

Solutions To Homework Set 06

from College Physics, 7th Ed

by Serway/Faughn

Dr Bhagat

[16-44] (a) The energy stored in one capacitor is $E = \frac{1}{2}CV^2$. Each capacitor has $V = 100$ volts because they are in parallel. Thus

$$\begin{aligned} E &= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 \\ &= \frac{1}{2}(25.0 \times 10^{-6} \text{ F})(100 \text{ V})^2 + \frac{1}{2}(5.00 \times 10^{-6} \text{ F})(100 \text{ V})^2 \\ &= 0.15 \text{ J} \end{aligned}$$

(b) If the capacitors are in *series*, then the effective capacitance is

$$\begin{aligned} C_{\text{eff}} &= \frac{C_1C_2}{C_1 + C_2} \\ &= \frac{(25.0 \times 10^{-6} \text{ F})(5.00 \times 10^{-6} \text{ F})}{25.0 \times 10^{-6} \text{ F} + 5.00 \times 10^{-6} \text{ F}} \\ &= 4.17 \times 10^{-6} \text{ F} \end{aligned}$$

This effective capacitor will need to have the same energy as part (a), so

$$\begin{aligned} E &= \frac{1}{2}C_{\text{eff}}V^2 \\ \frac{2E}{C_{\text{eff}}} &= V^2 \\ V &= \sqrt{\frac{2E}{C_{\text{eff}}}} \\ &= \sqrt{\frac{2(0.15 \text{ J})}{(4.17 \times 10^{-6} \text{ F})}} \\ &= 268 \text{ V} \end{aligned}$$

[16-49] We're given the area, $A = 175 \text{ cm}^2$ and the separation of the plates, $d = 0.0400 \text{ mm}$. Since the insulator is Teflon[®], we can look up the dielectric

constant and dielectric strength on page 557, $\kappa_{\text{Teflon}} = 2.1$, $\phi_{\text{Teflon}} = 60 \times 10^6 \text{ V/m}$. Now we can answer the question.

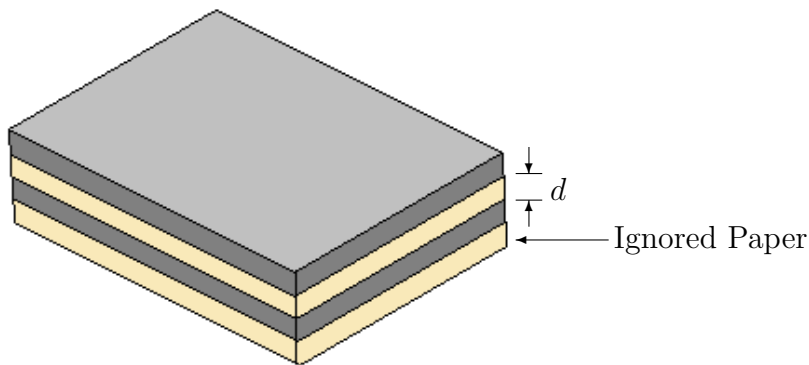
(a) $C = \frac{\kappa\epsilon_0 A}{d}$, so

$$\begin{aligned} C &= \frac{2.1 (8.85 \times 10^{-12} \text{ F/m}) (175 \text{ cm}^2)}{0.0400 \text{ mm}} \\ &= \frac{2.1 (8.85 \times 10^{-12} \text{ F/m}) (175 \times 10^{-4} \text{ m}^2)}{0.0400 \times 10^{-3} \text{ m}} \\ &= 8.13 \times 10^{-9} \text{ F} \\ &= 8.13 \text{ nF} \end{aligned}$$

(b)

$$\begin{aligned} V_{\text{break}} &= \phi_{\text{Teflon}} d \\ &= (60 \times 10^6 \text{ V/m}) (0.0400 \times 10^{-3} \text{ m}) \\ &= 2.4 \text{ kV} \end{aligned}$$

[16-50] The question is only asking us to calculate the capacitance of a parallel plate capacitor; for this purpose we may ignore the second sheet of paper



