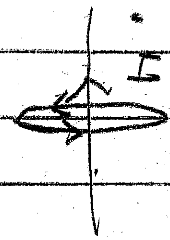
FORMULAE FOR WEEK 9

$$\sum_c \underset{\to}{E} \cdot \underset{\to}{\Delta A} = \frac{1}{\varepsilon_0} \sum Q_i$$

$$\sum_c \underset{\to}{B} \cdot \underset{\to}{\Delta l} = \mu_0 \sum I_i$$

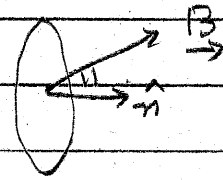
$$\underset{\to}{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$



$$\sum_c \underset{\to}{E_{NC}} \cdot \underset{\to}{\Delta l} = - \frac{\Delta \phi_B}{\Delta t} \quad , \quad \sum_c \underset{\to}{E_{NC}} \cdot \underset{\to}{\Delta A} = 0$$

$$\underset{\to}{F_B} = q \left[\underset{\to}{V} \times \underset{\to}{B} \right]$$

$$\phi_B = \underset{\to}{B} \cdot \underset{\to}{\Delta A} = B \Delta A \cos(\hat{n}, \underset{\to}{B})$$



$$P_W = I^2 R$$

Solutions Week 9.

9.1. (i) If you remember Gauss law, we have

$$\sum_C \vec{E}.\Delta\vec{A} = \frac{1}{\varepsilon_0}\Sigma Q_i$$

where the summation is over a closed surface.

In Analogy, if you write the same equation for the magnetic field, you will get

$$\sum_C \vec{B}.\Delta\vec{A} = \mu_0 \Sigma m_i$$

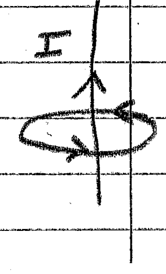
where, m_i are the magnetic charges or monopoles.

However, since $\sum_C \vec{B}.\Delta\vec{A} = 0$

$\Rightarrow \quad \Sigma m_i = 0$ always

This simply means there are no independent magnetic charges, and thus the elementary generators of $\vec{B}$ field are magnetic dipoles.

(ii) Since the $\vec{B}$ field lines are produced by magnetic dipoles, they always close on themselves, and thus there is no beginning or end.

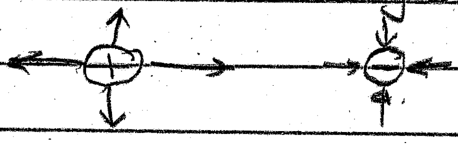


As regards $\vec{B}$ fields produced by currents, they also circulate around the currents.

9-2 A Coulomb $\underset{\to}{E}$-field is produced by a static charge

$$\underset{\to}{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

so it starts at +ive charges and ends at -ive charges.



yielding $\sum_c \underset{\to}{E}\cdot\Delta A = \frac{1}{\varepsilon_0}\sum Q_i$ (Gauss's Law)

A non-coulomb $\underset{\to}{E}$ field appears in every loop surrounding a region where the flux of $\underset{\to}{B}$ is varying as a function of time. Hence, $\underset{\to}{E}_{NC}$ lines close on themselves and it is the circulation of E_{NC} around a closed loop that is given by $\frac{\Delta\phi_B}{\Delta t}$.

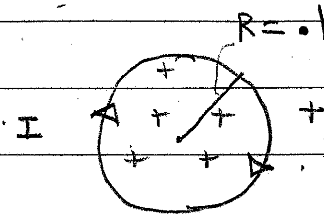
That is, $\sum_c \underset{\to}{E_{NC}}\cdot\underset{\to}{\Delta l} = -\frac{\Delta\phi_B}{\Delta t}$ (Faraday-Lenz).

The minus sign on the right hand side tells us that the sense of E_{NC} must be such as to OPPOSE the change in ϕ_B that gives rise to the $\underset{\to}{E}_{NC}$.

9-3 Since the E_{NC} lines close on themselves so the total flux of E_{NC} through any closed surface must be zero!

