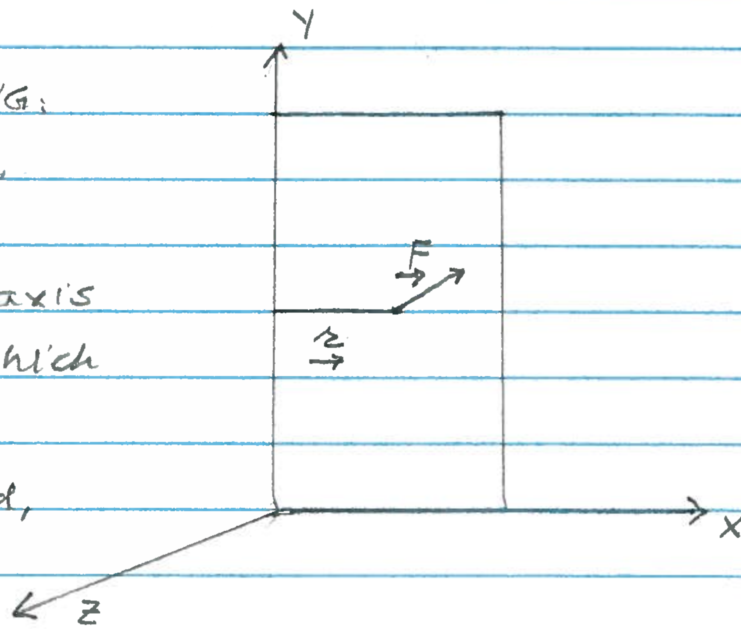
TORQUE

TORQUE: IS THE PHYSICAL AGENCY WHICH IS NECESSARY TO CAUSE ANGULAR ACCELERATION AND HENCE ROTATION ABOUT AN AXIS. WE WILL CONSIDER THE CASE OF ROTATION ABOUT A FIXED AXIS. TO HAVE A TORQUE ONE MUST APPLY A FORCE AT SOME DISTANCE FROM THE AXIS ABOUT WHICH ROTATION IS DESIRED.



CONSIDER THE FOLLOWING:

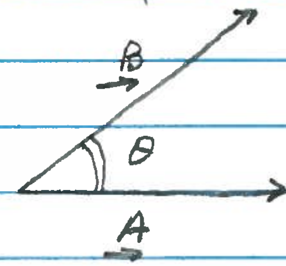
You want to open a door which is hinged along the y-axis. You pick a point which is some distance $\vec{r}$ from the hinge. Indeed, the larger the r the less push (force) you will need to cause the door to swing.

Next, you need to apply a force perpendicular to $\vec{r}$. If $\vec{F}$ is parallel to $\vec{r}$ the door will never open. Notice that $\vec{r} \parallel \hat{x}$, $\vec{F} \parallel -\hat{z}$ but door rotates about $\hat{y}$. Indeed the physical agency that causes the swing is the Torque vector

$$\vec{\tau}$$

which is parallel to $\hat{y}$. Amazing, $\vec{r}$ is horizontal, $\vec{F}$ is horizontal but $\vec{\tau}$ is vertical

We need a new concept in vector algebra such that multiplying two vectors produces a third vector which is perpendicular to both of them. Such a product is called a vector product or cross product



Given two vectors $\vec{A}$ and $\vec{B}$ with an angle

$$\theta = (\vec{A}, \vec{B})$$

between them, the vector product is written as

$$\vec{C} = (\vec{A} \times \vec{B})$$

The magnitude of C is

$$C = AB \sin(\vec{A}, \vec{B})$$

$\vec{C}$ is perpendicular to the AB plane. Which perpendicular?

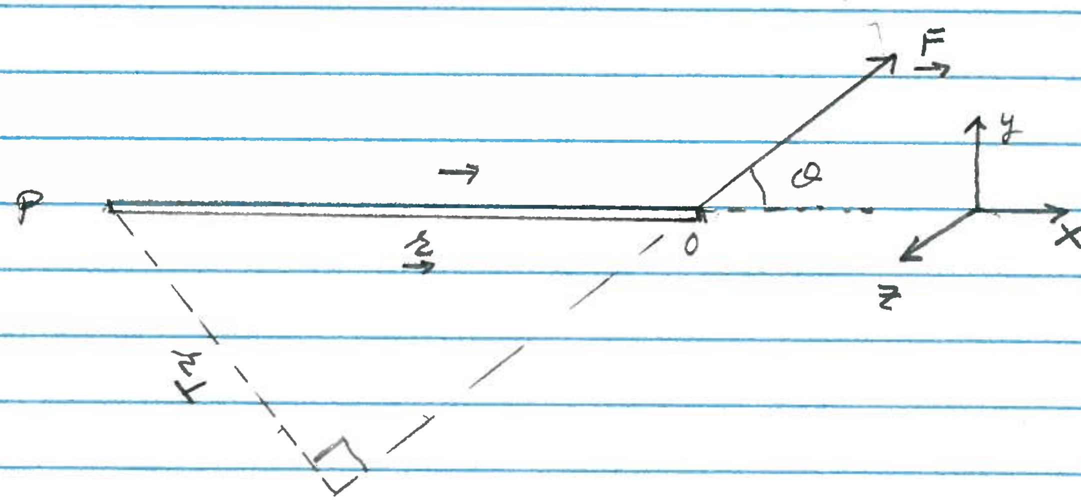
Right Hand Rule: stretch right hand

First vector $\quad \vec{A} \parallel$ Thumb

Second Vector $\quad \vec{B} \parallel$ Fingers

Product $\quad \vec{C} \perp$ Palm.

The Torque Vector can now be defined formally. A bar of length $\vec{l}$ can pivot (rotate) about an axis perpendicular to point P. We apply a force $\vec{F}$ as shown



Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

direction of $\vec{r}$ is always from pivot point P to point of application (0) of force $\vec{F}$.

Direction of $\vec{\tau}$ along $+\hat{z}$

Magnitude of $\tau = rF\sin\theta = rF_\perp = r_\perp F$

$F_\perp$ = component of $\vec{F}$ ∥ Bar

$r_\perp$ = Perpendicular distance between $\vec{F}$ (extended) and P [sometimes called moment arm].

Immediately one notices

$\vec{\tau}$ is zero if $\vec{F} \parallel \vec{r}$

$\vec{\tau}$ is maxim when $\vec{F} \perp \vec{r}$