As illustrated, the paper's thickness $d = 0.0250 \times 10^{-3} \text{ m}$ will be the separation of the aluminum plates. The area, A , of the plates will be given by $A = \ell w$ where the width is given, $w = 7.00 \text{ cm}$. We're also given the

dielectric constant, $\kappa = 3.70$ and the desired capacitance, $C = 9.50 \times 10^{-8}$ F. So, let's proceed:

$$\begin{aligned}
 C &= \frac{\kappa\epsilon_0 A}{d} \\
 &= \frac{\kappa\epsilon_0 \ell w}{d} \\
 Cd &= \kappa\epsilon_0 \ell w \\
 \ell &= \frac{Cd}{\kappa\epsilon_0 w} \\
 &= \frac{(9.50 \times 10^{-8} \text{ F})(0.0250 \times 10^{-3} \text{ m})}{3.70(8.85 \times 10^{-12} \text{ F/m})(7.00 \times 10^{-2} \text{ m})} \\
 &= .104 \text{ m} \\
 &= 10.4 \text{ cm}
 \end{aligned}$$

[16-54] The capacitors have a value C_p when in parallel, which means $C_1 + C_2 = C_p$. They have a value of C_s when in series, so that implies

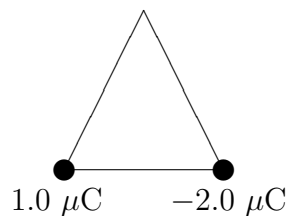
$$\begin{aligned}
 C_s &= \frac{C_1 C_2}{C_1 + C_2} \\
 &= \frac{C_1 C_2}{C_p}
 \end{aligned}$$

or $C_s C_p = C_1 C_2$.

We now have an expression for the sum, $C_1 + C_2 = C_p$, and the product, $C_1 C_2 = C_s C_p$. Due to this form, the capacitors will be

$$\begin{aligned}
 C_1 &= \frac{1}{2} \left(C_p + \sqrt{C_p^2 - 4C_s C_p} \right) \\
 C_2 &= \frac{1}{2} \left(C_p - \sqrt{C_p^2 - 4C_s C_p} \right)
 \end{aligned}$$

[16-56] The picture is reproduced below



(a) The potential due to $q_1 = 1.0 \mu\text{C}$ is

$$\begin{aligned}
 V_1 &= \frac{kq_1}{r_1} \\
 &= \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C})}{0.50 \text{ m}} \\
 &= 18 \times 10^3 \text{ V}
 \end{aligned}$$

(b) The potential due to $q_2 = -2.0 \mu\text{C}$ is

$$\begin{aligned}
 V_2 &= \frac{kq_2}{r_2} \\
 &= \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(-2.0 \times 10^{-6} \text{ C})}{0.50 \text{ m}} \\
 &= -36 \times 10^3 \text{ V}
 \end{aligned}$$

(c) The total potential at P is just the sum of the potential of the two charges:

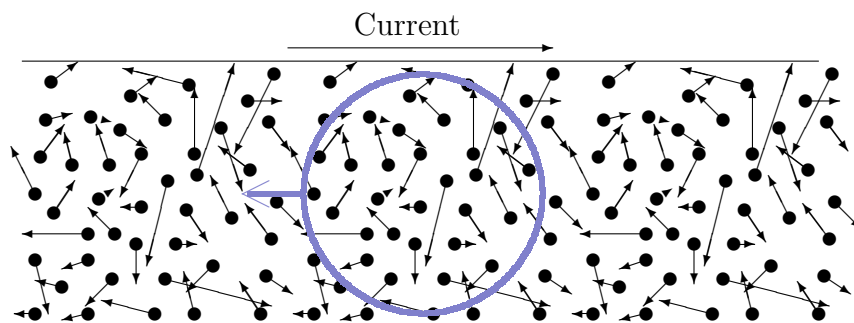
$$\begin{aligned}
 V_P &= V_1 + V_2 \\
 &= -18 \times 10^3 \text{ V}
 \end{aligned}$$

(d) The work done to bring a charge to P from infinity is just qV_P

$$\begin{aligned}
 W &= qV_P \\
 &= (3.0 \times 10^{-6} \text{ C})(-18 \times 10^3 \text{ V}) \\
 &= -54 \times 10^{-3} \text{ J}
 \end{aligned}$$

Notice the minus sign! We would actually *gain* energy by bringing in the $+3.0 \times 10^{-6}$ C charge.