EMF

9-4) $\mathcal{E} = -\dfrac{\Delta \Phi_B}{\Delta t}$



$I = 10A$

$R = 1\Omega$

Using Ohm's law, $\mathcal{E} = \Delta V = IR = 10A \times 1\Omega = 10V$

$\Phi_B = \vec{B} \cdot \Delta \vec{A} = \vec{B} \cdot \pi R^2 \hat{z}$ { ∵ the conducting ring lies on a plane $\Delta \vec{A} = \Delta A \hat{z}$

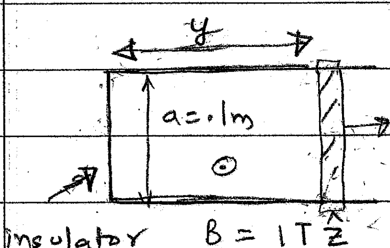
$= -B\pi R^2 \hat{z} \cdot \hat{z}$

$= -B\pi R^2$

$\mathcal{E} = \dfrac{-\Delta \Phi}{\Delta t} = -\dfrac{\Delta B}{\Delta t} \pi R^2$ { Area is a constant }

$\Rightarrow 10 = -\dfrac{\Delta B}{\Delta t} \times \pi \times (0.1)^2 \Rightarrow \dfrac{\Delta B}{\Delta t} = -318.5 \ T/s$

9.5)



$\vec{V} = 5 m/s \hat{y}$. In this case we don't have a closed circuit, so in steady state there won't be any current passing through the copper rod.

$\vec{B} = 1T\hat{z}$

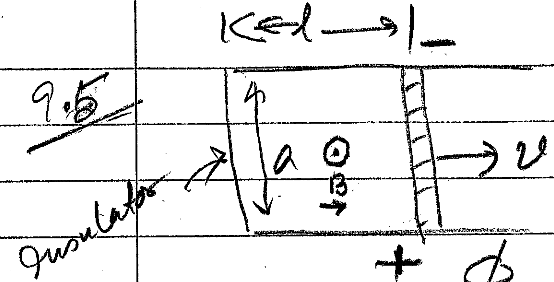
$\Phi = \Sigma \vec{B} \cdot \Delta \vec{A} = \vec{B} \cdot \vec{A}$ { Loop is in a plane $\Delta \vec{A} = \Delta A \hat{z}$

$= 1 \hat{z} \cdot ay\hat{z} \ Tm^2$ $\quad \vec{A} = ay\hat{z}$

$= ay$

$\mathcal{E} = -\dfrac{\Delta \Phi}{\Delta t} = -a\dfrac{\Delta y}{\Delta t} = -aV = -0.1 \times 5 = -0.5 V$

9.5



$\vec{B} = 1\,T\,\hat{z}$

$v = 5\,m/s$

$a = 0.1\,m$

$$\phi = BA = Bal$$

$$\therefore \frac{\Delta\phi}{\Delta t} = Ba\frac{\Delta l}{\Delta t} = Bav$$

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} = -Bav$$

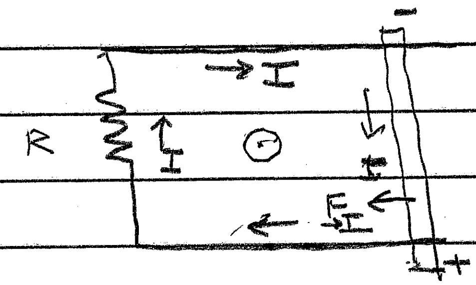
$$\therefore \mathcal{E} = -1 \times 0.1 \times 5 \qquad \left[\begin{matrix} + \text{ at bottom} \\ - \text{ at top} \end{matrix}\right]$$

$$= -0.5\ V$$

This will produce a "clockwise" *
current, hence the lower end is +ve
Because of the insulator, no actual current flows through the circuit, and hence you do not need to apply a force to move the rod [also see 9.6 for more discussion]
at a constant velocity

* As the bar moves to the right flux of $\vec{B}$ out of page is increasing. $\vec{E}_{NC}$ must counter this by producing a $\vec{B}$ field into the page. That requires a clockwise "current" [although there is no actual current]. The moving bar plays the role of a battery so current inside it must flow from − to + as indicated on the picture above

