PRESSURE IN A GAS

We now know that a Gas Exerts a pressure on the walls of its Container and by Combining the three Expts. on Pressure (P) Volume (V) and temperature (T) we can write the Equation for an Ideal or Perfect gas

$$PV = Nk_BT \qquad \text{— (A)}$$

where N = # of atoms in the system.

$k_B = 1.383 \times 10^{-23}$ J/K is Boltzmann's Constant and T is temperature measured on the Kelvin Scale $T \cong \theta + 273$, If θ is on Celsius scale.

We will now try to understand the origin of the pressure by approaching the problem from the microscopic viewpoint.

Question: What is a gas?

In our system we have N atoms (or molecules but for molecules we consider only translation) each of which has a mass m. The atoms have a finite size & so they Collide with one another as well as with the walls of the Container and for all practical purposes the collisions are totally elastic so both linear momentum and kinetic energy are conserved.

A useful Qty to define is number density

$$n_d = \frac{N}{V} \qquad [\rho = mn].$$

That is, n is # of atoms per m^3

At finite temperatures all the atoms are in motion. Their velocities are distributed over a wide range and the motions are entirely random. That is, if the gas is in a stationary container the average velocity of all the particles is precisely zero.

Mathematically we say: that of the n atoms/m^3

n_1 have vel. $\underset{\rightarrow}{v_1}$

n_2 have vel $\underset{\rightarrow}{v_2}$

$\vdots$

n_i have vel $\underset{\rightarrow}{v_i}$

$\vdots$

Such that $\quad \Sigma n_i = n.$

and

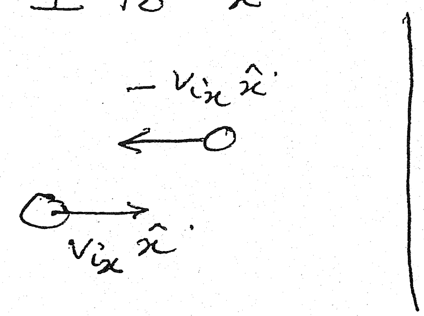
$$\boxed{\langle \underset{\rightarrow}{v} \rangle = \frac{\Sigma n_i \underset{\rightarrow}{v_i}}{\Sigma n_i} \equiv 0} \quad \text{— (B)}$$

Next, let us concentrate on the atoms with velocity $\underset{\rightarrow}{v_i}$. and in particular the x-component $+ v_{ix}\hat{x}$

and consider such an atom approaching a wall of the container $\perp$ to $\hat{x}$

The mom.$^{m.}$ before the collision is

$\underset{\rightarrow}{p_B} = m v_{ix} \hat{x}$



Collision is totally Elastic, Mom^{m} after is

$\underset{\rightarrow}{p_A} = -m v_{ix} \hat{x}$

Now $\underset{\rightarrow}{p_A} + (\underset{\rightarrow}{p}_{wall})_A = \underset{\rightarrow}{p_B} + (p_{wall})_B$.

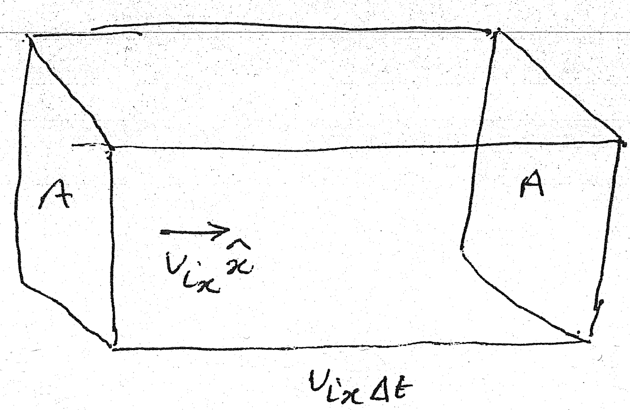
so $\boxed{\Delta p_{wall} = 2m v_{ix}.}$

That is, each time an $_\wedge$ i type atom, bounces off the wall, the wall picks up $2m v_{ix}$

Next, we need to know how many collisions occur

Take wall area A

Construct a parallelepiped of height $v_{ix}\Delta t$



All n_i particles travelling to the right contained in this volume will hit the wall in Δt secs.