[17-1] The situation is picture below, the dots are electrons and arrows indicate their direction of motion. Notice that the electrons are actually moving in various and sundry directions but that, on average, they move to left. This is represented by the **circle** which can be considered a “cloud” of electrons which slowly drifts to the left.

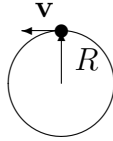


Given that the current is $80.0 \text{ mA} = 80 \times 10^{-3} \text{ A} = 80 \times 10^{-3} \text{ C/s}$ we need to know the charge of the electron, $e = 1.60 \times 10^{-19} \text{ C}$, and the time $t = (10 \text{ min})(60 \text{ s/min}) = 600 \text{ s}$. Thus, on average, the number of electrons going by a given cross-section (the movement of my “cloud”) is

$$\begin{aligned}
 N &= \frac{It}{e} \\
 &= \frac{(80 \times 10^{-3} \text{ C/s})(600 \text{ s})}{1.60 \times 10^{-19} \text{ C}} \\
 &= 3 \times 10^{20}
 \end{aligned}$$

Looks like I need to draw some more electrons :-)

[17-5] We just need to know the current along the circle, which is depicted below



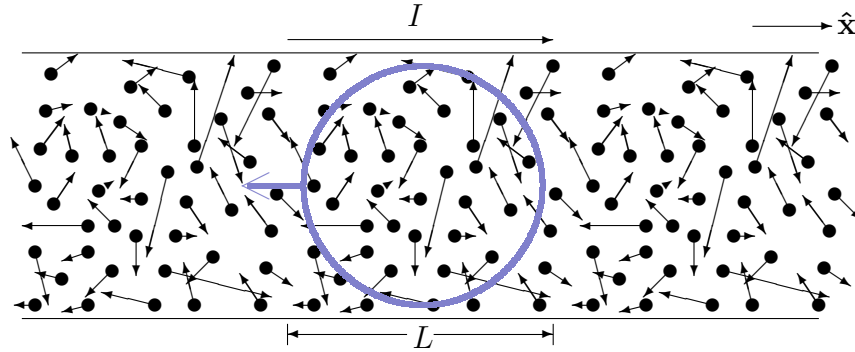
Since the electron's speed is known, $v = 2.19 \times 10^6$ m/s, and the total distance traveled is just the circumference of the circle, $d = 2\pi R$ with $R = 5.29 \times 10^{-11}$ m, we can find how long it takes the electron to go around once

$$\begin{aligned} T &= \frac{2\pi R}{v} \\ &= \frac{2\pi (5.29 \times 10^{-11} \text{ m})}{2.19 \times 10^6 \text{ m/s}} \\ &= 1.5 \times 10^{-16} \text{ s} \end{aligned}$$

Now we want to know the electron's period because that's how often it goes by a particular spot. Recall that the current just measures how much charge goes by, so in this case we have one electron going by every 1.5×10^{-16} s, thus:

$$\begin{aligned} I_{\text{eff}} &= \frac{e}{T} \\ &= \frac{1.60 \times 10^{-19} \text{ C}}{1.5 \times 10^{-16} \text{ s}} \\ &= 10.7 \times 10^{-3} \text{ A} \end{aligned}$$

[17-8] To do this we'll need to know the number of electrons in a given length L



But since we know the volume of that segment, we can find the total mass of aluminum in that section:

$$\begin{aligned} m_{\text{Al}} &= \rho V \\ &= \rho L A \end{aligned}$$

with $\rho = 2.7 \text{ g/cm}^3$ and $A = 4.0 \times 10^{-6} \text{ m}^2$, the cross-sectional area. Now the atomic mass, \mathcal{A} , of Al is 26.98 which means that there are 26.98 g Al/mole. Since we all recall Avogadro's Number, $N_A = 6.022 \times 10^{23}$, we can find the number of aluminum atoms in that volume

$$\begin{aligned} N_{\text{atoms}} &= \frac{m_{\text{Al}}}{\mathcal{A}} N_A \\ &= \frac{\rho L A N_A}{\mathcal{A}} \end{aligned}$$

Now, if each atom produces one free electron, then there are $N_e = N_{\text{atoms}}$ electrons wandering around in that volume. If each of these electrons has a velocity $-v_d \hat{x}$, then they travel the distance L in a time τ given by $\tau = \frac{L}{v_d}$.