9-6 Now there is a current in the bar



$R = 2\Omega$

so it must experience the force

$$\underline{F}_I = I[\underline{\Delta l} \times \underline{B}] = -IlB\hat{x}$$

Hence to move at constant $\underline{V}$, you must apply a force

$$\underline{F}_{app} = +IlB\hat{x}$$

$$I = \frac{VlB}{R} \qquad = +\frac{Vl^2B^2}{R}\hat{x}$$

The current is

$$I = \frac{\varepsilon}{R} = \frac{0.5}{2} = 0.25A$$

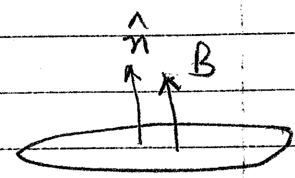
Incidentally $\underline{F}_{app}$ does $\underline{F}_{app} \cdot \underline{V}$ amount of work per second

Input Power $\quad P_W = \frac{V^2l^2B^2}{R}$.

and this is exactly equal to the Power absorbed by the resistor

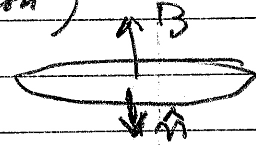
$$P_W = I^2R = \frac{V^2B^2l^2}{R^2} \cdot R$$

9.7. Initial magnetic flux



$$\phi_i = \pi r^2 B$$

Final flux (after rotation)



$$\phi_f = -\pi r^2 B$$

$\phi = \underset{\to}{B} \cdot \underset{\to}{A}$
$= BA \cos(\hat{n}, \underset{\to}{B})$

$$\therefore \frac{\Delta \phi}{\Delta t} = \frac{\phi_f - \phi_i}{\Delta t} = \frac{-2\pi r^2 B}{\Delta t}$$

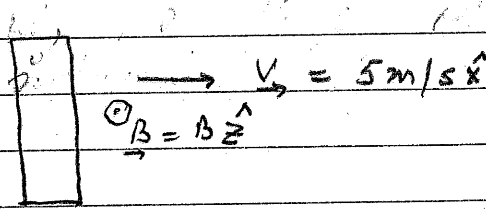
But, $\mathcal{E} = -\dfrac{\Delta \phi}{\Delta t}$

$$\therefore IR = \frac{2\pi r^2 B}{\Delta t}$$

or, $\dfrac{\Delta Q}{\Delta t} R = \dfrac{2\pi r^2 B}{\Delta t}$

$$\therefore \Delta Q = 2\pi r^2 B / R.$$

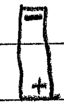
9.8. This is a case of motional EMF.



Electrons inside rod feel force

$$\underset{\to}{F_B} = q[\underset{\to}{v} \times \underset{\to}{B}] = +evB\hat{y}$$

They move up creating + charge at bottom − on top.



This generates an $\underset{\to}{E}$ field which causes electrons to feel force $\underset{\to}{F_E} = -eE\hat{y}$

9-8 Contd. Equilibrium will prevail when

$$\vec{F}_E + \vec{F}_B = 0.$$

That is

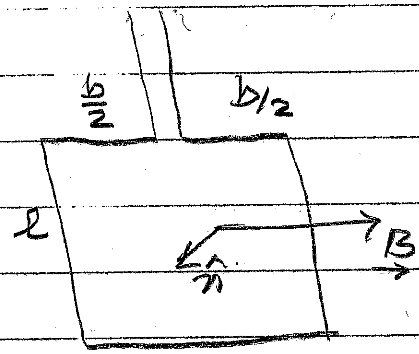
$$evB - eE = 0$$
$$E = vB.$$

which sets up an EMF

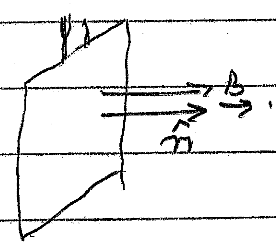
$$\mathcal{E} = vBl$$

exactly as in problem 9-5.

9-9 In order to make a generator we must rotate the coil at angular velocity ω about the y-axis. If so, the flux of B through the coil will vary with time and an $\vec{E}_{NC}$ will appear in the coil generating the emf $= \sum \vec{E}_{NC} \cdot \Delta \vec{l}$



Let us begin at $t=0$ with $\hat{n} \parallel \vec{B}$. As coil rotates the angle between $\hat{n}$ and $\vec{B}$ will vary as

