Types of MOTION OF A RIGID BODY

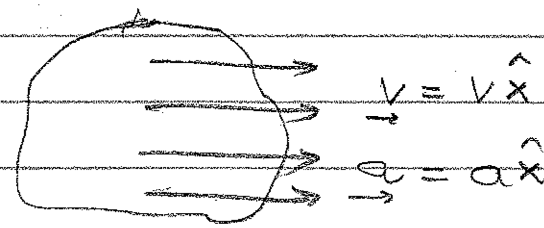
As you know a rigid Body consists of many mass points (m_i) located at different points ($\vec{r}_i$) but ($\vec{r}_i - \vec{r}_j$) is fixed so it neither changes shape nor size as it moves. This buys considerable simplification in describing the two types of motion it can have:

Translation: All the masses have the same linear velocity and the same linear acceleration

$\sum \vec{F}_i \neq 0$

$\sum \vec{\tau}_i = 0$

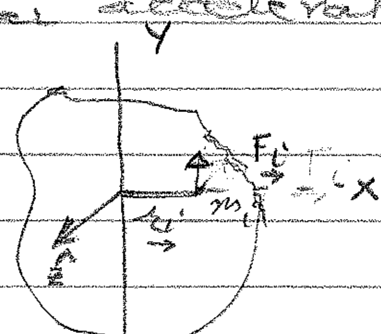
[Indeed $\vec{V} = \vec{V}_{c.m.}$]



Rotation about a fixed axis: Now the angular velocity and the angular acceleration are the same for all m_i

Now $\sum \vec{F}_i = 0$

$\sum \vec{\tau}_i \neq 0$.



For equilibrium we need Two conditions

$\vec{a} = 0$ and $\vec{\alpha} = 0$ So $\sum \vec{F}_i = 0$

$\sum \vec{\tau}_i = 0$

All torques taken about a single axis.

The Table below summarizes the equations when $\vec{a} \neq 0$, $\vec{\alpha} \neq 0$. (Dynamics)

Translation (one dim. X)	ROTATION (fixed axis z)
$\vec{x}$	$\vec{\theta}$
$\vec{v}$	$\vec{\omega}$
$\vec{a} = a\hat{x}$	$\vec{\alpha} = \alpha\hat{z}$, $\vec{a}_t = \alpha r_i \hat{\tau}$ *
$\vec{v} = (v_i + at)\hat{x}$	$\vec{\omega} = (\omega_i + \alpha t)\hat{z}$ $\vec{v}_t = \omega r_i \hat{\tau}$ *
$\vec{x} = (x_i + v_i t + \frac{1}{2} a t^2)\hat{x}$	$\vec{\theta} = (\theta_i + \omega_i t + \frac{1}{2} \alpha t^2)\hat{z}$
$v^2 = v_i^2 + 2a(x - x_i)$	$\omega^2 = \omega_i^2 + 2\alpha(\theta - \theta_i)$
	Displacement along arc
	$\Delta S = r \Delta\theta$
M (Mass)	$I = \sum m_i r_i^2$ (Moment of Inertia) **
$M\vec{a} = \sum \vec{F}_i$	$I\vec{\alpha} = \sum \vec{\tau}_i$
at that pt.	about same axis as
at that time	I **

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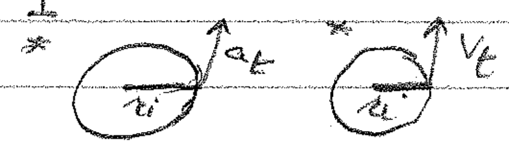

** I measures the manner in which the mass is distributed around the axis so $\sum \vec{\tau}_i$ must also be calculated using the same axis