SOLUTIONS – 11 FORMULAE

LINER MOMENTUM

$$\vec{p} = M\vec{v}$$

IMPULSE $\vec{J} = \vec{F}\,\Delta t$ [Const. $\vec{F}$]

$\vec{J}$ = Area under $\vec{F}$ vs. t graph.

2nd Law $$\frac{\Delta \vec{p}}{\Delta t} = \Sigma \vec{F}_i$$

or $$\frac{\vec{p}_f - \vec{p}_i}{t_f - t_i} = \langle \Sigma \vec{F}_i \rangle$$

CONSERVATION LAW FOR LINEAR MOMENTUM.

IF $\vec{F}_{ext} = 0$, Total (VECTOR) MOMENTUM IS CONSTANT.

$$(\Sigma \vec{p}_i)_{Aft} = (\Sigma \vec{p}_i)_{Bef.}$$

ANGULAR MOMENTUM

SINGLE MASS $\vec{L} = \vec{r} \times \vec{p} = \pm m r^2 \vec{\omega}$

Rigid Body $\vec{L} = I\vec{\omega}$

2nd Law $$\frac{\Delta \vec{L}}{\Delta t} = I\vec{\alpha} = \Sigma \vec{\tau}_i$$

CONSERVATION LAW: IF $\vec{\tau}_{ext} = 0$

$$\vec{L} = \text{Const.}$$

Two Body Collisions

Konservation of linear momentum

$$\underbrace{\vec{p}_1{}' + \vec{p}_2{}'}_{\text{After}} = \underbrace{\vec{p}_1 + \vec{p}_2}_{\text{Before}}$$

Special Cases

I Totally Inelastic $\quad \vec{V}_1{}' = \vec{V}_2{}'$

or $\quad \vec{V}_1 = \vec{V}_2$

II Totally Elastic General

Kinetic Energy is also conserved

$$\frac{1}{2} M_1 V_1'^2 + \frac{1}{2} M_2 V_2'^2 = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2$$

Totally Elastic Head-On Collision (x-axis)

$$\vec{V}_1{}' = \frac{M_1 - M_2}{M_1 + M_2} \vec{V}_1 + \frac{2 M_2 \vec{V}_2}{M_1 + M_2}$$

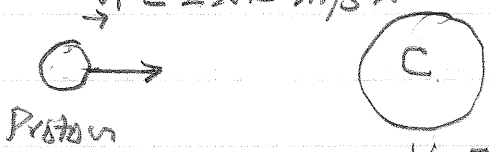
$$\vec{V}_2{}' = \frac{M_2 - M_1}{M_1 + M_2} \vec{V}_2 + \frac{2 M_1 \vec{V}_1}{M_1 + M_2}$$

(FROM LAST WEEK)

10-35

$\vec{v}_1 = 2\times10^7 \, m/s \, \hat{x}$

Before



$\vec{v}_2 = 0$

$M_1 = M_p$

$M_2 = 12 M_p$

Tot. Elastic Head on.

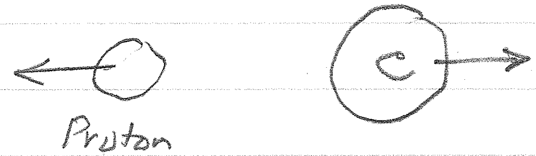
$$\vec{v}_1' = \frac{M_1 - M_2}{M_1 + M_2} \vec{v}_1 + \frac{2M_2 \vec{v}_2}{M_1 + M_2}$$

$$= \frac{M_p - 12M_p}{M_p + 12M_p} 2\times10^7 \, m/s \, \hat{x} = -1.69\times10^7 \, m/s \, \hat{x}$$

$$\vec{v}_2' = \frac{2M_1 \vec{v}_1}{M_1 + M_2} + \frac{M_2 - M_1}{M_1 + M_2} v_2$$