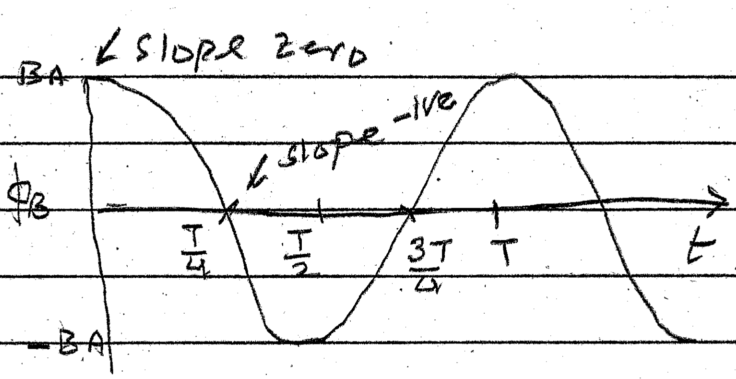


$$\theta = \omega t \qquad [\theta = \theta_0 + \omega t, \ \theta_0 = 0]$$

The flux of $\vec{B}$ is

$$\phi_B = \vec{B} \cdot \Delta \vec{A} = BA \cos(\hat{n}, \vec{B})$$
$$= BA \cos \omega t.$$

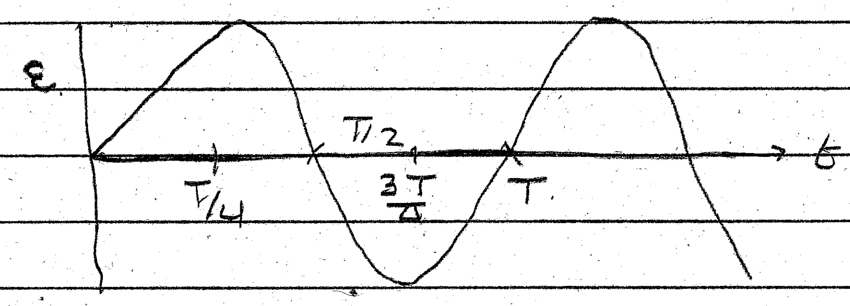
Plot ϕ_B as a function of t we get



$\left(T = \frac{2\pi}{\omega}\right)$

Now $\mathcal{E} = \Sigma_c \underline{E}_{NC} \cdot \Delta \vec{l} = -\frac{\Delta \phi_B}{\Delta t}$

That is the -ive slope of the ϕ_B vs time curve.

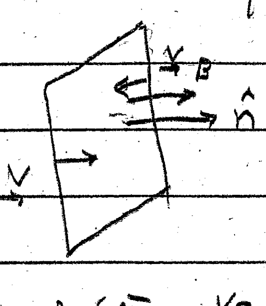


9-10 FROM The two diagrams of Prob 9-9. You can see that $\mathcal{E}$ is zero when ϕ_B is maximum and maximum when ϕ_B is zero.

One can ~~get~~ a better answer by recalling the result of Prob 9-8.

At the INSTANT WHEN ϕ_B is maximum

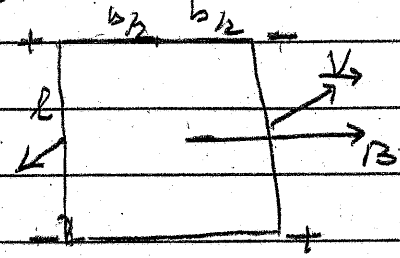
$\hat{n} \parallel \underline{B}$



$\vec{V} \parallel \vec{B}$

$\vec{F}_B = 0$

The velocities of the vertical arms are parallel to $\underline{B}$ SO NO MOTIONAL EMF when the flux is zero



$\underline{V} = R\omega \hat{\tau}$

$\therefore \underline{V} = \pm \frac{b}{2} \omega \hat{z}$

$\underline{V} \perp \underline{B}$ $\therefore [\underline{V} \times \underline{B}]$ is maximum

Emf is maximum $2vBl = 2 \frac{b}{2} \omega B l$

$= \omega B A$

A = area of coil.

At other positions $\underline{V}$ makes an angle θ with $\underline{B}$ so emf becomes $= \omega BA \sin\theta = \omega BA \sin \omega t$

as shown by the second graph in Prob. 9-9.

