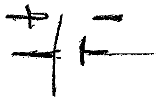
FORMULAE FOR WEEK 6 PROBLEMS

Battery 

output is Emf in VOLTS

Capacitor – parallel plate

Vac. or air $C_0 = \frac{\epsilon_0 A}{d}$

with Conductor of thickness ℓ

$$C_c = \frac{\epsilon_0 A}{d - \ell}$$

with Dielectric $C_k = \frac{k \epsilon_0 A}{d}$

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{QV}{2}$$

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

Current $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$

$$I = n_e e A v_D \qquad I = \vec{J} \cdot \vec{A}$$

$$I = \frac{V}{R}, \qquad R = \frac{\rho \ell}{A} = \frac{\ell}{\sigma A}$$

Resistors in Series – Current is common, potential drops add $R_s = \Sigma R_i$

Resistors in parallel – Voltage is common, currents add $\frac{1}{R_P} = \Sigma \frac{1}{R_i}$

Conductivity

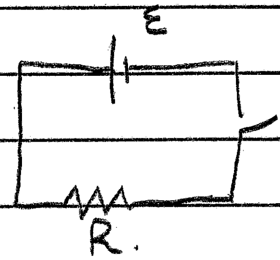
$$\sigma = \frac{n_e e^2 \tau}{m_e}$$

Power: $P_W = IV = I^2R = V^2/R$.

IDEAL BATTERY on R.

when switch is closed

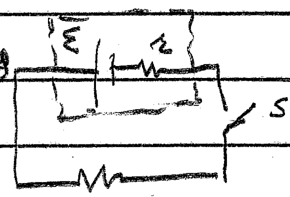
$$I = \frac{\mathcal{E}}{R}$$



Power absorbed in R

$$P_W = I^2 R.$$

REAL BATTERY - switch closed



$$I = \frac{\mathcal{E}}{R+r}$$

$$P_W = \left(\frac{\mathcal{E}}{R+r}\right)^2 R$$

Electrical EQUIVALENT OF HEAT:

4.18 Joules of Electrical Energy will produce same effect as 1 calorie of heat

SOLUTIONS.

6-1. EMF is the work done on a unit charge by the $\vec{E}$-field in a battery. It is measured by the potential difference provided by the battery between its positive and negative terminals. (1)

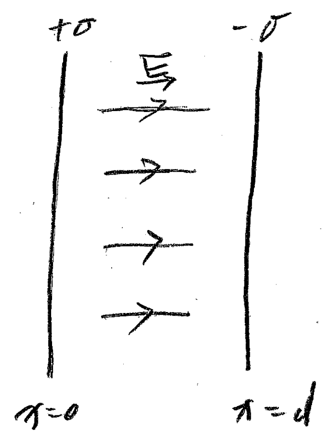
The unit is $\frac{J}{C}$ or Volt

6-2 [EMF IS NOT A FORCE]

6-2 The energy is stored in the electric field in the capacitor. By separating the positive and negative charges, which naturally attract, we increase potential energy of the system. This potential energy, stored in the $\vec{E}$-field, can be used to do work at a later time.

[When the plates have charges, the space between them is NOT Empty. It is filled with an $\vec{E}$-field]

6-3. Consider a parallel plate capacitor, with area A and separation d. The energy desity by definition, is the amount of energy stored per unit volume, i.e., $1\,m^3$.



Or, energy density $= \frac{U_E}{Vol}$, where U_E is the total energy stored in the electric field and vol. is the volume occupied by the E-field. For the parallel plate capacitor, $vol = Ad$

Hence, the energy density $= \frac{U_E}{Ad} = \frac{Q^2}{2C_0 Ad}$ where C_0 is the capacitance, given by $\frac{\varepsilon_0 A}{d}$ for a parallel plate capacitor.

