FORMULAE FOR WEEK 8 Problems

$\vec{F}_E = q\vec{E}$ $\qquad$ $\vec{F}_B = q[\vec{v} \times \vec{B}]$

$\vec{F}_I = I[\Delta\vec{\ell} \times \vec{B}]$ $\qquad$ $R = \frac{MV}{qB}$ $\qquad$ $\omega = \frac{qB}{M}$

$\vec{\mu} = I A\hat{n}$ $\qquad$ $\vec{\tau} = \vec{\mu} \times \vec{B}$

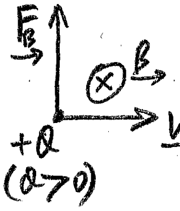
$\Delta\vec{B} = \frac{\mu_0}{4\pi} I \frac{[\Delta\vec{\ell} \times \vec{r}]}{r^3}$ $\qquad$ $\boxed{\sum_C \vec{B} \cdot \Delta\vec{L} = \mu_0 \sum I}$

$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ $\qquad$ $\boxed{\sum_C \vec{B} \cdot \Delta\vec{A} = 0}$

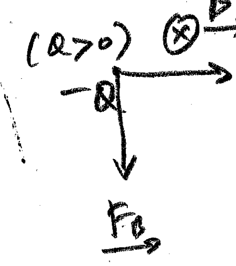
$U_B = -[\vec{\mu} \cdot \vec{B}]$

SOLUTION of HOMEWORK 8

(8-1)



If a positive charge moves in the area of magnetic field shown here, then the magnetic force will point upward (use right hand rule) $\vec{B} = -B\hat{z}$



If a negative charge moves in the area of magnetic field shown here, then the magnetic force will point downward. ($\vec{B} = -B\hat{z}$)

And if the particle is neutral, then it won't feel the magnetic interaction.

Therefore, particle ① mentioned in the problem has positive charge, particle ② is neutral, and particle ③ has negative charge.

(8-2) The current flows along the z-direction and $\vec{B}$ points to the $(-z)$-direction, so the cross product between them is zero. Therefore, the current in the conductor will experience no magnetic force.

(8-3) For the cyclotron, we have the relation $\omega = 2\pi f = \frac{qB}{M}$, or

$$B = \frac{2\pi f M}{q}$$

$$= \frac{2\pi \times 12\times 10^{6} \times 3.34\times 10^{-27}}{1.6\times 10^{-19}}\ \mathrm{T}$$

$$= 1.58\ \mathrm{T}$$

(8-4) (i) If $\vec{B} \parallel \hat{z}$, then the xy-plane is the plane of the path.

(ii) Angular velocity ω won't change because $\omega = \frac{V}{r} = \frac{qB}{M}$ doesn't depend on V.

(8-5) Since it is a positive charge, the electric field will apply an electric force upward to the charge. Hence the magnetic force should be downward for the proton to be undeflected.

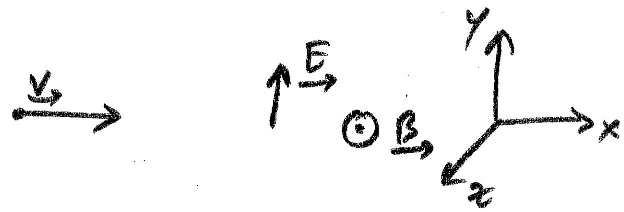
$$\underset{\rightarrow}{F_E} + \underset{\rightarrow}{F_B} = 0$$

$$eE\hat{y} + evB(-\hat{y}) = 0$$

$$v = \frac{E}{B}$$

$$B = \frac{E}{v} = \frac{100}{10^5}\ \mathrm{T} = 10^{-3}\ \mathrm{T} = 1\ \mathrm{mT}.$$

The direction of $\underset{\rightarrow}{B}$ is shown in the figure below, i.e. $\underset{\rightarrow}{B} = B\hat{z}$.



(8-6) The magnitude of magnetic moment is given by

$$|\underset{\rightarrow}{\mu}| = nIA$$

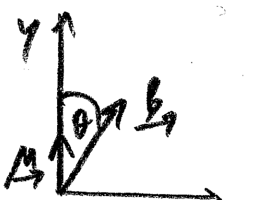
$$= 5 \times 10 \times \pi\left(\frac{50}{100}\right)^2\ \mathrm{A m^2}$$

$$= 39.3\ \mathrm{A \cdot m^2}$$

Since the current is counterclockwise seen from positive y-direction, we conclude

$$\underset{\rightarrow}{\mu} = (39.3\ \mathrm{A \cdot m^2})\ \hat{y}$$

(8-7) Suppose the $\underset{\rightarrow}{B}$-field is in the x-y plane, forms an angle 30° with the y-axis. The torque on the coil is