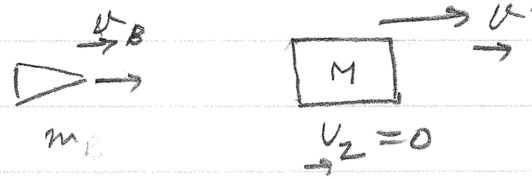
$$= \frac{2 M_p}{13 M_p} 2\times10^7 \, m/s \, \hat{x} = +0.31\times10^7 \, m/s \, \hat{x}$$

After

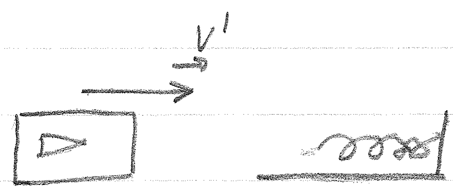


10-61

a) Totally Inelastic Collision. Only momentum is conserved



$$(m+M)\vec{v}' = m\vec{v}_B$$



Now it is Energy conservation

kinetic energy of $(m+M)$ goes to compress spring

$$K_f + P_f = K_i + P_i$$

$$0 + \frac{1}{2} k d^2 = \frac{1}{2}(m+M) v'^2 + 0.$$

so $$v'^2 = \frac{kd^2}{m+M} = \frac{m^2 v_B^2}{(m+M)^2}$$

$$v_B^2 = \frac{kd^2(m+M)}{m^2}$$

or $$\vec{v}_B = \sqrt{\frac{kd^2(m+M)}{m^2}}\ \hat{x}$$

b) $m = 5\times10^{-3}$ kg, $M = 2$ kg, $k = 50$ N/m $d = 0.1$m.

$$\vec{v}_B = \sqrt{\frac{50 \times 10^{-2} \times 2.005}{(5\times10^{-3})}} = 200\text{m/s}$$

c) $$K_{Bullet} = \frac{1}{2} \times 5\times10^{-3} \times (200)^2 \text{ J} \cong 100\text{ J}$$

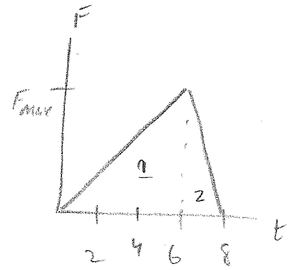
$$K_{m+M} = \frac{1}{2} \times 2.005 \times \frac{(5\times10^{-3})^2 \times (200)^2}{(2.005)^2} = 2.5\times10^{-2}\text{ J}$$

So the Bullet has lost nearly 0.9975 of its energy.

Week 11

Chapter 9 $\quad$ 4, 11, 13, 16, 22, 24, 27, 29, 30, 34, 35, 71.

4)



To find F_{max} note that impulse $\vec{J}$ is the area under the curve. Draw a $\perp$ from the vertex to the t axis. The area of the 2 Δ's is:

$(1/2)\, F_{max} \cdot 6s = A_1 \qquad A_2 = (1/2)\, F_{max} \cdot 2s$

Let

$\vec{J} = 6\ N\text{-}s\ \hat{x}$

$\Rightarrow A_1 + A_2 = 3\, F_{max} \cdot s + F_{max} \cdot s = 4\, F_{max} \cdot s$

Since $\vec{J} = 6.0\ N \cdot s\ \hat{x} \Rightarrow 6.0\ N \cdot \cancel{s} = 4 \cdot F_{max} \cdot s$

$\Rightarrow F_{max} = (3/2)\ N\, \hat{x}$

11) Mass of car is 1400 kg

$\underset{\rightarrow}{v} = 20\,m/s\ \hat{x}$

Rewriting Newton's second law $\vec{F} \cdot \Delta t = \Delta \vec{p} = m \Delta \vec{v}$

Now for the Barrels $\Delta t = 1.5\,s$ $\quad$ For the concrete $\Delta t = 0.1\,s$

In both cases $\Delta \vec{v} = -20\,m/s\ \hat{x}$ b/c the cars come to rest.