Energy density $= \frac{Q^2}{2\cdot \varepsilon_0 A/d \cdot A \cdot d} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A^2}$ (1)

Since ~~the~~ E-field strength $E = |\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$, we can express (1) as Energy density $= \frac{1}{2}\varepsilon_0 E^2$ as desired

6-4. Current I is defined as $\lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$ or the amount of charge that flows through a cross section per time interval Δt. ②

Hence, the amount of charge that flows through a cross-section, can be expressed as $\Delta Q = I \Delta t$ for a constant current.

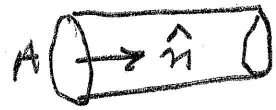
$\therefore \Delta Q = 1A \times 4min = 1C/s \times (4 \times 60s) = 240C$.

Since the charge of an electron is $e = -1.6 \times 10^{-19} C$,

$$\# \text{ of electrons} = \frac{\Delta Q}{|e|} = \frac{240C}{1.6 \times 10^{-19} C} = 1.5 \times 10^{21}.$$

6-5. For a cylindrical conductor/resistor, $I = \vec{J} \cdot \vec{A}$ where $\vec{A}$ is the cross-sectional area. So, $J = \frac{I}{A} = \frac{1A}{\pi (0.001m)^2} = 3.2 \times 10^5 A/m^2$.

$$\vec{J} = 3.2 \times 10^5 A/m^2 \hat{n}$$



6-6. For a cylindrical conductor, made from a material w/ conductivity σ,

$$R = \frac{V}{I} = \frac{E\ell}{JA} = \frac{E\ell}{(\sigma E)A} = \frac{\ell}{\sigma A}$$

$\therefore$ The resistance is directly proportional to length, and inversely proportional to cross-sectional area.

$A_A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi}{4} 1m^2 = 0.79 m^2$

$A_B = \pi r_{out}^2 - \pi r_{in}^2 = \pi (1mm^2 - 0.5mm^2) = \frac{3\pi}{4} \times 10^{-6} m^2 = 2.36 \times 10^{-6} m^2$

$\therefore \frac{R_A}{R_B} = \frac{A_B}{A_A} = 3 \times 10^{-6}$

6-7. Let us assume the wire is cylindrical.
By the equation obtained in 6-6, the resistance of the wire is $R = \frac{l}{\sigma A}$.
Since the volume of the wire is constant, lA = constant.
If the new length of the wire is three times the orginal length, then the new cross-sectional area must be $\frac{1}{3}$ the orginal.
Hence, the new resistance is

$$R' = \frac{l'}{\sigma A'} = \frac{3l}{\sigma (A/3)} = 9\frac{l}{\sigma A} = 9R = 54\Omega.$$

6-8. From 3-6, the number of Cu atoms per volume of $1 m^3$, is $8.4 \times 10^{28}/m^3$. Since each Cu atom contributes one free electron, the electron density $n_e = 8.4 \times 10^{28}/m^3$.
From the equation $I = n_e e A V_D$, we have

$$V_D = \frac{I}{n_e e A} = \frac{1A}{8.4\times10^{28}/m^3 \times 1.6\times10^{-19}C \times \pi(0.5mm)^2}$$

$$= \frac{1A}{8.4\times10^{28} \times 1.6\times10^{-19}C \times (\frac{\pi}{4}\times10^{-6}m^2)}$$

$$= 9.5\times10^{-5} m/s.$$

6-9. If the diameter increases by a factor of 2, the cross-sectional area increases by a factor of 4.
Since $V_D = \frac{I}{n_e e A}$, the drift velocity will decrease by a factor of 4.

6-10. From eq. (1) and $R = \frac{l}{\sigma A}$,

we have $V = \frac{I l}{\sigma A}$. Rearranging the terms, gives

$\frac{I}{A} = \sigma \frac{V}{l}$. Since $J = \frac{I}{A}$ and $E = \frac{V}{l}$, the expression becomes

$J = \sigma E$. Assuming $\vec{J}$ to be in the same direction as $\vec{E}$,

we can write this as $\vec{J} = \sigma \vec{E}$.