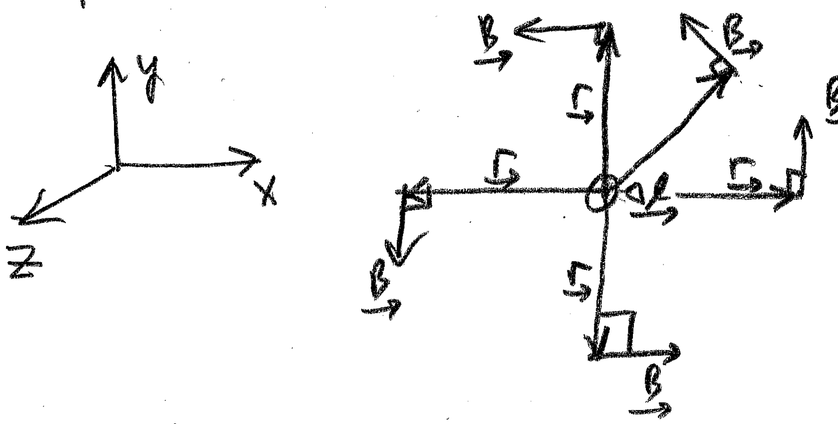
$$\underset{\rightarrow}{\tau} = \underset{\rightarrow}{\mu} \times \underset{\rightarrow}{B}$$

$$= |\underset{\rightarrow}{\mu}| \cdot |\underset{\rightarrow}{B}| \cdot \sin\theta \cdot (-\hat{z})$$

$$= (-1.96\ \mathrm{N \cdot m})\ \hat{z}$$

(8-8) The direction of the magnetic field $\vec{B} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\ell} \times \vec{r}}{r^3}$ is determined solely by the $\Delta\vec{\ell} \times \vec{r}$ part.

Using the Rt. hand Rule we can draw the figure of the field lines around the wire as below.



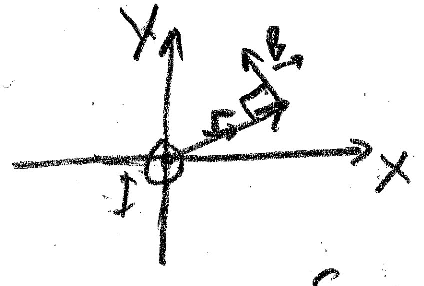
$\Delta\vec{\ell} \parallel$ Thumb

$\vec{r} \parallel$ Fingers

$\Delta\vec{\ell} \times \vec{r} \perp$ Palm

So the $\vec{B}$-field lines circulate around the current.

(8-9)



Using Ampere's law,

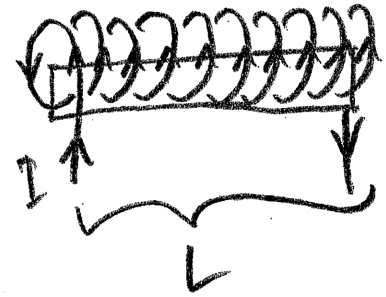
$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Since $\vec{B}$ is on the direction of $\hat{\phi}$, we have

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}.$$ [$\hat{\phi}$ circulates around

(8-10)



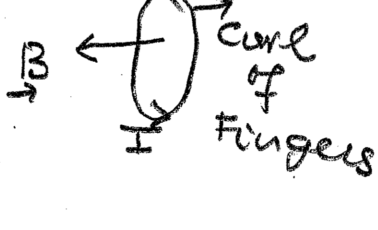
Using Ampere's law for the loop shown, we have

$$B \cdot L = \mu_0 \cdot NI$$

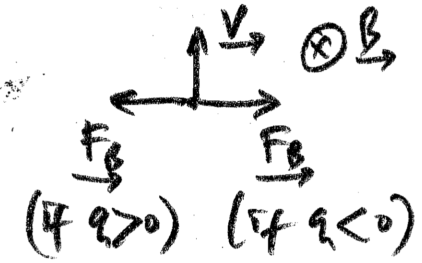
$$B = \mu_0 \frac{N}{L} I = \mu_0 n I$$

Use Rt. hand rule, we see that $\vec{B}$ points to $-\hat{x}$ direction. So,

$$\vec{B} = -\mu_0 n I \hat{x}.$$



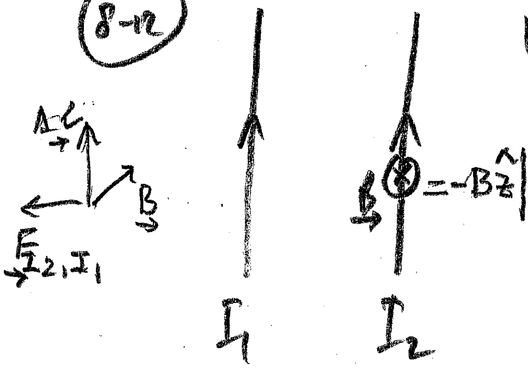
(8-11) If charge is positive, the magnetic force will point to the left, vice versa.