Hence in both cases $m \Delta \vec{v} = -1400\,kg \cdot 20\,m/s\,\hat{x} = -28{,}000\ kg\,m/s\ \hat{x}$

For Barrels:

$\vec{F} = \dfrac{m \Delta \vec{v}}{\Delta t}$

$= -\dfrac{28{,}000\,kg\,m/s\ \hat{x}}{1.5\,s} \approx -18{,}700\ N\,\hat{x}$

For concrete:

$\vec{F} = \dfrac{m \Delta \vec{v}}{\Delta t}$

$= -\dfrac{28{,}000\,kg\,m/s\ \hat{x}}{0.1\,s} = -280{,}000\ N\,\hat{x}$

13)

Momentum is conserved! $\Rightarrow$ $\sum \vec{P}_i = \sum \vec{P}_f$

$\vec{P}_{i_1} = m_1 \vec{v}_{i_1} = .100\,kg \cdot 1.20\,m/s\,\hat{x} = 0.12\ kg\,m/s\,\hat{x}$

$\vec{P}_{f_1} = m_1 \vec{v}_{f_1} = 0.100\,kg\,(-.85\,m/s)\hat{x} = -0.085\,kg\,m/s\,\hat{x}$

$\vec{P}_{f_2} = m_2 \vec{v}_{f_2} = 1\,kg\ \vec{v}_{f_2}$

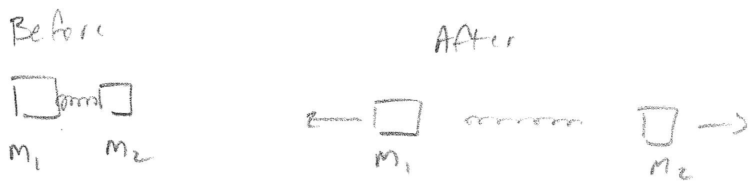
By momentum conservation

$\vec{P}_{i_1} = \vec{P}_{f_1} + \vec{P}_{f_2} \Rightarrow \vec{P}_{i_1} - \vec{P}_{f_1} = m_2 \vec{v}_{f_2} \Rightarrow \dfrac{\vec{P}_{i_1} - \vec{P}_{f_1}}{m_2} = \vec{v}_{f_2}$

Plugging in #'s

$\Rightarrow \dfrac{0.12\,kg\,m/s + 0.085\,kg\,m/s\,\hat{x}}{1\,kg} = \dfrac{0.205\,kg\,m/s\,\hat{x}}{1\,kg} = 0.205\,m/s\,\hat{x}$

16)



This is like an "explosive collision".

$m_1 = 2.3\,kg$

$m_2 = 5.3\,kg$

$\vec{v}_{i1} = \vec{v}_{i2} = 0\,m/s$

$v_{1f} = 6.0\,m/s\,\hat{x}$

$v_{2f} = ?$

Momentum must be conserved before + after. Since there is no momentum to begin with, there must be not net momentum after the blocks fly off

Hence $m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} = 0$

$\Rightarrow m_1 \vec{v}_{1f} = -m_2 \vec{v}_{2f}$

$\Rightarrow \vec{v}_{2f} = \dfrac{-m_1 \vec{v}_{1f}}{m_2} \Rightarrow v_{2f} = \dfrac{-2.3\,kg \cdot 6.0\,m/s\,\hat{x}}{5.3\,kg}$

$= -2.6\,m/s\,\hat{x}$

22) In the air the player can be pushed back by the ball since it is the only horizontal force acting on him. IT IS A TOTALLY INELASTIC COLLISION

Momentum must be conserved. we look at momentum in the $\hat{x}$ direction, After the collision the masses must be combined since the player is holding the ball.

