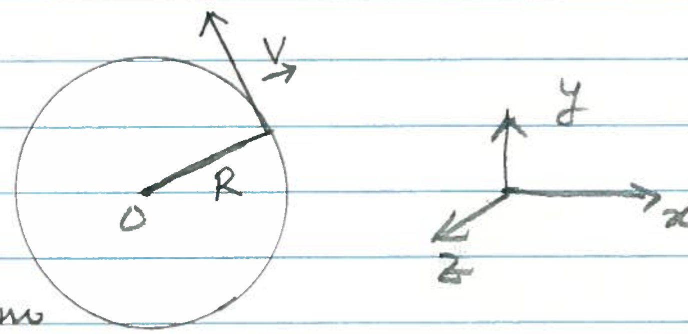
CONSERVATION OF ANGULAR MOMENTUM — KEPLERS LAWS.

A single mass m moving on a circle of radius R at a uniform velocity has a tangential velocity

$$\vec{v} = R\omega\hat{\tau}$$

It therefore has a linear momentum

$$\vec{p} = MR\omega\hat{\tau}$$



The angular momentum of this object is defined by

$$\vec{l} = \vec{r}\times\vec{p}$$

where $\vec{r} = R\hat{r}$ so $\vec{l}$ is $\perp$ to the plane of the circle. and will be along $\pm\hat{z}$

$$\vec{l} = \pm MR^2\omega\hat{z}$$

If a tangential force is applied to M

$$M\vec{a}_t = \vec{F}_t$$

and there will be a torque about z $\vec{\tau} = \vec{r}\times\vec{F}_t$

and it will have an angular acceleration $\vec{\alpha}$.

$$\vec{a}_t = R\alpha\hat{\tau}$$

Now

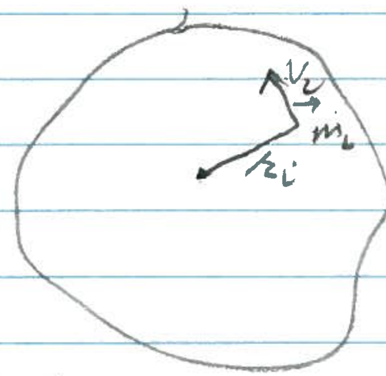
$$\vec{\tau} = \pm RMa\hat{z} = \pm MR^2\alpha\hat{z}$$

$$= \pm MR^2\frac{\Delta\omega}{\Delta t}\hat{z} = \frac{\Delta\vec{l}}{\Delta t}$$

That is, if you want angular momentum to change with time you must apply a torque.

Newton's law for rotation in terms of angular momentum.

Next, apply it to a rigid body rotation



$\vec{\omega}$ & $\vec{\alpha}$ are common

but ith mass has

$\vec{v}_t = r_i \omega \hat{\tau}$

ith mass has angular momentum

$\vec{l}_i = m_i r_i^2 \omega \hat{z}$ for c.c.w. rotation

right hand rule $\begin{cases} \vec{r}_i \text{ along thumb} \\ \vec{p}_i \text{ '' finger} \\ \vec{l}_i \perp \text{palm} \end{cases}$

Total angular momentum of Rigid Body

$\vec{L} = \sum m_i r_i^2 \vec{\omega}$

$= I \vec{\omega}$

compare this to the total linear momentum

$\vec{P} = M \vec{V}$

so again I replaces M and ω replaces V.

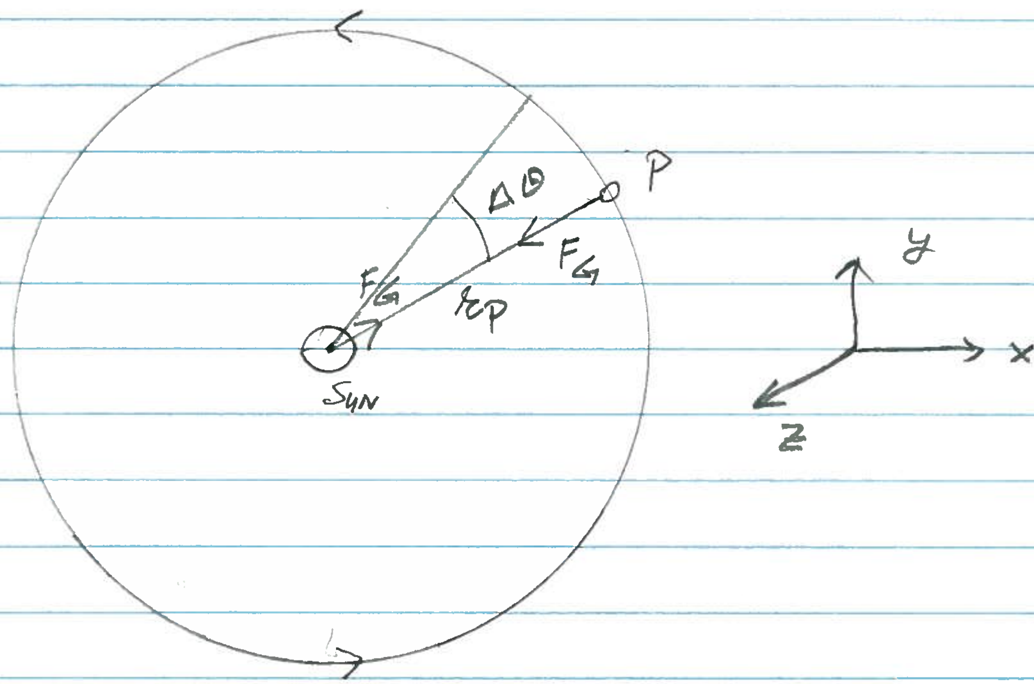
<u>CONSERVATION LAWS</u>

Linear Mom^m.	Angular Mom^m.
$\vec{F}_{ext} = 0$	$\vec{T}_{ext} = 0$
$\vec{P}$ = const.	$\vec{L}$ = Const.

Let us apply this to motion of planets around Sun in circular orbits

Kepler's laws: i) PLANETS MOVE IN PLANAR ORBITS.
ii) As planet goes around the sun the radius sweeps out equal areas in equal intervals of time.

Radius vector
$\vec{r} = r_p \hat{r}$



The only force acting on the planet is the Gravitational force due to the Sun

$$\vec{F}_G = -\frac{G M_s M_p}{r_p^2}\hat{r}$$

IF we take the torque about an axis through the Sun

$$\vec{\tau}_p = \vec{r} \times \vec{F}_G = 0 \qquad \text{because } [\hat{r} \times \hat{r}] = 0$$

Hence angular momentum of planet around this axis must be constant

$$\vec{L}_p = M_p r_p^2 \omega_p \hat{z}$$

Since $\vec{L}_p$ cannot change direction orbit must lie in xy-plane. [It is also a plane because $\vec{F}_G$ is only along $\hat{r}$].

Next, consider that the radius rotates through angle $\Delta\theta$ in time Δt.

Area swept out by $\vec{r}$ becomes

$$\Delta A = \frac{1}{2} r_p^2 \, \Delta\theta$$

and area swept per second

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r_p^2 \frac{\Delta\theta}{\Delta t}$$

$$= \frac{1}{2} r_p^2 \, \omega = \frac{1}{2} \frac{L_p}{M_p}$$

$$= \text{Const.}$$

because magnitude of L_p is constant.