So, from the particle's path, we know that the force should point to the right, implying that the charge is negative.

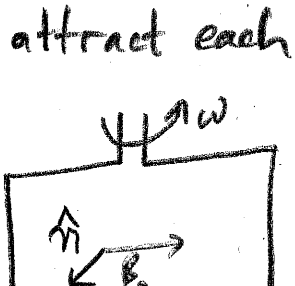
Since $\frac{v}{r} = \frac{|q|B}{m}$, then $r = \frac{mv}{|q|B}$. The larger the charge, the smaller the radius. So, the larger charge will land at P_1.

(8-12)



Let's find out the direction of magnetic force at the wire I_2. Since current I_1 produces $\vec{B}$-field in the place of wire of I_2 as being shown in the figure, the magnetic force that the current I_2 feels will point to the left. Similar argument can be made for I_1, where we conclude that the magnetic force felt by I_1 due to I_2 points to the right. So, these parallel currents I_1 and I_2 attract each other.

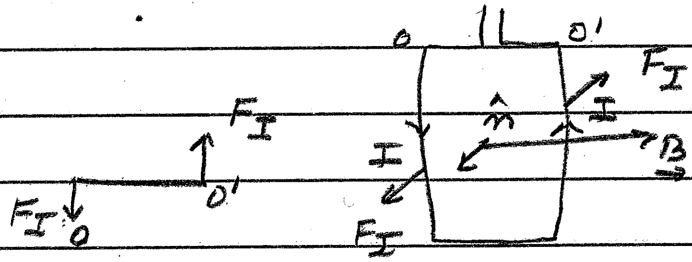
(8-13)



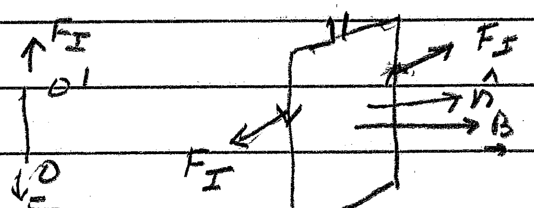
To make this coil rotates with angular velocity as shown, we need to flow the current in the counterclockwise direction. However, after this coil rotates by an angle $\frac{\pi}{2}$, we need to change the direction of the current to maintain the rotation of the coil. Therefore, we have to reverse the current every half cycle, whenever $\hat{n}$ becomes parallel or antiparallel to $\vec{B}$. The device which allows you to do that is called a Commutator.

8-13 When you go back to problems 8-6 and 8-7 you learn that when a coil carries a current it acquires a magnetic moment $\vec{\mu} = IA\hat{n}$ and a magnetic moment experiences a torque in a $\vec{B}$-field.

Here, we start with coil as



Torque is maximum
$\hat{n} \perp \vec{B}$ (IAB)



coil turns
$\hat{n} \parallel \vec{B}$
Torque is zero.

To keep it turning in the same direction reverse I, indeed reverse I every $\frac{1}{2}$ cycle and you have a d.c. motor.

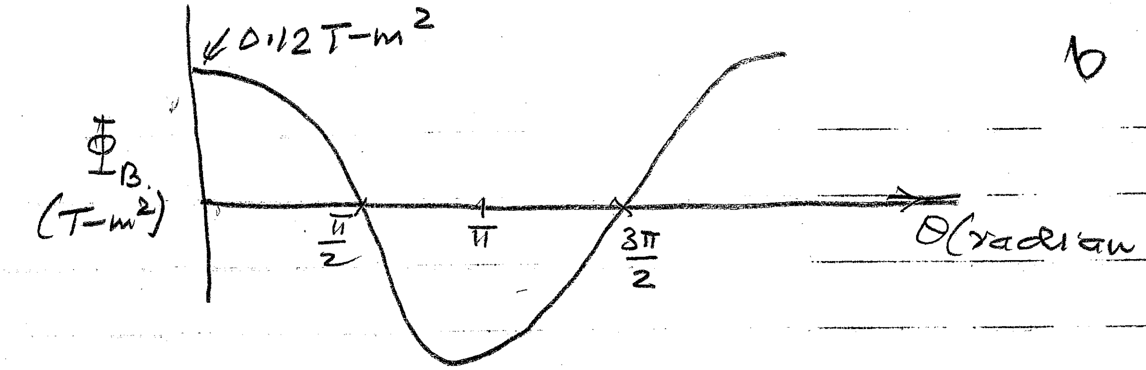
8-14 $\vec{A} = (2 \times 3)\, m^2 \hat{n}$