Therefore, the total amount of charge moving in that volume is $Q = N_e e = I \tau$, with the last equals coming from the fact that current measures charge per time passing by. So, now we have everything to find what we

need:

$$\begin{aligned}N_e e &= I\tau \\N_{\text{atoms}} e &= I\tau \\ \frac{\rho L A N_A e}{\mathcal{A}} &= I\tau \\ &= I \frac{L}{v_d} \\ \frac{\rho A N_A e}{\mathcal{A}} &= \frac{I}{v_d} \\ v_d &= \frac{\mathcal{A} I}{\rho A N_A e}\end{aligned}$$

thus

$$\begin{aligned}v_d &= \frac{(26.98 \text{ g Al/mole}) (5.0 \text{ A})}{(2.7 \text{ g Al/cm}^3) (4.0 \times 10^{-6} \text{ m}^2) (6.022 \times 10^{23} \text{ 1/mole}) (1.60 \times 10^{-19} \text{ C})} \\ &= \frac{(26.98 \text{ g Al/mole}) (5.0 \text{ C/s})}{(2.7 \times 10^6 \text{ g Al/m}^3) (4.0 \times 10^{-6} \text{ m}^2) (6.022 \times 10^{23} \text{ 1/mole}) (1.60 \times 10^{-19} \text{ C})} \\ &= 1.30 \times 10^{-4} \text{ m/s}\end{aligned}$$

[17-15] (a) Using Ohm's law, we have $R = \frac{V}{I} = \frac{12 \text{ V}}{0.40 \text{ A}} = 30 \Omega$

(b) $\rho \frac{L}{A} = R$ so $\rho = R \frac{A}{L} = (30 \Omega) \left(\frac{\pi (0.40 \times 10^{-2} \text{ m})^2}{3.2 \text{ m}} \right) = 4.7 \times 10^{-4} \Omega \cdot \text{m}$

[17-19] First we must find the resistivity, since that won't change $\rho =$

$R\pi r_0^2/L_0$. Now we want the new resistance

$$\begin{aligned} R' &= \rho \frac{L}{\pi r^2} \\ &= R \frac{\pi r_0^2}{L_0} \frac{L}{\pi r^2} \\ &= R \frac{r_0^2}{L_0} \frac{L}{r^2} \\ &= R \frac{L}{L_0} \left(\frac{r_0}{r}\right)^2 \end{aligned}$$

The problem told us $r = \beta r_0$ where $\beta = 0.25$, but what's L_0 ? Well, the volume of the material doesn't change so

$$\begin{aligned} V &= \pi r_0^2 L_0 \\ &= \pi r^2 L \\ &= \pi \beta^2 r_0^2 L \\ \pi r_0^2 L_0 &= \pi \beta^2 r_0^2 L \\ \frac{L_0}{\beta^2} &= L \end{aligned}$$

So

$$\begin{aligned} R' &= R \frac{L}{L_0} \left(\frac{r_0}{r}\right)^2 \\ &= R \frac{L_0/\beta^2}{L_0} \left(\frac{r_0}{\beta r_0}\right)^2 \\ &= \frac{R}{\beta^4} \\ &= \frac{R}{(0.25)^4} \\ &= 256R \\ &= 256 \Omega \end{aligned}$$

[17-21] To get the new current, we'll need to know the Temperature Coefficient of Resistivity for copper, which can be found in Table 17.1 on page 576. It states that $\alpha_{\text{Cu}} = 3.9 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$.