$M_{ball} = 0.14\,kg$

$M_{player} = 71\,kg$

Hence $\vec{P}_{ball\,i} = \vec{P}_{ball+player\,final} \Rightarrow M_{ball}\vec{V}_{ball\,initial} = M_{b+p}\vec{V}_{b+p\,final} \Rightarrow V_{b+p\,final} = \dfrac{M_{ball}V_{ball\,i}}{M_{b+p}}$

$$\Rightarrow V_{b+p\,f} = \frac{0.14\,kg \cdot 28\,m/s\,\hat{x}}{71\,kg + 0.14\,kg} = 0.055\,m/s\,\hat{x}$$

24) If we want the total momentum to be zero after collision, it must also be so before collision

$M_c = 2000\,kg$ $\quad \vec{V}_{ci} = 1\,m/s\,\hat{x}$

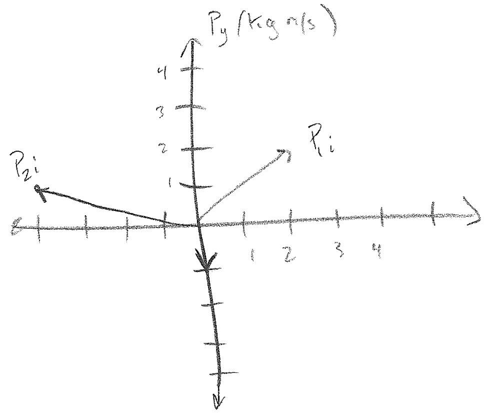
$M_w = 1000\,kg$ $\quad V_{wi} = ?$

$$\Rightarrow m_c\vec{V}_{ci} + m_w\vec{V}_{wi} = 0$$

$$\Rightarrow -\frac{m_c\vec{V}_{ci}}{m_w} = \vec{V}_{wi}$$

$$\Rightarrow \frac{-2000\,kg \cdot 1\,m/s}{1000\,kg} = -2\,m/s\,\hat{x}$$

27



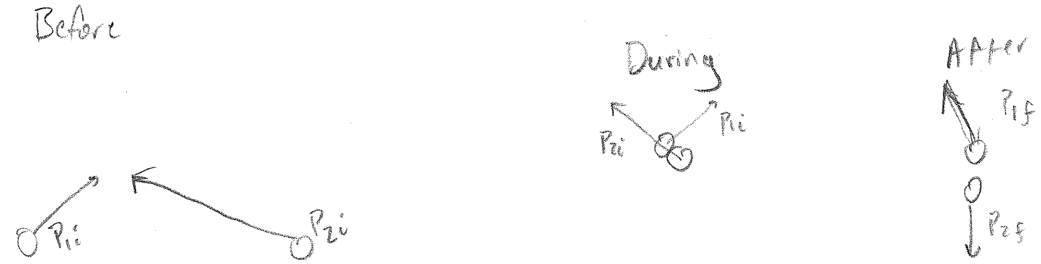
From the graph

$P_{1i} = 2\,\text{kgm/s}\,(\hat{x} + \hat{y})$

$P_{2i} = -4\,\text{kgm/s}\,\hat{x} + (\text{m/s})\text{kg}\,\hat{y}$

$P_{2f} = -1\,\text{kgm/s}\,\hat{y}$

Now, we must interpret the graph. It gives momenta but Not position. So we draw a picture:



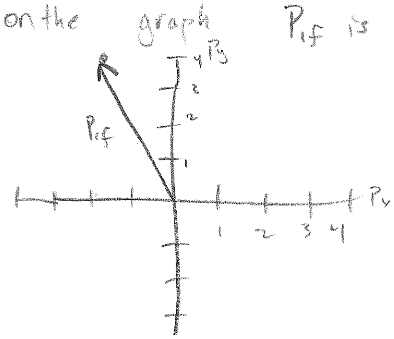
Since momentum must be conserved in both x + y directions, we have 2 eqns:

For x: $\Sigma P_{ix} = \Sigma P_{fx} = P_{1x_i} + P_{2x_i} = P_{1xf} + P_{2xf} \Rightarrow 2\,\text{kgm/s}\,\hat{x} - 4\,\text{kgm/s}\,\hat{x} = P_{1fx} = -2\,\text{kgm/s}\,\hat{x}$

For y $\quad P_{1yi} + P_{2yi} = P_{1yf} + P_{2yf} = 2\,\text{kgm/s} + 1\,\text{kgm/s} = -1\,\text{kgm/s} + P_{1yf}$

