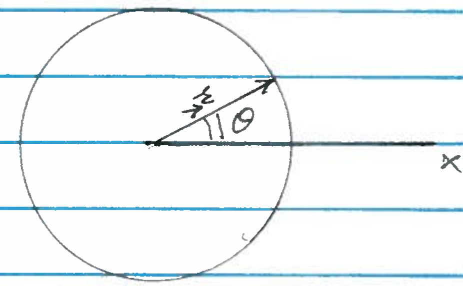
CIRCULAR MOTION WHEN SPEED IS NOT CONSTANT

Upto now we have been considering circular motion where the speed was constant so we could define period T, and write



$$s = \frac{2\pi R}{T}$$

$\vec{r} = R\hat{r}$ rotating by ω radian per second

$\vec{v} = R\omega\hat{t}$ " " " " " "

$\vec{a}_c = -R\omega^2\hat{r}$ " " " " " "

$\vec{\omega} = \pm \frac{\Delta\theta}{\Delta t}\hat{n} \longrightarrow$ Constant.

If we want to think of how the angle θ changes with time we can construct a table

Let $\omega = 0.1$ rad/s

time (sec)	Δθ (rad)
0	0
1	0.1
2	0.2
t	0.1t

and write

$$\vec{\theta} = (\theta_i + \omega t)\hat{n}$$

where θ_i is angle at $t=0$ exactly as we wrote

$\vec{x} = (x_i + vt)\hat{x}$ some time ago.

Next, we want to consider a situation where speed is not constant. That means that the angular speed is also not constant. We will not change the direction of $\vec{\omega}$, only its magnitude and define angular acceleration vector

$$\underset{\to}{\alpha} = \frac{\Delta \underset{\to}{\omega}}{\Delta t} \qquad [\ L^0 T^{-2} \quad rad/s^2 \text{ vector}]$$

and α measures the change in ω per sec so now

$$\underset{\to}{\omega} = \pm(\omega_i + \alpha t)\hat{n} \qquad [\text{compare } \underset{\to}{v} = (v_i + at)\hat{x}]$$

where ω_i is angular velocity at $t=0$.

And following the same steps as before

$$\underset{\to}{\theta} = \pm(\theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2)\hat{n} \qquad [\text{compare } \underset{\to}{x} = (x_i + v_i t + \tfrac{1}{2}\alpha t^2)\hat{x}$$

So Kinematic EQNs are

Linear Motion (One Dim)	Angular motion
x	θ
$\underset{\to}{a} = a\hat{x}$	$\underset{\to}{\alpha} = \alpha\hat{n}$
$\underset{\to}{v} = (v_i + at)\hat{x}$	$\underset{\to}{\omega} = (\omega_i + \alpha t)\hat{n}$
$\underset{\to}{x} = (x_i + v_i t + \frac{1}{2}at^2)\hat{x}$	$\underset{\to}{\theta} = (\theta_i + \omega_i t + \frac{1}{2}\alpha t^2)\hat{n}$
$v^2 = v_i^2 + 2a(x - x_i)$	$\omega^2 = \omega_i^2 + 2\alpha(\theta - \theta_i)$

To cause an acceleration $\underset{\to}{a}$, Newton taught us that we must provide a force at that pt. at that time

$$M\underset{\to}{a} = \Sigma \underset{\to}{F_i} \quad \text{(at that pt. at that time)}.$$

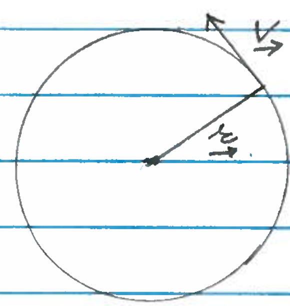
What do we need to cause angular acceleration $\underset{\to}{\alpha}$? A new physical agency which we will develop next.

Before we go there let us note that we still have

$$\underset{\rightarrow}{r} = R\hat{r}$$
$$\underset{\rightarrow}{v} = R\omega\,\hat{\tau}$$
$$\underset{\rightarrow}{a_c} = -R\omega^2\,\hat{r}$$

but they no longer rotate at constant rates and the magnitudes of v and a_c are now varying with time.

Indeed now in addition to centripetal acceleration we have a TANGENTIAL ACCELERATION



$$\underset{\rightarrow}{a_t} = R\frac{\Delta\omega}{\Delta t}\,\hat{\tau} = R\alpha\,\hat{\tau}$$

and in accord with Newton's law we not only need a centripetal force

$$\underset{\rightarrow}{F_c} = -MR\omega^2\hat{r}$$

but also a tangential force

$$\underset{\rightarrow}{F_t} = M a_t\,\hat{\tau}$$

which leads to a new physical agency.