# of collisions in time $\Delta t = \frac{1}{2} n_i v_{ix} A \Delta t$

Note the $\frac{1}{2}$ factor. It is put in because random motion requires that of the n_i particles with velocity v_{ix} on the average half the particles are going left (right).

Hence, in time Δt the wall will pick up from i type particles the mom^m.

$$(\Delta p_i)_{wall} = m\, n_i\, v_{ix}^2\, A\, \Delta t.$$

Newton's law tells us that $\frac{\Delta p}{\Delta t}$ is a FORCE.
so wall experiences a force due to i particles

$$F_i = m\, n_i\, v_{ix}^2\, A.$$

and a pressure

$$P_i = \frac{F_i}{A} = m\, n_i\, v_{ix}^2.$$

Total pressure

$$P = \Sigma P_i = m \;\Sigma n_i\, v_{ix}^2$$

$$= m\, n_d \langle v_x^2 \rangle \qquad \left[\langle v_x^2 \rangle = \frac{\Sigma n_i v_{ix}^2}{n_d}\right].$$

Since there is no preferred direction

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

also

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

hence

$$\boxed{P = \frac{1}{3} n m \langle v^2 \rangle} \longrightarrow \text{C}$$

NOTICE $\langle \underset{\sim}{v} \rangle \equiv 0$

$$\langle v^2 \rangle \neq 0.$$

Look at (C) again

$$P = \frac{1}{3} m n_d \langle v^2 \rangle$$

$$= \frac{2}{3} n_d \frac{1}{2} m \langle v^2 \rangle = \frac{2}{3} \frac{N}{V} \frac{1}{2} m \langle v^2 \rangle$$

But $\frac{1}{2} m \langle v^2 \rangle$ = Av. kinetic Energy of the atom of mass m.

so $\boxed{P = \frac{2}{3} n \langle K \rangle = \frac{2}{3} \frac{N}{V} \langle K \rangle.}$ (D)

That is, pressure of a gas is Equal to $\frac{2}{3}$ rds of the kinetic Energy of the particles per unit volume

Next, compare our microscopic result (D) with result of Expt (A)

$$\frac{2}{3} N \langle K \rangle = N k_B T.$$

or $\boxed{\langle K \rangle = \frac{3}{2} k_B T.}$ (E)

IF A GAS IS SITTING AT A TEMPERATURE OF T KELVIN THE AVERAGE KINETIC ENERGY OF MOTION OF ITS ATOMS IS $\frac{3}{2} k_B T$ [IN MOLECULES THIS WOULD REFER TO TRANSLATION OR MOTION OF C.M.]

Notice, the answer in Eq (E) DOES NOT DEPEND ON MASS OF ATOMS. Hence if we have a mixture of two gases in ≡m at T the <u>kinetic Energies are identical.</u>

An equally interesting quantity is the so-called ROOT-MEAN-SQUARE (or r.m.s.) speed

$$V_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3 k_B T}{m}}$$

Now indeed if two gases are in ≡m the r.m.s. speed of the lighter atoms is higher. For example, in a mixture of He [$m_{He} = 4 m_p$, m_p = mass of proton] and A [$m_A = 40 m_p$] the He atoms have an rms speed which is higher by a factor of $\sqrt{10} \sim 3.162$.

A typical value for V_{rms}

$$T = 300K$$

$$m_{He} = 4 \times 1.6 \times 10^{-27} Kg$$

$$k_B = 1.38 \times 10^{-23} J/K.$$

$$V_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4 \times 1.6 \times 10^{-27}}} \cong 1.4 \times 10^3 m/s.$$