20 = 30J$ $\Rightarrow$ obeyed

Engine (b) $DQ_H = 10J$, $DQ_C = -7J$ $DW_{out} = 4J$

$10 - 7 = 3J$, $DW = 4J$ $\Rightarrow$ violated

Engine (c) $DQ_H = 30J$ $DQ_C = -20J$ $W_{out} = 10J$

$30 - 20 = 10J$ $\Rightarrow$ obeyed

(b) To check if second law of thermodynamics is violated, simply check $e \leq 1 - \frac{T_C}{T_H}$ is violated. The right hand side is Carnot engine's efficiency,

$$e_{carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{300K}{600K} = 0.50$$

Engine (a): $e = 1 - \frac{Q_C}{Q_H} = \frac{30J}{50J} = 0.6 > 0.5$ $\Rightarrow$ violated

Engine (b): $e = 1 - \frac{Q_C}{Q_H} = \frac{4J}{10J} = 0.4 < 0.5$ $\Rightarrow$ obeyed

Engine (c): $e = 1 - \frac{Q_C}{Q_H} = \frac{10J}{30J} = 0.33 < 0.5$ $\Rightarrow$ obeyed

11-45 By conservation of energy,

$$\overset{\text{initial}}{KE_i + \cancel{PE_i}_{0}} = \overset{\text{final}}{\cancel{KE_f}_{0} + PE_f}$$

$$\Rightarrow \frac{3}{2}k_B T = mgh$$

$$\Rightarrow h = \frac{3k_B T}{2mg} = \frac{3\,(1.38\times10^{-23}\,\mathrm{J\,K^{-1}})\,(300\,\mathrm{K})}{2\,(32\times1.66\times10^{-27}\,\mathrm{kg})\,(9.8\,\mathrm{m\,s^{-2}})}$$

$$= 1.19\times10^{4}\ \mathrm{m}$$

11-50 (a) $$e = \frac{DW_{out}}{DQ_H} = \frac{W_{out}}{DQ_C + W_{out}} = \frac{10\,\mathrm{J}}{15\,\mathrm{J}+10\,\mathrm{J}} = 0.40$$

(b) The Carnot engine's efficiency is

$$e_{carnot} = 1 - \frac{T_C}{T_H}$$

By second law of thermodynamics,

$$e \leq e_{carnot}$$

$$0.40 \leq 1 - \frac{T_C}{T_H}$$

$$T_H \geq \frac{T_C}{0.6} = \frac{293\,\mathrm{K}}{0.6} = 489\,\mathrm{K}$$

11-59 The maximum efficiency is that of the Carnot engine

$$e_{max} = 1 - \frac{T_C}{T_H}$$

where $T_C = 3^\circ C = (3+273)\,K = 276\,K$

$T_H = 27^\circ C = (27+273)\,K = 300\,K$

Then $$e_{max} = 1 - \frac{276}{300} = 0.08$$