$\Rightarrow P_{1yf} = 4\,\text{kgm/s}$

So on the graph P_{1f} is



$$\vec{P}_{1f} = -2\,\text{kg-m/s}\,\hat{x} + 4\,\text{kg-m/s}\,\hat{y}$$

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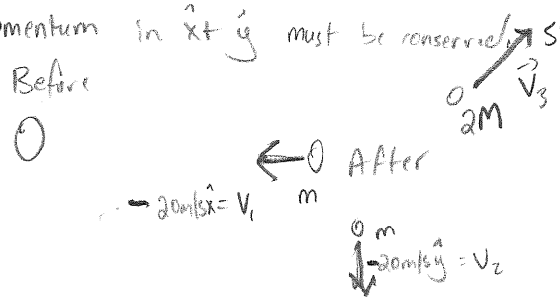
29 Momentum in $\hat{x} + \hat{y}$ must be conserved. Since it is initially zero, it must remain so in both x + y directions.



In x direction $mV_1 + 2mV_{3x} = 0 \Rightarrow V_{3x} = -\dfrac{\cancel{m}V_1}{2\cancel{m}}$

$= \dfrac{20\,\text{m/s}}{2}\hat{x} = 10\,\text{m/s}\,\hat{x}$

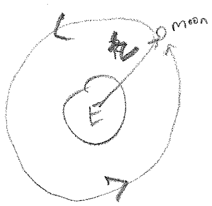
(continued on next page)

29 continued

In the y direction $mV_2 + 2m V_{3y} = 0 \Rightarrow V_{3y} = -\frac{\cancel{m} V_2}{2\cancel{m}} = \frac{20 m/s \, \hat{y}}{2} = 10 m/s \, \hat{y}$

So the third piece has velocity $\vec{v} = 10 m/s (\hat{x} + \hat{y})$

30)



$R = 3.8 \times 10^{8}$ meters $\quad M_{moon} = 7.4 \times 10^{22}$ kg

Angular momentum is given by $\vec{\ell} = \vec{r} \times \vec{p}$

For a particle (assume the moon is a particle) $\vec{\ell} = m r^2 \omega \hat{z}$

ω is $\frac{2\pi}{T}$ where T is the period.

Hence by substitution $\ell = \frac{mR^2 \cdot 2\pi}{T}$, T is 28 days

$$\vec{\ell} = \frac{7.4 \times 10^{22} \times (3.8 \times 10^8)^2 \times 2 \times \pi}{28 \times 24 \times 3600} \text{ kg-m}^2/\text{sec} \cdot \hat{z}$$

$$= +2.8 \times 10^{34} \text{ kg-m}^2/\text{sec } \hat{z}$$

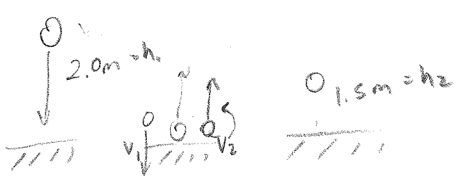
35) The angular momentum must be conserved.

$\vec{L}_f = \vec{L}_i \qquad L_i = I_i \omega_i = I_f \omega_f = L_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$

$I_i = 0.80 kg m^2$
$\omega_i = 5.0 rev/s$
$I_f = 3.2 kg m^2$

$$\Rightarrow \omega_f = \frac{0.80 \cancel{kg m^2} \cdot 5.0 rev/s}{3.2 \cancel{kg m^2}} = 1.25 rev/s$$

34)



$\vec{V_i} = -V_i \hat{y}$

$\vec{V_f} = +V_f \hat{y}$

By how much does the velocity change when the ball is in contact with the ground?

When the ball hits the ground its velocity is given by

$PE_i = KE_f \Rightarrow \cancel{m} g h_1 = \frac{1}{2} \cancel{m} v^2 \Rightarrow V_1^2 = 2gh_1$

When the ball rebounds its velocity can be found the same way:

$V_2^2 = 2gh_2$

(continued on next page)

39) continued.