9-11 The definition of Inductance is

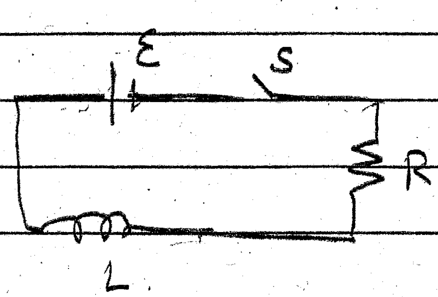
$$L = \frac{-\mathcal{E}}{\frac{\Delta i}{\Delta t}}$$

$\mathcal{E}$ is V, $\frac{\Delta i}{\Delta t}$ is $\frac{I}{T}$ so $\frac{VT}{I}$

$$R = \frac{V}{I}$$

$$\frac{L}{R} = \frac{\frac{VT}{I}}{\frac{V}{I}} \rightarrow T$$

9-12 In the L-R circuit when switch is closed the



battery wants to establish a current through R. L opposes this.

L controls the rate of change of current, so the larger L is, the longer it will take for current to be established.

$L = -\frac{\mathcal{E}}{\frac{\Delta i}{\Delta t}}$

R controls the eventual current $\left(\frac{\mathcal{E}}{R}\right)$, so larger R is, the smaller the current required to complete the process and the quicker it will happen. That is why the characteristic time is $\frac{L}{R}$.

$i_{max} = \frac{\mathcal{E}}{R}$

9-13 Once the Inductor has a current in it, there is a B-field in it and the work done by the battery is stored in the B-field. [See Next problem] [compare with the case of the capacitor, there U_E went into the E-field (problem 6-2)

9.14. We have $U_B = \frac{1}{2} L I^2$

For a solenoid of n turns per unit length and total length l,

Self inductance $L = \mu_0 n^2 A l$

And, $B = \mu_0 n I$

then, $U_B = \frac{1}{2} L I^2$

$= \frac{1}{2} \cdot \mu_0 n^2 A l \cdot I^2$

$= \frac{1}{2\mu_0} (\mu_0 n I)^2 \cdot A l$

$= \frac{1}{2\mu_0} B^2 \cdot V.$ (volume $V = Al$)

$\therefore$ Energy stored per unit volume in B is $u_B = \frac{B^2}{2\mu_0}$.

9.15 (i) time constant $\tau = L/R$

$\therefore \tau = \frac{10^{-3}}{10} = 10^{-4}$ s.

(ii) $I = I_0 (1 - e^{-t/\tau})$.

But, $I = 0.9 I_0$

$\therefore 0.9 = 1 - e^{-t/\tau}$

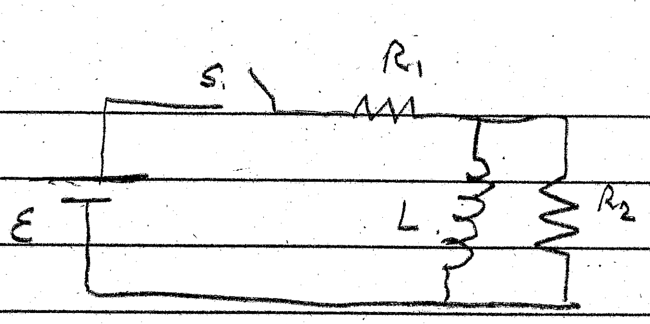
or, $e^{-t/\tau} = 0.1$

$\therefore -t/\tau = -2.3$

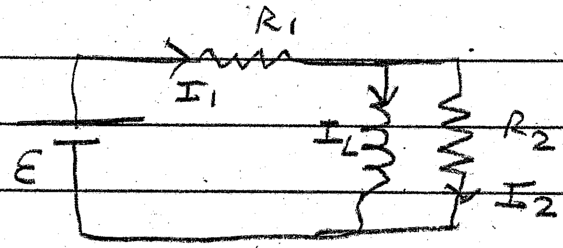
$\therefore t = 2.3\tau$

$= 2.3 \times 10^{-4}$ s. Ans.

9-16



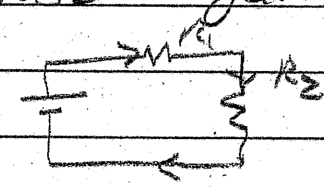
When you close switch current begins to flow



and the junction rule is

$$I_L + I_2 = I_1 \quad \rightarrow (1)$$

at $t = 0^+$ there is no current in the inductor, the battery has just begun to do its thing so $I_L = 0$



$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2}$$

A long time later current through L is constant so there is NO potential

$V_L = 0$

$V_{R_2} = 0$

drop across L, R_2 is parallel to L so $V_{R_2} = 0$, $I_2 = 0$

$$I_1 = I_L = \frac{\mathcal{E}}{R_1}$$