6-11. From 3-6, $n = 8.4 \times 10^{28}/m^3$. Since $\sigma = \frac{n e^2 \tau}{m}$, we have

$$\tau = \frac{m\sigma}{ne^2} = \frac{5.9 \times 10^7 (\Omega \cdot m)^{-1} \cdot 9 \times 10^{-31} kg}{(8.4 \times 10^{28}/m^3) \times (1.6 \times 10^{-19} C)^2}$$

$$= 2.5 \times 10^{-14} s.$$

6-12 (i) The resistance of each section of the wire can be found using $R = \frac{\rho l}{A}$ where ρ is the resistivity of the material.

$$R_{Cu} = \frac{1.7\times10^{-8}\,\Omega\cdot m \times 1m}{\frac{\pi}{4}(10^{-3}m)^2} = 0.022\,\Omega.$$

$$R_{Fe} = \frac{10\times10^{-8}\,\Omega\cdot m \times 1m}{\frac{\pi}{4}(10^{-3}m)^2} = 0.127\,\Omega.$$

Since the two sections are connected in series,

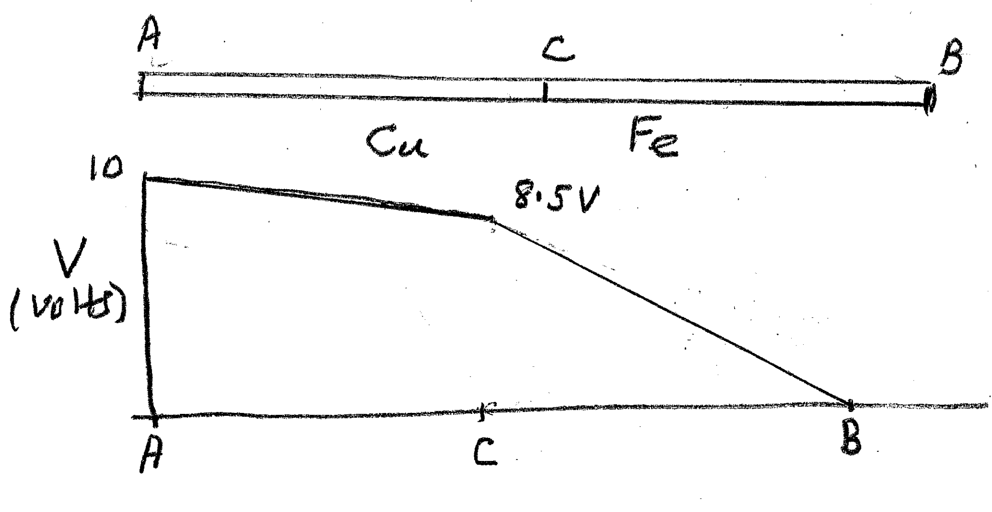
$$R_{total} = R_{Cu} + R_{Fe} = 0.149\,\Omega$$

(ii) $$I = \frac{V}{R_{total}} = \frac{10V}{0.149\Omega} = 67A$$

(iii) The potential at C can be found, by finding the potential difference between C and B. (We assume the potential at B is 0) Since we have found the current through the iron part of the wire, and the resistance R_{Fe}, we can find the potential difference by

$$V = IR_{Fe} = 67A \times 0.127\Omega = 8.5V.$$

Hence, the potential at C is 8.5V.



6-13. The amount of energy dissipated by the wire per unit time, (i.e Power), is given by

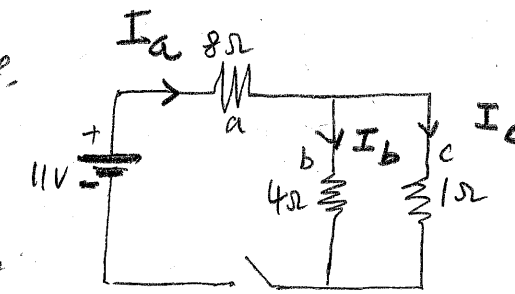
$$P = \underbrace{I^2 R_{total}}_{①} = \underbrace{\frac{V^2}{R_{total}}}_{②}$$ (Both ① and ② will work for our purposes, since both V and I are known)

$$P = \frac{V^2}{R_{total}} = \frac{(10\,V)^2}{0.149\,\Omega} = 671\,W = 671\,J/s.$$

To find the amount of thermal energy generated in 1 hour, we can simply multiply P by # of seconds in an hour, i.e.

$$W = Pt = 671\,W \times (3600\,s) = 2.4 \times 10^6\,J$$

6-14. To find the equivalent resistance, we observe that resistors b, c are connected in parallel, and that this part of the circuit is in series w/ resistor a.