Hence the ΔV or change in momenta is given by

$$\Delta V = V_2 - V_1 = \sqrt{2gh_2}\,\hat{y} - -\sqrt{2gh_1}\,\hat{y} = \sqrt{2gh_2} + \sqrt{2gh_1}$$

Now $\quad J = m\Delta V$

J is given by the area under the triangle, so $J = \dfrac{\Delta t\, F_{max}}{2} \Rightarrow$

$$\frac{\Delta t\, F_{max}}{2} = m\Delta V \Rightarrow \frac{2m\Delta V}{\Delta t} = f_{max} = \frac{2 \cdot 0.2kg \cdot \left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)}{0.005\,s}$$

$$\Rightarrow \frac{2 \cdot 0.2kg \cdot \left(\sqrt{2 \cdot 9.8m/s^2 \cdot 2m} + \sqrt{2 \cdot 9.8m/s^2 \cdot 1.5m}\right)}{0.005\,s}$$

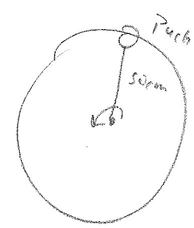
$$= \frac{2 \cdot 0.2kg \cdot (6.26m/s + 5.42)}{0.005} = \frac{0.4\,kg \cdot (11.68)m/s}{0.005\,s} = 935N \text{ the max force.}$$

$$\vec{F}_{Max} = 935N\hat{y} \downarrow \uparrow$$

70) The string supplies centripetal force to maintain the motion.

Now $\quad L = I\omega \quad , I = m.R^2 \Rightarrow \quad L = mR^2\omega \qquad L = 3.00kg\,m^2/s \quad , m = 0.2kg \quad R = 0.5m$

$$\Rightarrow \omega = \frac{L}{I} = \frac{L}{mR^2}$$



But centripetal force is given by $\quad F_c = m\omega^2 R$

$$\Rightarrow F_c = m\frac{L^2}{m^2R^4}R = \frac{L^2}{mR^3} = \frac{(3.00kg\,m^2/s)^2}{(0.5m)^3 \cdot 0.2kg} = 3.6 \times 10^2 N$$

71. The two weights provide the centripetal force. $\vec{F}_c = -\frac{MV^2}{R}\hat{r}$

a) $0.4 \times 9.8 = \frac{0.10 \times V^2}{0.2}$

$V = \sqrt{\frac{0.4 \times 9.8 \times 0.2}{0.1}}$

$= 2.8 \text{ m/s}$

71. ||| Figure P9.71 shows a 100 g puck revolving in a 20-cm-radius circle on a frictionless table. The string passes through a hole in the center of the table and is tied to two 200 g weights.

a. What speed does the puck need to support the two weights?

b. The lower weight is a light bag filled with sand. Suppose a hole is poked in the bag and the sand slowly leaks out while the puck is revolving. What will be the puck's speed and the radius of its trajectory after all of the sand is gone?

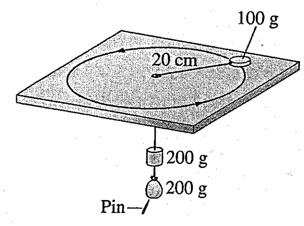


FIGURE P9.71

b) After all of the sand is gone now

$0.2 \times 9.8 = \frac{0.1 \times V_f^2}{R_f} \rightarrow (1)$ V_f, R_f final values

Also, since Force on puck is along radius Torque is zero so its angular momentum is same as before.

$L_{Bef} = 0.1 \times 2.8 \times 0.2 \text{ kg-m/s}.$
$= 0.1 \times V_f \times R_f \rightarrow (2)$

Eq (1) $V_f^2 = 19.6 R_f$

(2) $V_f = \frac{0.56}{R_f}$

so $V_f^3 = 10.98 \,(\text{m/s})^3$, $V_f = 2.2 \text{ m/s}.$

$R_f = \frac{0.56}{2.2} = 0.25 \text{ m}.$