$\vec{B} = 0.2T\, \hat{z}$

As before flux is defined as

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos(\vec{B}, \hat{n})$$

so Φ_B follows a Cosine function as $\hat{n}$ turns and the angle θ changes with time



b

Max

$\Phi_B = 0.12\ T\text{-}m^2$

Min

$\Phi_B = -0.12\ T\text{-}m^2$

8-15 Because $\underline{B}$ field lines close on themselves, total flux of $\underline{B}$ through any closed surface is always zero

$$\Sigma_c \vec{B} \cdot \Delta \vec{A} = 0$$

So the flux of $\vec{B}$ through the other 3 ~~faces~~ must be $-90\mu T\text{-}m^2$

8-16 "Bar Magnet"

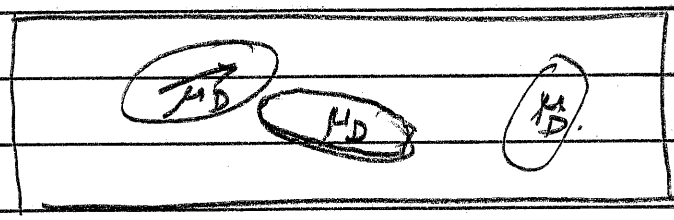
Yes, let us begin with an electron $\mu_B = 9.27 \times 10^{-24}\ A\text{-}m^2$

Put Electrons in an atom (nucleus also has μ but it is much smaller)

Arrange that Els. donot align their μ's like so ↑↓↑↓ because then $\mu_A = 0$.

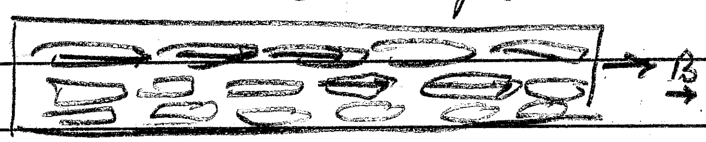
Fe^{3+} is a good ion, Els. are arranged as ↑↑↑↑↑, $\mu_A = \frac{5}{2}\mu_B$.

Put atoms together in a solid. At high temperatures thermal effects make μ_A's wobble so we need to cool solid down. Then in some materials such as Fe, Ni, Co, Quantum mechanics tells us that a new kind of force comes into play which causes the atomic moments to line up forming domains



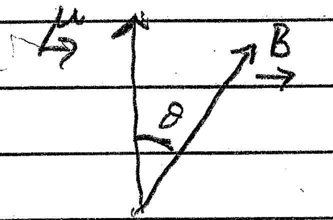
A typical domain may have millions of atoms so μ_D is very large. Such a material is called a Ferromagnet. If we put on a $\underset{\to}{B}$ field, all the μ_D's will line up parallel to $\underset{\to}{B}$.

The last step to make a bar magnet is that the material must have what is called an "Easy" axis, that is the μ_D's like to align along this axis and it would cost a lot of energy to pull μ_D's away from this axis. suppose for our bar Easy axis is along $\hat{x}$. Apply $\underset{\to}{B} \parallel \hat{x}$ – domains align



When $\underset{\to}{B}$ is removed, domains stay put, you have a Bar magnet. A magnet is the most common manifestation of Quantum phenomena!

8-17 In a $\vec{B}$ field the piece of Fe acquires a magnetic moment as all the domains are aligned. The potential energy comes because torque is $\vec{\tau} = \vec{\mu} \times \vec{B}$.



and so $\tau = -\mu B \sin\theta \, \hat{z}$

as before, $\Delta U_E = -\vec{\tau} \cdot \Delta\vec{\theta}$

$\Delta\vec{\theta} = \Delta\theta\,\hat{z}$

and so

$$U_E = -[\vec{\mu} \cdot \vec{B}]$$

Near a magnet $\vec{B}$ is a function of position, it increases as one gets closer, so the piece of Fe can reduce its potential energy (become more negative) by moving toward the magnet.

The bottom line is that a dipole experiences <u>zero</u> FORCE IF $\vec{B}$ is constant.