Hence, the equivalent resistance is

$$R_{eq} = R_a + R_{b\parallel c} = R_a + \frac{R_b R_c}{R_b + R_c}$$
$$= 8\,\Omega + \frac{4\Omega \cdot 1\Omega}{(4+1)\Omega}$$
$$= 8\,\Omega + 0.8\,\Omega = 8.8\,\Omega.$$

(i) & (ii) The current through resistor a, is simply the total current

$$I_a = I_{total} = \frac{V}{R_{eq}} = \frac{11V}{8.8\Omega} = 1.25A.$$

The potential drop across resistor a is given by

$$V_a = I_a R_a = 1.25A \times 8\,\Omega = 10\,V.$$

(continued.)

By Kirchhoff's Loop rule, the potential drop across R_A and R_B must satisfy $11V - V_a - V_b = 11V - V_a - V_c = 0$

$\therefore V_b = V_c = 11V - 10V = 1V$.

The current through resistors b and c can be found using

$$I_b = \frac{V_b}{R_b} = \frac{1V}{4\Omega} = 0.25A$$

$$I_c = \frac{V_c}{R_c} = \frac{1V}{1\Omega} = 1A.$$

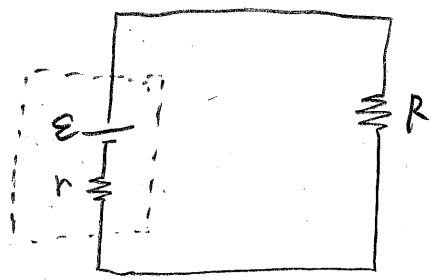
Note that $I_b + I_c = 1.25A = I_a$, satisfying Kirchhoff's junction rule.

6-15. The total resistance in the circuit is given by

$R_{total} = r + R$ since the two resistors are in series.

The total current in the circuit is

$$I = \frac{\varepsilon}{R_{total}} = \frac{\varepsilon}{R+r}.$$



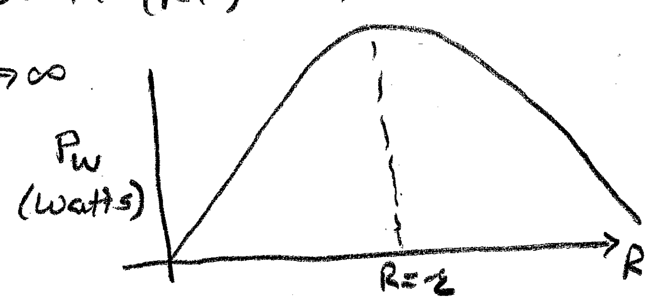
The power dissipated in R is

$$P = I^2 R = \frac{\varepsilon^2 R}{(R+r)^2}$$

i) If $R \ll r$, the term r^2 dominates in the denominator $(R+r)^2$, and $P \approx \frac{\varepsilon^2 R}{r^2}$. For small R, $P \to 0$.

ii) If $R \gg r$, the term R^2 dominates in $(R+r)^2$ and

$$P \approx \frac{\varepsilon^2 R}{R^2} = \frac{\varepsilon^2}{R} \to 0 \quad \text{as } R \to \infty$$



6-16. In 6-12, we found the resistance of each section of the wire to be

$R_{Cu} = 0.022\Omega$ and

$R_{Fe} = 0.127\Omega$.

Since the two sections are now in parallel, the resistance between A and B is given by

$$R_{AB} = \frac{R_{Cu} \cdot R_{Fe}}{R_{Cu} + R_{Fe}} = 0.019\Omega.$$