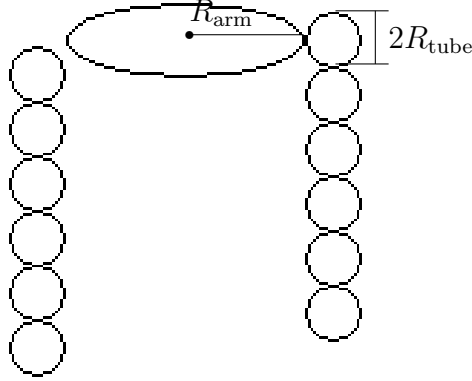
Now, while in Death Valley, Bill Hiker notes that Ohm's law states that $V = I_{DV}R_{DV}$, where the subscript DV indicates that the measurement is made in Death Valley. Of course in Antarctica we have the same voltage, so $V = I_{Ant}R_{Ant} = I_{DV}R_{DV}$. We also know the temperatures: $T_{DV} = 58.0\text{ }^{\circ}\text{C}$ and $T_{Ant} = -88.0\text{ }^{\circ}\text{C}$, but more importantly we can get the temperature difference,

$$\begin{aligned}\Delta T &= T_{Ant} - T_{DV} \\ &= -88.0\text{ }^{\circ}\text{C} - 58.0\text{ }^{\circ}\text{C} \\ &= -143.0\text{ }^{\circ}\text{C}\end{aligned}$$

Since we know how the Resistance changes, $R_{Ant} = R_{DV}(1 + \alpha_{Cu}\Delta T)$, we can solve for the new current:

$$\begin{aligned}I_{DV}R_{DV} &= I_{Ant}R_{Ant} \\ &= I_{Ant}R_{DV}(1 + \alpha_{Cu}\Delta T) \\ I_{DV} &= I_{Ant}(1 + \alpha_{Cu}\Delta T) \\ I_{Ant} &= \frac{I_{DV}}{1 + \alpha_{Cu}\Delta T} \\ &= \frac{1.000\text{ A}}{1 + (3.9 \times 10^{-3}\text{ }^{\circ}\text{C}^{-1})(-143.0\text{ }^{\circ}\text{C})} \\ &= 2.26\text{ A}\end{aligned}$$

[17-29] The basic idea is illustrated below:



If there are N wrappings, then we must have that $L_0 = 2N\pi R_{\text{arm},0}$, and so when R_{arm} changes, we necessarily have L change. The problem gives us both L and L_0 ; $L_0 = 100.00 \times 10^{-2}$ m and $L = 100.04 \times 10^{-2}$ m as well as $R_{\text{tube}} = 0.500 \times 10^{-3}$ m. We can use the solution to problem [17-19] to get the change in resistance of mercury, with one small change: we showed that $L = \frac{L_0}{\beta^2}$ which tells us that $\beta^2 = \frac{L_0}{L}$ and this is the quantity we were given.

(a) but before we do that, let's find the resistance of mercury:

$$\begin{aligned}
 R &= \rho_{\text{Hg}} \frac{L_0}{A_0} \\
 &= \rho_{\text{Hg}} \frac{L_0}{\pi R_{\text{tube},0}^2} \\
 &= (9.4 \times 10^{-7} \text{ } \Omega \cdot \text{m}) \frac{100.00 \times 10^{-2} \text{ m}}{\pi (0.500 \times 10^{-3} \text{ m})^2} \\
 &= 1.20 \text{ } \Omega
 \end{aligned}$$

(b) Okay, let's go for the fractional change in resistance. We know from [17-19] that $R = R_0/\beta^4 = R_0/(\beta^2)^2$. The fractional change in resistance is

defined as

$$\begin{aligned} f &= \frac{R - R_0}{R_0} \\ &= \frac{R_0/(\beta^2)^2 - R_0}{R_0} \\ &= \frac{1}{(\beta^2)^2} - 1 \\ &= \frac{1 - (\beta^2)^2}{(\beta^2)^2} \\ &= \frac{1 - \left(\frac{L_0}{L}\right)^2}{\left(\frac{L_0}{L}\right)^2} \\ &= \frac{1 - \left(\frac{100.00 \text{ cm}}{100.04 \text{ cm}}\right)^2}{\left(\frac{100.00 \text{ cm}}{100.04 \text{ cm}}\right)^2} \\ &= 8.00 \times 10^{-4} \end{aligned}$$

[17-33] Since a 15 A breaker will trip when the current exceeds its rating, the maximum number of bulbs allowed will be when the current is at exactly 15 A. If there are N bulbs, then the power consumed by each is $P_{\text{tot}} = NP_{\text{bulb}}$ and this power is the power in the circuit, so $P_{\text{tot}} = IV$. Thus

$$\begin{aligned} P_{\text{tot}} &= IV \\ NP_{\text{bulb}} &= IV \\ N &= \frac{IV}{P_{\text{bulb}}} \\ &= \frac{(15 \text{ A})(120 \text{ V})}{100 \text{ W}} \\ &= 18 \end{aligned}$$

So 18 bulbs is it; anymore and the breaker trips.

[17-39] If the power loss per meter is $\gamma = 2.00 \text{ W/m}$ then a length L would

radiate a total power of $P = \gamma L$. Since the wire is just a resistor, this must be equal to the current squared times the resistance, or $P = \gamma L = I^2 R$.

Now copper's resistance is given by its resistivity as $R = \rho_{\text{Cu}} \frac{L}{A}$. Thus our length of cable has $\gamma L = I^2 \rho_{\text{Cu}} \frac{L}{A}$. Of course A is just a circular cross-section, so $A = \pi r^2$ with r the radius of the cable—the quantity we desire. Now some algebra

$$\begin{aligned}
 \gamma L &= I^2 \rho_{\text{Cu}} \frac{L}{A} \\
 &= I^2 \rho_{\text{Cu}} \frac{L}{\pi r^2} \\
 \gamma &= \frac{I^2 \rho_{\text{Cu}}}{\pi r^2} \\
 r^2 &= \frac{I^2 \rho_{\text{Cu}}}{\pi \gamma} \\
 r &= \sqrt{\frac{I^2 \rho_{\text{Cu}}}{\pi \gamma}} \\
 &= \sqrt{\frac{(300 \text{ A})^2 (1.7 \times 10^{-8} \Omega \cdot \text{m})}{\pi (2.00 \text{ W/m})}} \\
 &= 1.56 \times 10^{-2} \text{ m}
 \end{aligned}$$

[17-41] Let the number of clocks be $N = 270 \times 10^6$, and the power consumed by each clock be given as $P_{\text{clock}} = 2.50 \text{ W}$. Then total energy consumed by all the clocks in a time τ is $E_{\text{tot}} = NP_{\text{clock}}\tau$.

This energy has to come from somewhere, and we assume it all comes from coal combustion. If the efficiency is $\eta = 0.25$, then it is necessary that $E_{\text{tot}} = \eta E_{\text{coal}}$.

Now the energy from coal combustion will be determined by L_{comb} , the heat of combustion for coal. We are given this value, $L_{\text{comb}} = 33.0 \times 10^6 \text{ J/kg}$. So, if m_{coal} kilograms of coal are used to produce the energy, $E_{\text{coal}} = m_{\text{coal}} L_{\text{comb}}$.

Putting this all together yields

$$\begin{aligned}E_{\text{tot}} &= NP_{\text{clock}}\tau \\ &= \eta E_{\text{coal}} \\ &= \eta m_{\text{coal}} L_{\text{comb}} \\ NP_{\text{clock}}\tau &= \eta m_{\text{coal}} L_{\text{comb}} \\ m_{\text{coal}} &= \frac{NP_{\text{clock}}\tau}{\eta L_{\text{comb}}}\end{aligned}$$

we were asked to find the amount of coal burned in an hour meaning $\tau = 1 \text{ h} = 3600 \text{ s}$ and then

$$\begin{aligned}m_{\text{coal}} &= \frac{(270 \times 10^6) (2.50 \text{ W}) (3600 \text{ s})}{0.25 (33.0 \times 10^6 \text{ J/kg})} \\ &= 2.95 \times 10^5 \text{ kg} \\ &= 295 \text{ metric tons}\end{aligned}$$